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RESEARCH ARTICLE

Solving neutrosophic multi-objective linear fractional programming problem using central measures

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Abstract

This article presents different kinds of mean techniques to solve multi-objective linear fractional programming problems (MOLFPP) in a neutrosophic environment. Here in these mean techniques, the MOLFPP is converted into a single objective linear programming problem (SOLPP) and then we obtain the optimal solution by the simplex method in a neutrosophic environment. The proposed method is illustrated with the help of a numerical example.

Keywords: Neutrosophic multi-objective linear fractional programming problem, Neutrosophic triangular number, Harmonic averaging techniques, Advanced harmonic averaging techniques.

Subject Classifications: 90C29, 90C70.

Introduction

Linear fractional programming problem has attracted great interest due to their usefulness in various fields, such as production planning, financial and corporate planning, health care services, and hospital management. Methods for solving the problem were proposed by several researchers (Charanes *et al*. 1962), (Sulaiman *et al*. 2006). These approaches depend on transforming the linear fractional programming problem into an equivalent linear programming problem and solving the multi-objective linear fractional programming problem (MOLFPP) using mean values.

In this paper, we develop a technique to solve neutrosophic (MOLFPP). The algorithm is explained in detail with the help of the numerical example. We use the simplex method to generate the optimal solution. The rest of the paper is structured as follows: In section two, preliminaries are given. In section three, MOLFPP in a neutrosophic environment is given. In section four, different techniques and procedures for determining a combined single objective function are explained in detail. Numerical examples are included in section five. Finally, the conclusion is given in section six.

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Materials and Methods

We recall some necessary definitions and results to make out the main thought.

Definition: Single valued triangular neutrosophic number (SVTNN). A (SVTNN) is defined by $\tilde{A}^* = \{(a_1^l, b_1^m, c_1^u); \tau_a, i_a, \omega_a\}$ whose three membership functions for the truth, indeterminacy, and a falsity of x are given by:

$$\tau_{\tilde{a}^n}(x) = \begin{cases} \dfrac{(x - a_1^l)\tau_a}{b_1^m - a_1^l} & (a_1^l \le x < b_1^m) \\ \tau_a & (x = b_1^m) \\ \dfrac{(c_1^u - x)\tau_a}{c_1^u - b_1^m} & (b_1^m \le x < c_1^u) \\ 0 & \text{Otherwise,} \end{cases}$$

$$i_{\tilde{a}^n}(x) = \begin{cases} \dfrac{(b_1^m - x)i_a}{b_1^m - a_1^l} & (a_1^l \le x < b_1^m) \\ i_a & (x = b_1^m) \\ \dfrac{(x - c_1^u)i_a}{c_1^u - b_1^m} & (b_1^m \le x < c_1^u) \\ 1 & \text{Otherwise,} \end{cases}$$

$$\omega_{\tilde{a}^n}(x) = \begin{cases} \dfrac{(b_1^m - x)\omega_a}{b_1^m - a_1^l} & (a_1^l \le x < b_1^m) \\ \omega_a & (x = b_1^m) \\ \dfrac{(x - c_1^u)\omega_a}{c_1^u - b_1^m} & (b_1^m \le x < c_1^u) \\ 1 & \text{Otherwise,} \end{cases}$$

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where $0 \le \tau_{\tilde{a}^n}(x) + i_{\tilde{a}^n}(x) + \omega_{\tilde{a}^n}(x) \le 3, \quad x \in \tilde{a}^n$. Additionally, when $a_1^l > 0$, $\tilde{a}^n$ is called a positive SVTNN. Similarly, when $a_1^l < 0$, $\tilde{a}^n$ becomes a negative SVTNN.

Definition: Let $\tilde{a}^n = \{(a_1^l, b_1^m, c_1^u); \tau_a, i_a, \omega_a\}$ and $\tilde{b}^n = \{(a_2^l, b_2^m, c_2^u); \tau_b, i_b, \omega_b\}$ be two SVTNN's and $\gamma \neq 0$. Then

