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RESEARCH ARTICLE

Comparative analysis of inverted pendulum control

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Abstract

The main motive of this paper is to balance the inverted pendulum system (nonlinear model) using controllers and to compare the results obtained from using different controllers. The aim is to determine which controller provides best results with respect to the cart's position and pendulum's angle. The controllers used in this paper are PI, PD, PID. The inverted pendulum model is modeled using Simscape and the simulation results are obtained using MATLAB

Keywords: Inverted-pendulum, nonlinear system, PID controller, optimal control, Simscape.

Introduction

In modern control theory, balancing the inverted pendulum is quite fascinating. It is inherently unstable, so balancing the inverted pendulum is the ultimate goal. One of the most important and difficult problems faced is having an inherently unstable system (Prasad et al., 2014). In the inverted pendulum model, when given a disturbance, the angle between the normal and the pendulum increases and at one particular instance, the pendulum falls. So the aim of balancing is to decrease the angle between the pendulum and the normal after the disturbance is given, the pendulum moves front and back accordingly to balance the pendulum hence preventing the falling. Ensuring the system is robust and efficient and stable while in a closed loop is practically very hectic. This unstable system, however, has maintained its usefulness because of its nonlinear nature and is now used in illustrating many ideas in the field of nonlinear control (Wang, 2015). The inverted pendulum model is used in several applications. Here in this paper, the Segway model is discussed. The 3d modeling of the Segway model is done using Simscape and that model is being controlled using PI, PD, PID and LQR controller and the results are tabulated and compared.

The main control objective here is stabilizing the mechanism by the application of force to the cart around the unstable equilibrium point (Ibañez et al., 2005). Maintaining an equilibrium position in this system is considered a challenge as the equilibrium position is unstable (Lam, 2004). Considering various fuzzy based nonlinear system stability assurance approaches, there are different advantageous methods.

Based on the comparison made by Muskinja, the latest algorithms for the fuzzy swinging can be compared with a strategy based on energy utilization (Muskinja & Tovornik, 2006). The energy model is also derived from a model of the Inverted pendulum based on its nonlinear mathematical aspects. This system is well-known as a model for attitude control, particularly in the aerospace industry. It does, however, have its own set of problems. Due to its principles, it has a shortcoming of its own; it is highly nonlinear and an insecure open-loop system (Nasir et al., 2008). A PID controller is a feedback control loop controller frequently utilized in industrial control systems. The PID controller is responsible for transient and steady state responses (Prasad et al., 2012).

A PID controller is the most popular method of obtaining feedback for controlling. It computes an inaccurate value based on the two values that is fed in. This inaccuracy can be eliminated using a process variable, a desired set point that alters the parameters per the model's demands (Yadav et al., 2011). In this paper the 3D model of the inverted pendulum is modeled using SimScape (Figure 1). The model is controlled using different controllers and the results obtained from the MATLAB simulation are tabulated and compared.

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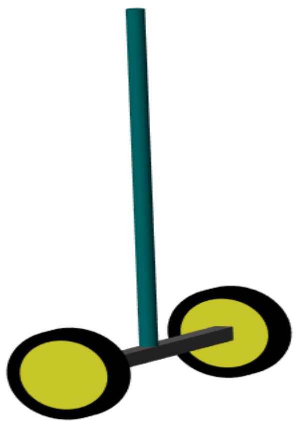

Figure 1: Model of inverted pendulum developed in simulink

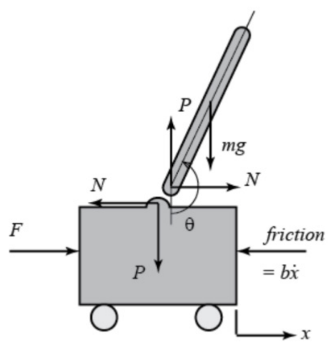

Figure 2: Free body diagram of inverted pendulum on a cart

Mathematical Model

Considering the forces acting in the horizontal direction of the cart, we get

$$M\ddot{x} + b\dot{x} + N = F \qquad (1)$$

The reaction force N in the horizontal direction is,

$$N = m\ddot{x} + ml\ddot{\theta} \cos\cos\theta - ml\dot{\theta}^2 \sin\sin\theta \qquad (2)$$

