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RESEARCH ARTICLE

Extended Kalman filter-based prognostic of actuator degradation in two tank system

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Abstract

Rapid growth in the industries need an effective predictive maintenance policy. Failure in the equipment decreases the production rate and thereby, causing a loss to the industry. The equipment, especially the actuator, is operated continuously in the industries to achieve the desired production rate. Actuator is the key element which undergoes degradation due to frequent control actions. However, degradation is mainly influenced by different operating conditions and other environmental factors. This decreases the lifetime of the equipment and also it increases the maintenance cost. This problem is addressed by carrying out the reliability studies on the actuator by using Gamma process. It is used to describe the system degradation.

In this work, Gamma process based actuator modelling is used to study the deterioration in the actuator. The gamma parameters such as shape and scale parameters are the deciding factors describing the system's degradation level. It is then applied to two tank feedback control system. Extended version of Kalman filter estimates the state of noisy measurements which describes the fault trend characteristics in the system. Finally, the evolution of actuator capacity inpresence of fault is analyzed and simulated in MATLAB environment.

Keywords: Predictive Maintenance, Extended Kalman Filter, Gamma Process.

Introduction

Dynamic system is a system which undergoes changes in state throughout time. In a real time, there are numerous possibilities of changes. These changes create either a positive or negative impact to the system. The negative impact includes degradation in the system which is caused by faults in the system. These faults are undesired phenomenon occurring in the system due to abnormal behavior or deviations. The faults are classified into many categories based upon its nature. It is classified as additive fault, multiplicative fault and process fault (Moulahi, & Ben Hmida, 2022).

Additive faults is the most occurring fault which involves a sudden raise or drop from its operating condition (Shahraki, Yadav and Liao, 2017). It is caused by the additive behavior of unwanted phenomena leading to fault in the system.

The second type of fault is caused by gradual changes or variance in the measurements. The third type of fault contains fault in the process itself. It may occur due to plant leakage or improper working of coolant (Moulahi, & Ben Hmida, 2022). The subdivisions of fault includes single fault or multiple faults. Single fault shows the effect of residual of single fault at a time.

Multiple fault shows that multiple faults occur under the influence of residual. In general, it is difficult to differentiate the fault from nuisances (Riascos-Ochoa, Sánchez-Silva and Klutke, 2016). This lead to the emerging growth in the fault diagnosis and control.

The faults affect the actuator and sensor most frequently (Sikorska, Hodkiewicz and Ma, 2011) . A feedback control system provides a good servo and regulatory operation. Though feedback control system provides good performance, a special technique is needed to handle the faulty systems in a special fashion (Moulahi and Hmida, 2020). In traditional days, to overcome this faulty nature redundancy is used. But, it increases the system's cost and weight to be maintained fault-free. This in turn increases the maintenance cost of the equipment. According to the varying needs of industries, the production rate changes frequently (Moulahi & Ben Hmida, 2022). This needs the component to adapt to varying industrial demands. The

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actuator is one of the main component which undergoes degradation due to its sudden inability to provide control action to the plant (Mabrouk, Moulahi and Ben Hmida, 2020). This causes the entire control system to get collapsed.

Fault Detection and Isolation (FDI) is emerging area that addresses the detection of faults, isolation and identification of faults (Singleton, Strangas and Aviyente, 2014). State Estimation is a broad research area that estimates noisy measurements' state. The combination of these provides better understanding of the states of the system, measurements and the possible faults in the system.

Gamma Modelling

The degradation modeling of the two tank non-interacting feedback level control system with actuator modelling is discussed in this section. First principle modeling is the widely used method to know the dynamics of the system.

Actuator Modelling

One of the most used process for actuator modeling is gamma process. It is based on Levy process containing independent increments over time Riascos-Ochoa, Sánchez-Silva and Klutke, 2016). These increments which are non-negative in nature. The gamma distribution function is strictly increasing over time. It is the preferable method in case of deterioration in motor pump, which is caused by wear and tear, corrosion of metals and erosion in the impeller. It mainly consists of two parameters such as shape parameter 'a' and scale parameter 'b'. The shape parameter describes the degradation effect on the system performance (Cao *et al.*, 2020). The scale parameter tells about action of random factors caused by the environment as well as the human. These parameters are non-negative, positive real numbers (Moulahi, & Ben Hmida, 2022). The random degradation quantity d(t) with stochastic nature of the gamma parameters given by,

$$d(t)=dG(t)\sim G(d|a,b)>0 \qquad (1)$$

where, G(d|a,b) denotes the gamma process with degradation parameters 'a' and 'b'.