- Addition
  $\tilde{a}^n + \tilde{b}^n = \{(a_1^l + a_2^l, b_1^m + b_2^m, c_1^u + c_2^u); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b\}$
- Subtraction
  $\tilde{a}^n - \tilde{b}^n = \{(a_1^l - c_2^u, b_1^m - b_2^m, c_1^u - a_2^l); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b\}$
- Multiplication
  $$\tilde{a}^n . \tilde{b}^n = \begin{Bmatrix} Min(a_1^l a_2^l, a_1^l c_2^u, c_1^u a_2^l, c_1^u c_2^u), b_1^m b_2^m, Max(a_1^l a_2^l, a_1^l c_2^u, c_1^u a_2^l, c_1^u c_2^u); \\ \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b \end{Bmatrix}$$
- Division
  $$\frac{\tilde{a}^n}{\tilde{b}^n} = \left\{ Min\left(\frac{a_1^l}{a_2^l}, \frac{a_1^l}{c_2^u}, \frac{c_1^u}{a_2^l}, \frac{c_1^u}{c_2^u}\right), \frac{b_1^m}{b_2^m}, Max\left(\frac{a_1^l}{a_2^l}, \frac{a_1^l}{c_2^u}, \frac{c_1^u}{a_2^l}, \frac{c_1^u}{c_2^u}\right); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b \right\}$$
- Scalar Multiplication
  $$\gamma \tilde{a}^n = \begin{cases} \{(\gamma a_1^l, \gamma b_1^m, \gamma c_1^u); \tau_a, i_a, \omega_a\}, (\gamma > 0) \\ \{(\gamma c_1^u, \gamma b_1^m, \gamma a_1^l); \tau_a, i_a, \omega_a\}, (\gamma < 0). \end{cases}$$

MOLFPP in Neutrosophic Environment

The ratio objective function that has a numerator and denominator and is defined as follows:

$$\text{Max } \tilde{Z}_k \text{ and Min } \tilde{Z}_k = \frac{\tilde{c}_i^k \tilde{x}_i^T + \tilde{\alpha}_i^k}{\tilde{D}_i^k(\tilde{x}_i^T + \tilde{1})} \quad \forall \begin{cases} Max\ \tilde{Z}_k & if\ k = 1,2,\dots,r \\ Min\ \tilde{Z}_k & if\ k = r+1, r+2,\dots,s \end{cases} \quad (1)$$

subject to $\tilde{A}_{ij}\tilde{x}_i \le \tilde{B}_i$ (2)

$\tilde{x}_i \ge \{(0,0,0); 1,0,0\}$ (3)

where $\tilde{x}_i^T$ and $\tilde{x}_i$ is a n-dimensional vector of decision variables;

r is the number of objective functions that is to be maximized;

$s - r$ is the number of objective functions that is to be minimized;

$\tilde{C}_i$ and $\tilde{D}_i (\forall i = 1,2,\dots,r,r+1,\dots,s)$ are n-dimensional vector of SVTNN;

$\tilde{A}_{ij}$ is a $m \times n$-matrix of co-efficient (SVTNN). ($i = 1,2,\cdots,n$; $j = 1,2,\cdots,m$);

$\tilde{\alpha}_i$ is a neutrosophic constant; $\tilde{1} = \{(1,1,1); 1,0,0\}$

Results

Different Kind of Techniques

Suppose we obtained a single value corresponding to each of the objective functions of it being optimized individually subject to the constraints (2) and (3) as follows:

$$\text{Max } \tilde{Z}_k \text{ and Min } \tilde{Z}_k = \frac{\tilde{c}_i^k \tilde{x}_i^T + \tilde{\alpha}_i^k}{\tilde{D}_i^k(\tilde{x}_i^T + \tilde{1})}$$

$$\forall \begin{cases} \text{Max } \tilde{Z}_k & if \quad k = 1,2,\dots,r \\ \text{Min } \tilde{Z}_k & if \quad k = r+1, r+2,\dots,s \end{cases}$$

where $\tilde{Z}_1^{Max}, \tilde{Z}_2^{Max}, \dots, \tilde{Z}_r^{Max}, \tilde{Z}_{r+1}^{Min}, \dots, \tilde{Z}_s^{Min}$ are the optimal values of the objective functions.

Here we discuss about the various kind of techniques.

$$|\tilde{Z}_k^{Max}| = \tilde{\varphi}_k^{Max}; \quad and \quad |\tilde{Z}_k^{Min}| = \tilde{\varphi}_i^{Min};$$

$$SM = \sum_{k=1}^{r} \frac{\text{Max } \tilde{Z}_k}{\tilde{\varphi}_k^{\text{Max}}} \quad and \quad SN = \sum_{k=r+1}^{s} \frac{\text{Min } \tilde{Z}_k}{\tilde{\varphi}_k^{\text{Min}}}$$