Substitute (2) in (1),

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta} \cos\cos\theta - ml\dot{\theta}^2 \sin\sin\theta = F \qquad (3)$$

To get another equation, we are adding the forces perpendicular to the pendulum,

$$P \sin\sin\theta + N \cos\cos\theta - mg \sin\sin\theta = ml\ddot{\theta} + m\ddot{x} \cos\cos\theta \qquad (4)$$

Summing the moments about the centroid of the pendulum, we get,

$$-Pl \sin\sin\theta - Nl \cos\cos\theta = I\ddot{\theta} \qquad (5)$$

Combining (4) & (5), we get

$$(I + ml^2)\ddot{\theta} + mgl \sin\sin\theta = -ml\ddot{x} \qquad (6)$$

$$\cos\cos\theta = \cos\cos(\pi + \emptyset) \approx -1 \qquad (7)$$

$$\sin\sin\theta = \sin\sin(\pi + \emptyset) \approx -\emptyset \qquad (8)$$

$$\dot{\theta}^2 = \dot{\emptyset}^2 \approx 0 \qquad (9)$$

Substituting (7), (8), (9) in (6),
We get two equations which are linear,

$$(I + ml^2)\ddot{\emptyset} - mgl\emptyset = ml\ddot{x} \qquad (10)$$

$$(M + m)\ddot{x} + b\dot{x} - ml\ddot{\emptyset} = u \qquad (11)$$

By applying Laplace transform to the linearized equations (10) & (11) assuming initial conditions as zero, we can obtain the transfer function

$$(I + ml^2)\emptyset(s)s^2 - mgl\emptyset(s) = mlX(s)s^2 \qquad (12)$$

$$(M + m)X(s)s^2 + bX(s)s - ml\emptyset(s)s^2 = U(s) \qquad (13)$$

Solving equation (12) for X(s),

$$X(s) = \left[\frac{I + ml^2}{ml} - \frac{g}{s^2}\right]\emptyset(s) \qquad (14)$$

Substitute (14) in (12),

$$(M + m)\left[\frac{I + ml^2}{ml} - \frac{g}{s^2}\right]\emptyset(s)s^2 + b\left[\frac{I + ml^2}{ml} - \frac{g}{s^2}\right]\emptyset(s)s - ml\emptyset(s)s^2 = U(s) \qquad (15)$$

The transfer function is,

$$\frac{\emptyset(s)}{U(s)} = \frac{\frac{ml}{q}s^2}{s^4 + \frac{b(I + ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \qquad (16)$$

Where q is,

$$q = [(M + m)(I + ml^2) - (ml)^2] \qquad (17)$$

The transfer function with respect to pendulum,

$$P_{pend}(s) = \frac{\emptyset(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{b(I + ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}} \left[\frac{rad}{N}\right] \qquad (18)$$

The transfer function with respect to cart,

$$P_{cart}(s) = \frac{X(s)}{U(s)} = \frac{\frac{(I + ml^2)s^2 - gml}{q}}{s^4 + \frac{b(I + ml^2)}{q}s^3 - \frac{(M + m)mgl}{q}s^2 - \frac{bmgl}{q}s} \left[\frac{m}{N}\right] \qquad (19)$$

Linear and Nonlinear controller

PID Controller

The PID controller is a feedback-based control loop. This controller is also called as a three-term controller. This controller is frequently and conveniently utilized in various different industrial systems wherein the system demands continuous point to point control. It has 3 terms Proportional, Integral and Derivative term

- The proportional term generates an output that is at a constant ratio level to the current error value. In order to change the proportional response value, the error is multiplied by a gain constant, which is represented by the term Kp. The rise time of the model is minimized and the response speed significantly increased with the aid of the Proportional controllers. This controller does not make any changes to the phase response of the system.
- The instantaneous error over a given period of time is totaled to obtain the Integral. These integral yields

previously unresolved offset that is accumulated over a given period of time. The errors caused due to the invariance in time is rectified with the help of the integral controller. This causes a phase lag thus there is thus there is no magnitude change in the output.