The actuator capacity degradation is given by,

$$Ka(t)=Kao-d(t) \qquad (2)$$

where, Ka(t) represents the actuator capacity.

The main properties of gamma modelling includes (Cao *et al.*, 2020),

- dG(t=0)=0 with probability one
- Increments dG(t2)-dG(t1) has gamma process distribution
- Degradation process dG(t) has increments that are independent

The gamma probability distribution function is expressed as follows,

$$f_{d_G}(x)=\frac{b^{a(t_2-t_1)}}{\Gamma(a(t_2-t_1))}x^{a(t_2-t_1)^{-1}}\exp(-bx) \qquad (3)$$

The gamma model function is expressed as (Ling, Tsui and Balakrishnan, 2014),

$$\Gamma(a(t_2-t_1))=\int_0^{+\infty}x^{a(t_2-t_1)^{-1}}\exp(-x)\,dx \qquad (4)$$

The distribution of dG(t) has the following form,

$$f(d_G(t))\sim G(d_G(t)|a(t),b) \qquad (5)$$

The mean and the variance of gamma process are,

$$E(d_G(t)=\frac{a(t)}{b} \qquad (6)$$

$$Var(d_G(t))=\frac{a(t)}{b^2} \qquad (7)$$

Modelling of Two Tank System

The operation of two tank non-interacting level control system associated with degradation modeling is discussed. It uses mathematical equations to deal with the mathematical relationship of the feedback system and the actuator failure of the two tank level control system.

System Description

The system consists of two tanks which are connected in a non-interacting fashion. The inlet flow rate is applied to Tank-1 which changes its liquid level. The actuator is the motor pump that pumps the liquid in to the tanks (Moulahi, & Ben Hmida, 2022). The input to second tank is the outlet from Tank 1.

To measure the height of the Tank-1, the level measurement sensor is used in feedback. Practically, the set-point is changed often to meet the industrial demands. This causes the actuator to exert extra control to maintain the liquid at the desired level.

During this phase, the actuator undergoes a degradation causing the entire closed loop system to collapse and provides the poor performance (Le Son, Fouladirad and Barros, 2014). Figure 1 shows the closed loop two tank level control system.

Using first principle model, the liquid level in tank-1 given by (Moulahi, & Ben Hmida, 2022),

$$\frac{dh_1(t)}{dt}=\frac{1}{A_1}q_{in}(t)-\frac{K_{v_i}}{A_1}\sqrt{2gh_1(t)} \qquad (9)$$

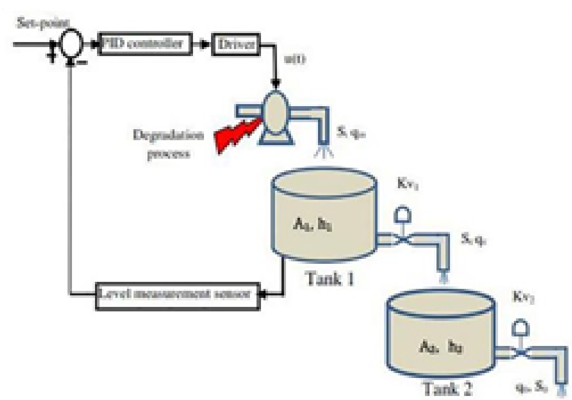

Figure 1: Closed Loop Level Control of a Two Tank System

Liquid level into tank-2 given by,

$$\frac{dh_2(t)}{dt} = \frac{K_{v_1}}{A_1}\sqrt{2gh_1(t)} - \frac{K_{v_2}}{A_2}\sqrt{2gh_2(t)} \quad (10)$$

The relation between flow rate and actuator control effort is given by,

$$\frac{d_{q_{in}(t)}}{dt} = -\frac{1}{\tau_a}q_{in}(t) + \frac{K_a}{\tau_a}u_a(t) \quad (11)]$$

where τa is motor pump time constant and ua(t) is actuator control effort.