- Arithmetic average technique
  $$\tilde{m}_1 = \min(\tilde{\varphi}_i^{Max}); \quad \tilde{m}_2 = \min(\tilde{\varphi}_i^{Min}); \quad AV_2 = \frac{\tilde{m}_1 + \tilde{m}_2}{2}$$
  $$\text{Max } \tilde{Z} = \frac{SM - SN}{AV}$$
- New arithmetic average technique
  $$\tilde{m}_1 = \min(\tilde{\varphi}_k^{Max}); \quad \tilde{m}_2 = \min(\tilde{\varphi}_k^{Min}); \quad AV_N = \frac{\tilde{m}_1 + \tilde{m}_2}{s};$$
  Where s is the number of objective function.
  $$Max\ \tilde{Z} = \frac{SM - SN}{AV_N}$$
- Harmonic average technique
  $$H_{av_1} = H_{av}(\tilde{\varphi}_k^{Max}) = \sum_{i=1}^{r} \frac{1}{\tilde{\varphi}_k^{Max}}; \quad H_{av_2} = H_{av}(\tilde{\varphi}_k^{Min}) = \sum_{i=r+1}^{s} \frac{1}{\tilde{\varphi}_k^{Min}};$$
  $$S_1 = \frac{SM}{H_{av_1}}; \quad S_2 = \frac{SN}{H_{av_2}} \quad Max\ \tilde{Z} = S_1 - S_2$$
- Advanced harmonic average technique
  $$\tilde{m}_1 = \min(\tilde{\varphi}_k^{Max}); \quad \tilde{m}_2 = \max(\tilde{\varphi}_k^{Min}); \quad AH_{av} = \frac{2\tilde{m}_1\tilde{m}_2}{\tilde{m}_1 + \tilde{m}_2}$$
  $$Max\ \tilde{Z} = \frac{SM - SN}{AH_{av}}$$

Procedure for Determining a Combined Single Objective Function

The Neutrosophic MOLFPP can be determined by following the appropriate procedure. The summary of this process is as follows:

- Find the optimal solution for individual objective functions which is to be maximized or minimized by using a modified simplex algorithm for fractional linear programming problems.
- If the solution obtained in step 1 is feasible, proceed to steps 3; otherwise, use dual simplex techniques to remove infeasibility.
- There are different techniques that can be used to derive the objective function.
  - Calculate neutrosophic SOLFPP equation, using $Max\ \tilde{Z} = (SM - SN)/AV$,
    where $AV = \frac{\tilde{m}_1 + \tilde{m}_2}{2}$ is arithmetic mean.
  - Calculate neutrosophic SOLFPP equation, using $Max\ \tilde{Z} = (SM - SN)/AV_N$,
    where $AV_N = \frac{\tilde{m}_1 + \tilde{m}_2}{s}$ is new arithmetic average. And then s is the number of objective function.
  - Calculate neutrosophic SOLFPP equation, using $Max\ \tilde{Z} = S_1 - S_2$
    where $S_1 = \frac{SM}{H_{av_1}}$ and $S_2 = \frac{SM}{H_{av_2}}$
  - Calculate neutrosophic SOLFPP equation, using $Max\ \tilde{Z} = \frac{SM - SN}{AH_{av}}$
    where $AH_{av} = \frac{2\tilde{m}_1\tilde{m}_2}{\tilde{m}_1 + \tilde{m}_2}$ is advanced harmonic average technique.
- Now find the optimal solution for combined objective functions obtained from four method. Using the same constraints.

- If the solution obtained in step 4 is feasible then stop otherwise use dual simplex technique to remove infeasibility.

Discussion with Numerical Example

Consider MOLFPP in Neutrosophic environment.

$$Max\ \tilde{Z}_1 = \frac{\{(4,5,6);.4,.6,.8\}\tilde{x}_1 + \{(2,3,4);.5,.4,.7\}\tilde{x}_2}{\{(1,2,3);.3,.5,.6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$Max\ \tilde{Z}_2 = \frac{\{(8,9,10);.3,.2,.6\}\tilde{x}_1 + \{(4,5,6);.4,.7,.5\}\tilde{x}_2}{\{(3,6,9);.3,.5,.6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$Max\ \tilde{Z}_3 = \frac{\{(3,4,5);.6,.4,.3\}\tilde{x}_1 - \{(2,3,4);.5,.6,.9\}\tilde{x}_2}{\{(1,2,3);.3,.5,.6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$Max\ \tilde{Z}_4 = \frac{\{(2,3,4);.4,.7,.5\}\tilde{x}_1 + \{(1,2,3);..5,.4,.2\}\tilde{x}_2}{\{(2,4,6);.3,.5,.6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

subject to

$$\{(1,3,7);.4,.6,.7\}\tilde{x}_1 + \{(3,6,9);.5,.7,.3\}\tilde{x}_2 \le \{(14,18,24);.6,.3,.5\}$$

$$\{(8,9,10);.6,.5,.4\}\tilde{x}_1 + \{(1,2,4);.3,.6,.9\}\tilde{x}_2 \le \{(20,27,32);.3,.4,.5\}$$

and $\tilde{x}_1, \tilde{x}_2 \ge \{(0,0,0);1,0,0\}$

Solution: Here, find the optimal solution for individual objective function which is to be maximized or minimized by using modified simplex algorithm for fractional linear programming problems in a neutrosophic environment.