- The derivative gain, expressed by the term Kd, is utilized greatly in determining the error slope over a given period of time by multiplying this rate of change of the slope by Kd and as a result, obtaining the Process error. Kd expresses the derivative term's magnitude and represents the term's contribution to the entire control action. A differential controller is required to minimize transient errors such as overshoot and oscillation of plant output. However, this can lead to serious instability in noisy environments. A phase lead is provided to the system's output and compared to the input with no changes in the output.

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \frac{de(t)}{dt}$$

Comparing P, PI, PD controllers,

P Controller

- Implementation is easy in case of P controller
- In contrary P controller has steady state error and long settling time

PI Controller

- No steady state error is present
- Stability is more with PI controller

PD Controller

- It is easy to stabilize the system with PD controller
- It is faster than P controller
- It can even work with the presence of high noise in the system

Simulation

Simulation Output using PID Controller

A PID controller is a general form of controller. Any controller can be achieved from this configuration by adjusting the gains of the three control actions. The magnitude and the change in the phase either lead or lag can be made available through the general controller model. The Simulink model of inverted pendulum is given in Figure 4. Figure 3 shows the Angle of the pendulum, Figure 5 shows the Position of cart wrt time and Figure 6 shows the Plot of Tuned response.

Simulation Output using PI Controller

A PI controller facilitates in decreasing the steady state errors and rise time of the system. This can be used to change the magnitude and lag the phase together. Figure 7 shows the Angle of the pendulum with respect to time (PI), Figure 8 shows the Position of cart with respect to time and Figure 9 shows the Plot of Tuned response using PI controller .

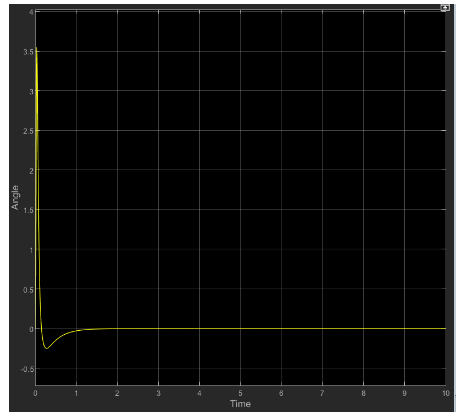

Figure 3: Angle of the pendulum with respect to time (PID)