The discretized system equations are as follows,

$$h_1(k+1) = h_1(k) + T_e(\frac{q_{in}(k)}{A} - T_e(\frac{K_{v_1\sqrt{2g}}\sqrt{h_1(k)}}{A}) \quad (12)$$

$$h_2(k+1) = h_2(k) + T_e(\frac{K_{v_1\sqrt{2g}}\sqrt{h_1(k)}}{A}) - T_e(\frac{K_{v_2\sqrt{2g}}\sqrt{h_2(k)}}{A}) \quad (13)$$

$$q(k+1) = q(k) - \frac{T_e}{\tau_a}q(k) + K_a\frac{T_e}{\tau_a}u(k) \quad (14)$$

The reformulated non linear model is as follows:

$$x(k+1) = A(k)x(k) + C\sqrt{x(k)} + B(k)u(k) + v(k) \quad (15)$$

$$y(k) = h_1(k) + w(k) \quad (16)$$

The process noise vector is represented as,

$$v(k) = (v_1(k) \quad v_2(k) \quad v_3(k) \quad v_4(k))^T \quad (17)$$

The measurement noise vector is represented as,

$$w(k) = (w_1(k) \quad w_2(k) \quad w_3(k) \quad w_4(k))^T \quad (18)$$

The state space matrices are found to be,

$$A = \begin{pmatrix} 1 & 0 & \frac{T_e}{A} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1-\frac{T_e}{A} & \frac{T_e}{A}u \\ 0 & 0 & 0 & 0 \end{pmatrix} B = \begin{pmatrix} 0 \\ 0 \\ \frac{K_a T_e}{\tau_a} \\ 0 \end{pmatrix} C = \begin{pmatrix} \frac{-K_{v_1}T_e\sqrt{2g}}{A} & 0 & 0 & 0 \\ \frac{K_{v_1}T_e\sqrt{2g}}{A} & \frac{-K_{v_2}T_e\sqrt{2g}}{A} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Methodology

The algorithm used to find the estimation of actuator degradation is discussed. An efficient Extended Kalman Filter can provide information about the trend of the innovation term in the system. This requires an estimation filter algorithm ((Moulahi and Hmida, 2020).

Extended Kalman Filter

Extended Kalman Filter (EKF) is designed to linearize the system's non-linearity around its current state and covariance. It uses analytical methods like Taylor's series, Jacobian to linearize the non-linearity (Bresselet *al.*, 2016).

Non-Linear System Dynamics:

$$X_K = f(X_{K-1}, U_{K-1}) + W_{K-1} \quad (19)$$

$$Z_K = h(X_K) + V_K \quad (20)$$

where,

Wk and Vk are the process noise and the measurement noise, respectively.

Xk, Uk, Zk are the system state, input and sensor measurements.

Table 1: Two Tank System Parameters

Parameters	*Description*
A_i	Cross-sectional area of the tank (i = 1, 2)
h_i	Height of fluid level in the tank (i = 1, 2)
p_0	Atmospheric pressure (Pa)
p_i	Bottom pressure (Pa)
ρ	Density of the fluid (Kg/cu.m)
K_{v1}, K_{v2}	Valve parameters v1 and v2
q_{in}	Flow rate (cu.m /s)
u(t)	Command signal

Table 2: Two tank specifications

Physical parameters	*Numerical values*
A1=A2=A	25 sq.m
K_{v1}	0.004 sq.m
K_{v2}	0.002 sq.m
g	9.82m/s
τ_a	1s
Initial conditions	h1(0)= 5m, h2(0)=2m Kao=5gpm
PID Tuning parameters	Kp=4 Ti=19 T_d=1.5

EKF uses a non-linear function 'f' to estimate the predicted state from the past estimate and the non-linear function 'h' uses the predicted state to compute the predicted measurement.

Since 'f' and 'h' cannot be applied directly to covariance, instead a matrix of first order partial derivatives is calculated. At each time step, the Jacobian is computed with the current predicted states.

The EKF involves two phases namely,

- Predict
- Update

Predict phase uses the past estimate to find the estimate at current time step.

Time Update Equations

Predicted state,

$$\hat{X}_{K|K-1} = f(\hat{X}_{K-1}, U_{K-1}) \quad (21)$$

Predicted Covariance

$$\hat{P}_{K|K-1} = F_K\,\hat{P}_{K-1}\,F_K{}^T + Q_K \quad (22)$$

Update phase is used to improve the prediction to achieve a more refined estimate for the same current time step.