- First objective function

$$Max\ \tilde{Z}_1 = \frac{\{(4,5,6);.4,.6,.8\}\tilde{x}_1 + \{(2,3,4);.5,.4,.7\}\tilde{x}_2}{\{(1,2,3);.3,.5,.6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

subject to

$$\{(1,3,7);.4,.6,.7\}\tilde{x}_1 + \{(3,6,9);.5,.7,.3\}\tilde{x}_2 \le \{(14,18,24);.6,.3,.5\}$$

$$\{(8,9,10);.6,.5,.4\}\tilde{x}_1 + \{(1,2,4);.3,.6,.9\}\tilde{x}_2 \le \{(20,27,32);.3,.4,.5\}$$

and $\tilde{x}_1, \tilde{x}_2 \ge \{(0,0,0);1,0,0\}$

Using modified simplex algorithm for fractional linear programming problem in neutrosophic environment, we solve this problem and then we get the optimal solution is

$$\mathrm{Max}\ \tilde{Z}_1 = \{(0.53, 1.87, 8);.3,.6,.8\}$$

$$\tilde{x}_1 = \{(2,3,4);.3,.5,.5\}$$

$$\tilde{x}_2 = \{(0,0,0);1,0,0\}$$

- Second objective function

$$\mathrm{Max}\ \tilde{Z}_2 = \frac{\{(8,9,10);.3,.2,.6\}\tilde{x}_1 + \{(4,5,6);.4,.7,.5\}\tilde{x}_2}{\{(3,6,9);.3,.5,.6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

subject to

$$\{(1,3,7);.4,.6,.7\}\tilde{x}_1 + \{(3,6,9);.5,.7,.3\}\tilde{x}_2 \le \{(14,18,24);.6,.3,.5\}$$

$$\{(8,9,10);.6,.5,.4\}\tilde{x}_1 + \{(1,2,4);.3,.6,.9\}\tilde{x}_2 \le \{(20,27,32);.3,.4,.5\}$$

and $\tilde{x}_1, \tilde{x}_2 \ge \{(0,0,0);1,0,0\}$

Using modified simplex algorithm for fractional linear programming problem in neutrosophic environment, we solve this problem and then we get the optimal solution is

$$\mathrm{Max}\ \tilde{Z}_2 = \{(0.35, 1.12, 4.44);.3,.5,.6\}$$

$$\tilde{x}_1 = \{(2,3,4);.3,.5,.5\}$$

$$\tilde{x}_2 = \{(0,0,0);1,0,0\}$$

- Third objective function

$$\mathrm{Max}\ \tilde{Z}_3 = \frac{\{(3,4,5);.6,.4,.3\}\tilde{x}_1 - \{(2,3,4);.5,.6,.9\}\tilde{x}_2}{\{(1,2,3);.3,.5,.6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

subject to

$$\{(1,3,7);.4,.6,.7\}\tilde{x}_1 + \{(3,6,9);.5,.7,.3\}\tilde{x}_2 \le \{(14,18,24);.6,.3,.5\}$$

$$\{(8,9,10);.6,.5,.4\}\tilde{x}_1 + \{(1,2,4);.3,.6,.9\}\tilde{x}_2 \le \{(20,27,32);.3,.4,.5\}$$

and $\tilde{x}_1, \tilde{x}_2 \ge \{(0,0,0);1,0,0\}$

Using modified simplex algorithm for fractional linear programming problem in neutrosophic environment, we solve this problem and then we get the optimal solution is

$$\mathrm{Max}\ \tilde{Z}_3 = \{(0.4, 1.5, 6.66);.3,.5,.6\}$$

$$\tilde{x}_1 = \{(2,3,4);.3,.5,.5\}$$

$$\tilde{x}_2 = \{(0,0,0);1,0,0\}$$

- Fourth objective function

$$\mathrm{Max}\ \tilde{Z}_4 = \frac{\{(2,3,4);.4,.7,.5\}\tilde{x}_1 + \{(1,2,3);..5,.4,.2\}\tilde{x}_2}{\{(2,4,6);.3,.5,.6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

subject to

$$\{(1,3,7);.4,.6,.7\}\tilde{x}_1 + \{(3,6,9);.5,.7,.3\}\tilde{x}_2 \le \{(14,18,24);.6,.3,.5\}$$

$$\{(8,9,10);.6,.5,.4\}\tilde{x}_1 + \{(1,2,4);.3,.6,.9\}\tilde{x}_2 \le \{(20,27,32);.3,.4,.5\}$$

and $\tilde{x}_1, \tilde{x}_2 \ge \{(0,0,0);1,0,0\}$

Using modified simplex algorithm for fractional linear programming problem in neutrosophic environment, we solve this problem and then we get the optimal solution is

$$\mathrm{Max}\ \tilde{Z}_4 = \{(0.13, 0.56, 2.66);.3,.7,.6\}$$

$$\tilde{x}_1 = \{(2,3,4);.3,.5,.5\}$$

$$\tilde{x}_2 = \{(0,0,0);1,0,0\}$$

The results of the optimization process are summarized in Table 1.