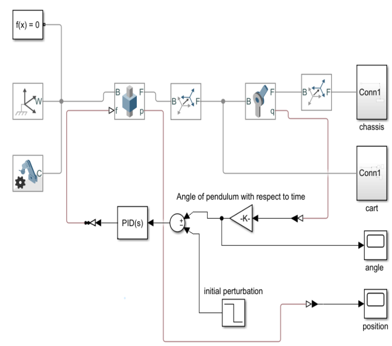

Figure 4: Simulink model of inverted pendulum system

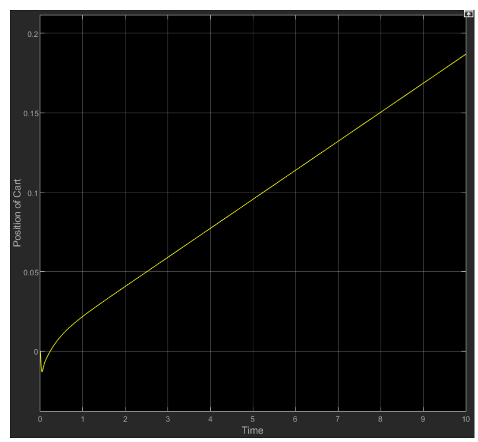

Figure 5: Position of cart with respect to time

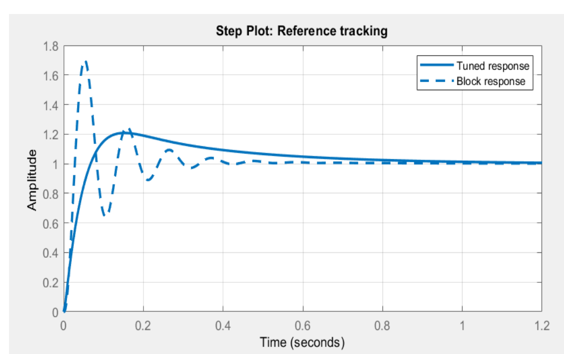

Figure 6: Plot of Tuned response using PID controller

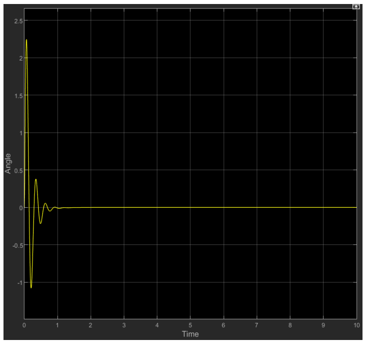

Figure 7: Angle of the pendulum with respect to time (PI)

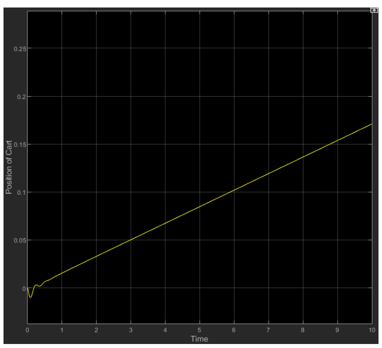

Figure 8: Position of cart with respect to time

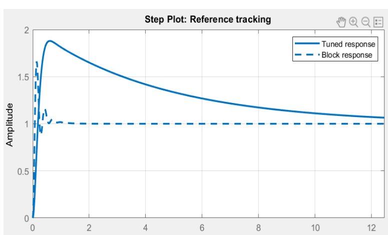

Figure 9: Plot of Tuned response using PI controller

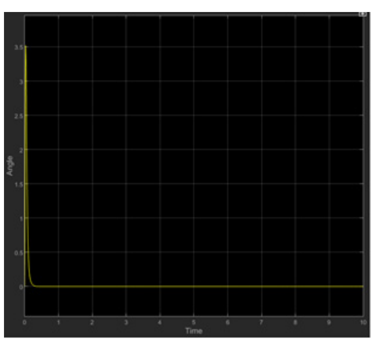

Figure 10: Angle of pendulum with respect to time

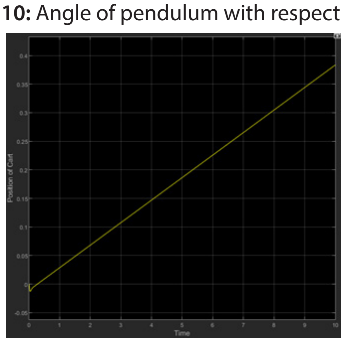

Figure 11: Position of cart with respect to time

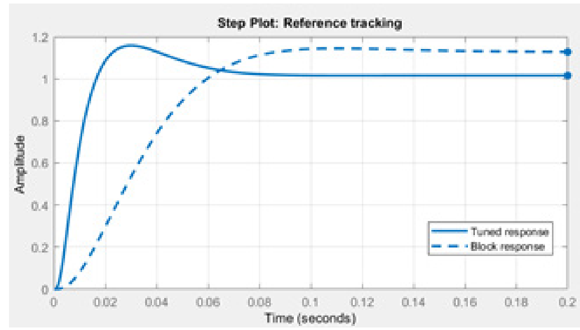

Figure 12: Plot of Tuned response using PD controller

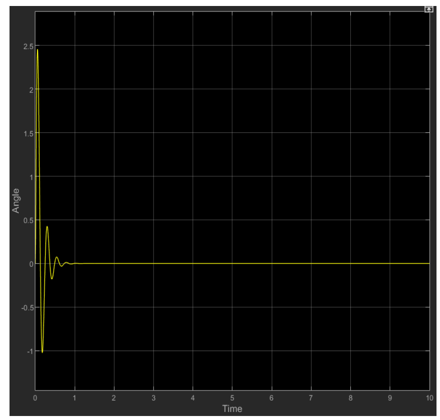

Figure 13: Angle of the pendulum with respect to time

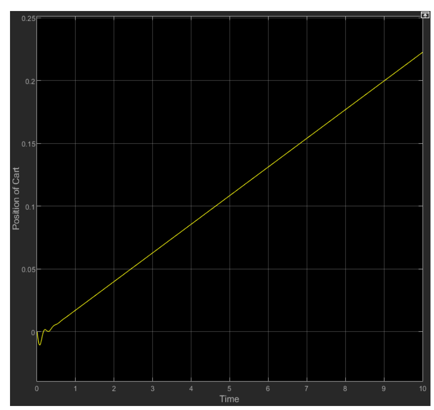

Figure 14: Position of the cart with respect to time

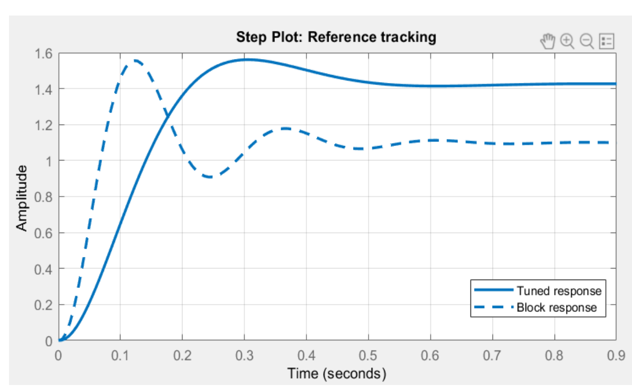

Figure 15: Plot of Tuned response using P controller

Table 1: Summary of the parameters in different controllers

Parameters	*P*	*PI*	*PD*	*PID*
Peak response	1.56	1.88	1.16	1.21
Overshoot percentage (%)	9.4	87.9	14.1	20.8
Rise time (s)	0.143	0.126	0.0111	0.0492
Settling time (s)	0.461	0.126	0.0674	0.866

Simulation Output Using PD Controller

A PD controller reduces the transients like rise time, overshoot, and oscillations in the output. It is useful for changing the magnitude and at the same time adding lead phase to the output. Figure 10 shows the Angle of the pendulum with respect to time (PI), Figure 11 shows the Position of cart with respect to time and Figure 12 shows the Plot of Tuned response using PD controller. The table 1 shows the comparison of P, PI, PD and PID Controller

Simulation Output Using P Controller

P controller have many disadvantages like offset error is present, fine controlling is tedious, leading to system instability for a large gain. Figure 13 shows the Angle of the pendulum with respect to time (PI), Figure 14 shows the Position of cart with respect to time and Figure 15 shows the Plot of Tuned response using P controller

Conclusion

This paper compares controllers P, PI, PD, PID, and the parameters tabulated. With only P controller, there are many disadvantages like offset error is present, fine controlling is tedious, leading to system instability for a large gain. With PI controller the overshoot is very high, the response is sluggish if a sudden disturbance is given to the system. With PD controller the high frequency noise can be amplified in a system. P controller is easy to implement, PD controller is easy to stabilize and it has faster response than P controller, PI controller has no steady state error but it has high overshoot percentage The above-discussed Controllers have both advantages and disadvantages they are good in some characteristics and bad in some other characteristics. Therefore PID controller with proper tuning is used in many applications. Steady state error is zero, moderate peak overshoot, moderate stability can be achieved by using PID controller. In an inverted pendulum, both PI and PID give similar performances, but PI have a maximum peak overshoot percentage than PID controller. From the simulation results, we can conclude that using PID controller to control the inverted pendulum is recommended with proper tuning of Kp, Ki and Kd.

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