In order to determine the neutrosophic single LFPP, we now have all of the necessary data.

Let $\tilde{W} = \frac{\tilde{C}_1^1}{\tilde{\varphi}_1^{\mathrm{Max}}} + \frac{\tilde{C}_1^2}{\tilde{\varphi}_2^{\mathrm{Max}}} + \frac{\tilde{C}_1^3}{\tilde{\varphi}_3^{\mathrm{Max}}} + \frac{\tilde{C}_1^4}{\tilde{\varphi}_4^{\mathrm{Max}}}$; $\tilde{X} = \frac{\tilde{C}_2^1}{\tilde{\varphi}_1^{\mathrm{Max}}} + \frac{\tilde{C}_2^2}{\tilde{\varphi}_2^{\mathrm{Max}}} + \frac{\tilde{C}_2^3}{\tilde{\varphi}_3^{\mathrm{Max}}} + \frac{\tilde{C}_2^4}{\tilde{\varphi}_4^{\mathrm{Max}}}$;

$\tilde{\alpha} = \frac{\tilde{\alpha}_1}{\tilde{\varphi}_1^{\mathrm{Max}}} + \frac{\tilde{\alpha}_2}{\tilde{\varphi}_2^{\mathrm{Max}}} + \frac{\tilde{\alpha}_3}{\tilde{\varphi}_3^{\mathrm{Max}}} + \frac{\tilde{\alpha}_4}{\tilde{\varphi}_4^{\mathrm{Max}}}$; $\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3, \tilde{\alpha}_4 = \{(0,0,0);1,0,0\}$

Table 1: Results of the example by using modified simplex technique

k	$\tilde{Z}_k$, $k = 1, \dots r, r+1, \dots, s.$	$(\tilde{x}_1, \tilde{x}_2)$	$\lvert\tilde{Z}_k^{\mathrm{Max}}\rvert = \tilde{\varphi}_k^{\mathrm{Max}}$	$\lvert\tilde{Z}_k^{\mathrm{Min}}\rvert = \tilde{\varphi}_k^{\mathrm{Min}}$
1	$\{(0.53, 1.87, 8);.3,.6,.8\}$	$\begin{pmatrix}\{(2,3,4);.3,.5,.5\},\\ \{(0,0,0);1,0,0\}\end{pmatrix}$	$\{(0.53, 1.87, 8);.3,.6,.8\}$	–
2	$\{(0.35, 1.12, 4.44);.3,.5,.6\}$	$\begin{pmatrix}\{(2,3,4);.3,.5,.5\},\\ \{(0,0,0);1,0,0\}\end{pmatrix}$	$\{(0.35, 1.12, 4.44);.3,.5,.6\}$	–
3	$\{(0.4, 1.5, 6.66);.3,.5,.6\}$	$\begin{pmatrix}\{(2,3,4);.3,.5,.5\},\\ \{(0,0,0);1,0,0\}\end{pmatrix}$	$\{(0.4, 1.5, 6.66);.3,.5,.6\}$	–
4	$\{(0.13, 0.56, 2.66);.3,.7,.6\}$	$\begin{pmatrix}\{(2,3,4);.3,.5,.5\},\\ \{(0,0,0);1,0,0\}\end{pmatrix}$	$\{(0.13, 0.56, 2.66);.3,.7,.6\}$	–

$\tilde{\varphi}_1^{\text{Max}} = \{(0.53, 1.87, 8); .3, .6, .8\}$

$\tilde{D}_1^1 = \{(1, 2, 3); .3, .5, .6\}$

$\tilde{C}_1^1 = \{(4, 5, 6); .4, .6, .8\}$

$\tilde{C}_2^1 = \{(2, 3, 4); .5, .4, .7\}$

$\tilde{\varphi}_2^{\text{Max}} = \{(0.35, 1.12, 4.44); .3, .5, .6\}$

$\tilde{D}_1^2 = \{(3, 6, 9); .3, .5, .6\}$

$\tilde{C}_1^2 = \{(8, 9, 10); .3, .2, .6\}$

$\tilde{C}_2^2 = \{(4, 5, 6); .4, .7, .5\}$

$\tilde{\varphi}_3^{\text{Max}} = \{(0.4, 1.5, 6.66); .3, .5, .6\}$

$\tilde{D}_1^3 = \{(1, 2, 3); .3, .5, .6\}$

$\tilde{C}_1^3 = \{(3, 4, 5); .6, .4, .3\}$

$\tilde{C}_2^3 = \{(-4, -3, -2); .5, .6, .9\}$

$\tilde{\varphi}_4^{\text{Max}} = \{(0.13, 0.56, 2.66); .3, .7, .6\}$

$\tilde{D}_1^4 = \{(2, 4, 6); .3, .5, .6\}$

$\tilde{C}_1^4 = \{(2, 3, 4); .4, .7, .5\}$

$\tilde{C}_2^4 = \{(1, 2, 3); .5, .4, .2\}$

$\tilde{\alpha} = \{(0, 0, 0); 1, 0, 0\}$

$$SM = \sum_{k=1}^{4} \frac{\tilde{Z}_k^{\text{Max}}}{\tilde{\varphi}_i^{\text{Max}}} = \frac{\tilde{W}\tilde{x}_1 + \tilde{X}\tilde{x}_2 + \tilde{\alpha}}{common\ term \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$SM = \frac{\{(10.66, 13.5, 16.33); .3, .7, .8\}\tilde{x}_1 + \{(-0.17, 2.66, 5.5); .4, .7, .9\}\tilde{x}_2}{\{(1, 2, 3); .3, .5, .6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$SN = \{(0, 0, 0); 1, 0, 0\}$$

For Arithmetic Average Technique

Arrange $\tilde{\varphi}_k^{\text{Max}}$ as

$\{(0.53, 1.87, 8); .3, .6, .8\}$, $\{(0.35, 1.12, 4.44); .3, .5, .6\}$, $\{(0.4, 1.5, 6.66); .3, .5, .6\}$, $\{(0.13, 0.56, 2.66); .3, .7, .6\}$

$\tilde{m}_1 = \{(0.13, 0.56, 2.66); .3, .7, .6\}$

$\tilde{\varphi}_k^{\text{Min}}$ as $\tilde{m}_2 = \{(0, 0, 0); 1, 0, 0\}$

$$AV = \frac{\tilde{m}_1 + \tilde{m}_2}{2} = \frac{\{(0.13, 0.56, 2.66); .3, .7, .6\} + \{(0, 0, 0); 1, 0, 0\}}{2} = \{(0.06, 0.28, 1.33); .3, .7, .6\}$$

$$\text{Max } \tilde{Z} = \frac{SM - SN}{AV}$$

$$SM = \frac{\{(10.66, 13.5, 16.33); .3, .7, .8\}\tilde{x}_1 + \{(-0.17, 2.66, 5.5); .4, .7, .9\}\tilde{x}_2}{\{(1, 2, 3); .3, .5, .6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$SN = \{(0, 0, 0); 1, 0, 0\}$$

$$\frac{SM - SN}{AV} = \frac{\{(10.66, 13.5, 16.33); .3, .7, .8\}\tilde{x}_1 + \{(-0.17, 2.66, 5.5); .4, .7, .9\}\tilde{x}_2}{\{(0.06, 0.28, 1.33); .3, .7, .6\} \times \{(1, 2, 3); .3, .5, .6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$\text{Max } \tilde{Z} = \frac{\{(8.01, 48.21, 272.16); .3, .7, .8\}\tilde{x}_1 + \{(-2.83, 9.5, 91.66); .3, .7, .9\}\tilde{x}_2}{\{(1, 2, 3); .3, .5, .6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

subject to

$\{(1, 3, 7); .4, .6, .7\}\tilde{x}_1 + \{(3, 6, 9); .5, .7, .3\}\tilde{x}_2 \le \{(14, 18, 24); .6, .3, .5\}$

$\{(8, 9, 10); .6, .5, .4\}\tilde{x}_1 + \{(1, 2, 4); .3, .6, .9\}\tilde{x}_2 \le \{(20, 27, 32); .3, .4, .5\}$

and $\tilde{x}_1, \tilde{x}_2 \ge \{(0, 0, 0); 1, 0, 0\}$

Using modified simplex algorithm for fractional linear programming problem in neutrosophic environment, we solve this problem and then we get the optimal solution is

$\text{Max } \tilde{Z} = \{(1.06, 18.07, 362.88); .3, .7, .8\}$

$\tilde{x}_1 = \{(2, 3, 4); .3, .5, .5\}$

$\tilde{x}_2 = \{(0, 0, 0); 1, 0, 0\}$

For New Arithmetic Average Technique

Arrange $\tilde{\varphi}_k^{\text{Max}}$ as

$\{(0.13, 0.56, 2.66); .3, .7, .6\}$, $\{(0.35, 1.12, 4.44); .3, .5, .6\}$, $\{(0.4, 1.5, 6.66); .3, .5, .6\}$, $\{(0.53, 1.87, 8); .3, .6, .8\}$

$\tilde{m}_1 = \{(0.13, 0.56, 2.66); .3, .7, .6\}$

$\tilde{\varphi}_k^{\text{Min}}$ as $\tilde{m}_2 = \{(0, 0, 0); 1, 0, 0\}$

$$AV_N = \frac{\tilde{m}_1 + \tilde{m}_2}{4} = \frac{\{(0.13, 0.56, 2.66); .3, .7, .6\} + \{(0, 0, 0); 1, 0, 0\}}{4} = \{(0.03, 0.14, 0.66); .3, .7, .6\}$$

$$\text{Max } \tilde{Z} = \frac{SM - SN}{AV_N}$$

$$SM = \frac{\{(10.66, 13.5, 16.33); .3, .7, .8\}\tilde{x}_1 + \{(-0.17, 2.66, 5.5); .4, .7, .9\}\tilde{x}_2}{\{(1, 2, 3); .3, .5, .6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$SN = \{(0, 0, 0); 1, 0, 0\}$$

$$\frac{SM - SN}{AV_4} = \frac{\{(10.66, 13.5, 16.33); .3, .7, .8\}\tilde{x}_1 + \{(-0.17, 2.66, 5.5); .4, .7, .9\}\tilde{x}_2}{\{(0.03, 0.14, 0.66); .3, .7, .6\} \times \{(1, 2, 3); .3, .5, .6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$\text{Max } \tilde{Z} = \frac{\{(16.15, 96.42, 544.33); .3, .7, .8\}\tilde{x}_1 + \{(-5.66, 19, 183.33); .3, .7, .9\}\tilde{x}_2}{\{(1, 2, 3); .3, .5, .6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

subject to

$\{(1, 3, 7); .4, .6, .7\}\tilde{x}_1 + \{(3, 6, 9); .5, .7, .3\}\tilde{x}_2 \le \{(14, 18, 24); .6, .3, .5\}$

$\{(8, 9, 10); .6, .5, .4\}\tilde{x}_1 + \{(1, 2, 4); .3, .6, .9\}\tilde{x}_2 \le \{(20, 27, 32); .3, .4, .5\}$

and $\tilde{x}_1, \tilde{x}_2 \ge \{(0, 0, 0); 1, 0, 0\}$

Using modified simplex algorithm for fractional linear programming problem in a neutrosophic environment, we solve this problem and then we get the optimal solution is

$\text{Max } \tilde{Z} = \{(2.15, 36.15, 725.77); .3, .7, .8\}$

$\tilde{x}_1 = \{(2, 3, 4); .3, .5, .5\}$

$\tilde{x}_2 = \{(0, 0, 0); 1, 0, 0\}$

For Harmonic Average (H_{av}) Technique

$$H_{av_1} = H_{av}(\tilde{\varphi}_k^{\text{Max}}) = \frac{1}{\{(0.53, 1.87, 8); .3, .6, .8\}} + \frac{1}{\{(0.35, 1.12, 4.44); .3, .5, .6\}} + \frac{1}{\{(0.4, 1.5, 6.66); .3, .5, .6\}} + \frac{1}{\{(0.13, 0.56, 2.66); .3, .7, .6\}}$$

$$= \{(0.12, 0.53, 1.88); .3, .6, .8\} + \{(0.22, 0.89, 2.85); .3, .5, .6\} + \{(0.15, 0.66, 2.5); .3, .5, .6\} + \{(0.37, 1.78, 7.69); .3, .7, .6\}$$

$$= \{(0.86, 3.86, 14.92); .3, .7, .8\}$$

$$H_{av_2} = H_{av}(\tilde{\varphi}_k^{\text{Min}}) = \{(0, 0, 0); 1, 0, 0\}$$

$$S_1 = \frac{SM}{H_{av_1}} = \frac{\{(10.66, 13.5, 16.33); .3, .7, .8\}\tilde{x}_1 + \{(-0.17, 2.66, 5.5); .4, .7, .9\}\tilde{x}_2}{\{(0.86, 3.86, 14.92); .3, .7, .8\} \times \{(1, 2, 3); .3, .5, .6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$= \frac{\{(0.71, 3.49, 18.98); .3, .7, .8\}\tilde{x}_1 + \{(-0.19, 0.68, 6.39); .3, .7, .9\}\tilde{x}_2}{\{(1, 2, 3); .3, .5, .6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$S_2 = \frac{SN}{H_{av_2}} = \{(0, 0, 0); 1, 0, 0\}$$

$$\text{Max } \tilde{Z} = S_1 - S_2 = \frac{\{(0.71, 3.49, 18.98); .3, .7, .8\}\tilde{x}_1 + \{(-0.19, 0.68, 6.39); .3, .7, .9\}\tilde{x}_2}{\{(1, 2, 3); .3, .5, .6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

subject to

$\{(1, 3, 7); .4, .6, .7\}\tilde{x}_1 + \{(3, 6, 9); .5, .7, .3\}\tilde{x}_2 \le \{(14, 18, 24); .6, .3, .5\}$

$\{(8, 9, 10); .6, .5, .4\}\tilde{x}_1 + \{(1, 2, 4); .3, .6, .9\}\tilde{x}_2 \le \{(20, 27, 32); .3, .4, .5\}$

and $\tilde{x}_1, \tilde{x}_2 \ge \{(0, 0, 0); 1, 0, 0\}$

Using modified simplex algorithm for fractional linear programming problem in neutrosophic environment, we solve this problem and then we get the optimal solution is

$\text{Max } \tilde{Z} = \{(0.09, 1.3, 1.58); .3, .7, .8\}$

$\tilde{x}_1 = \{(2, 3, 4); .3, .5, .5\}$

$\tilde{x}_2 = \{(0, 0, 0); 1, 0, 0\}$

For Advanced Harmonic Average technique

Arrange $\tilde{\varphi}_k^{\text{Max}}$ as

$\{(0.13, 0.56, 2.66); .3, .7, .6\}$, $\{(0.35, 1.12, 4.44); .3, .5, .6\}$, $\{(0.4, 1.5, 6.66); .3, .5, .6\}$, $\{(0.53, 1.87, 8); .3, .6, .8\}$

$\tilde{m}_1 = \min(\tilde{\varphi}_k^{\text{Max}}) = \{(0.13, 0.56, 2.66); .3, .7, .6\}$

Table 2: Comparison between results of the numerical results

S.No.	*Techniques*	Max $\tilde{Z}$
1	Arithmetic average technique	$\{(1.06, 18.07, 362.88); .3, .7, .8\}$
2	New arithmetic average technique	$\{(2.15, 36.15, 725.77); .3, .7, .8\}$
3	Harmonic average (H_{av}) technique	$\{(0.09, 1.3, 1.58); .3, .7, .8\}$
4	Advanced harmonic average (AH_{av}) technique	We can't derive objective function.

$\tilde{\varphi}_k^{\text{Min}}$ as $\tilde{m}_2 = \max(\tilde{\varphi}_k^{\text{Min}}) = \{(0,0,0); 1,0,0\}$

$$AH_{av} = \frac{2\tilde{m}_1\tilde{m}_2}{\tilde{m}_1 + \tilde{m}_2} = \frac{2 \times \{(0.13, 0.56, 2.66); .3, .7, .6\} \times \{(0,0,0); 1,0,0\}}{\{(0.13, 0.56, 2.66); .3, .7, .6\} + \{(0,0,0); 1,0,0\}}$$

$$= \frac{\{(0,0,0); .3, .7, .6\}}{\{(0.13, 0.56, 2.66); .3, .7, .6\}} = \{(0,0,0); .3, .7, .6\}$$

$$\text{Max } \tilde{Z} = \frac{SM - SN}{AH_{av}} = \frac{\{(10.66, 13.5, 16.33); .3, .7, .8\}\tilde{x}_1 + \{(-0.17, 2.66, 5.5); .4, .7, .9\}\tilde{x}_2}{\{(0,0,0); .3, .7, .6\} \times \{(1,2,3); .3, .5, .6\} \times (\tilde{x}_1 + \tilde{x}_2 + \tilde{1})}$$

$$\text{Max } \tilde{Z} = \infty$$

Advanced harmonic average technique, the single objective is derived from $\frac{2\tilde{m}_1\tilde{m}_2}{\tilde{m}_1+\tilde{m}_2}$, where $\tilde{m}_1 = \min(\tilde{\varphi}_k^{\text{Max}})$, $\tilde{m}_2 = \max(\tilde{\varphi}_k^{\text{Min}})$. Here $\tilde{m}_2$ is obtained from the minimization objective function of the given problem. Here in our example, there is no minimization function. So, we can't derive a single objective function for this problem. So for this example, the advanced harmonic average technique is not suitable for solving. The solution of the numerical example for different mean techniques is in Table 2.

Conclusion

In this paper, we defined the arithmetic average, new arithmetic average, harmonic average and advanced harmonic average techniques for conversion. Advanced Harmonic average (AH_{av}) technique is non-determinable due to the non-availability of the minimization function. From the remaining techniques, new arithmetic average technique gives the best solution.

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