Measurement Update Equations

Updated state,

$$\hat{X}_{K|K} = \hat{X}_{K|K-1} + K_K(Z_K - H_K\hat{X}_{K|K-1}) \quad (23)$$

Updated Covariance

$$\hat{P}_{K|K} = (I - K_K\, H_K)\, \hat{P}_{K|K-1} \quad (24)$$

Kalman gain

$$K_K = \hat{P}_{K|K-1} {H_K}^T \left(H_K \hat{P}_{K|K-1} {H_K}^T + R_K\right)^{-1} \quad (25)$$

The main advantages of using EKF is due to the following reasons (Singleton, Strangas and Aviyente, 2014):

- EKF may be applied to estimation of non-linear systems with small non-linearities.
- There is no need of any tuning parameters.
- EKF holds good for system with Gaussian distribution.
- Preferred mainly in non-linear dynamical systems
- Used in machine learning applications
- EKF also used in dual estimation, where both states and parameters of the system are estimated.
- Applied to land data assimilation, 2D hydrodynamics and oceanography.

Results and Discussion

Understanding the dynamics of the system is the first and foremost step to carry out the work related to control and fault diagnosis.

It describes the originality of the system and its capability to react to varying inputs and disturbances. In general, the control system is classified into two types based on whether the feedback is present or not.

- Open Loop System
- Closed Loop System

Open loop response is used to study the system's behavior without any external feedback whereas the closed loop response primarily deals with feedback mechanism. Feedback in turn classified into positive feedback and negative feedback.

Positive feedback raises the system's effective input considered by adding the input signal with the feedback signal. Negative feedback drops the effective input by subtracting feedback signal from input signal. Mostly, negative feedback is preferred.

It is inferred from Figure 2 that the system does not settle. The open loop response shows that the system is unstable.

From Fig.3, it is clear that,

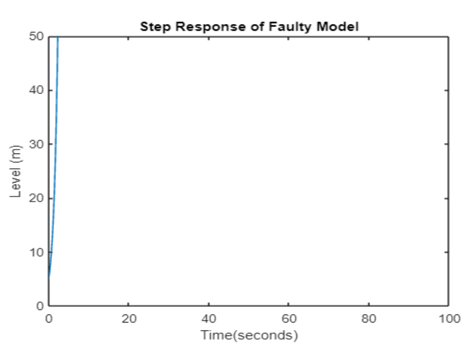

Figure 2: Open loop response

- The system undergoes a sudden overshoot between t=8 and t=9 sec.
- The system also has a transport lag for about 6 seconds.
- It is also found that due to degradation in the actuator capacity, the level goes for a negative values and settles at a certain value indicating the actuator is failure.

It is evident from Fig.4 and Fig.5 that the EKF estimates the internal system states. It shows the deviation between actual state and estimated state around an operating point. In addition to that, Fig.5 shows the stochastic nature of the system.

It is noticeable from Fig.6 that the actuator capacity decreases over the time due to the presence of degradation in the system. In this case, it is assumed that the degradation is mainly caused by frequent changes in set-point.

It is shown from Fig.7 that the degradation starts to build up at the initial stage of the process itself, indicating the actuator capacity.

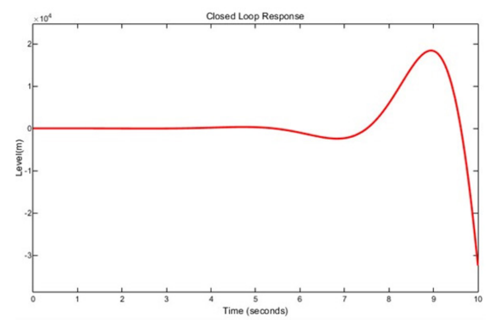

Figure 3: Closed loop response

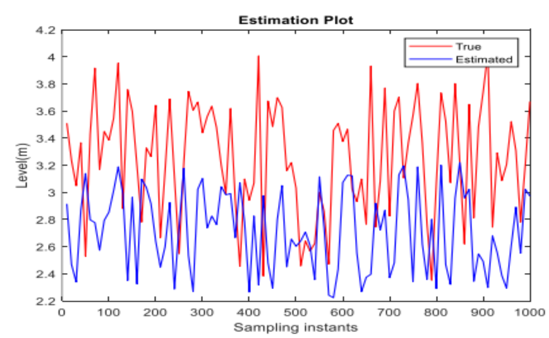

Figure 4: State estimation using EKF

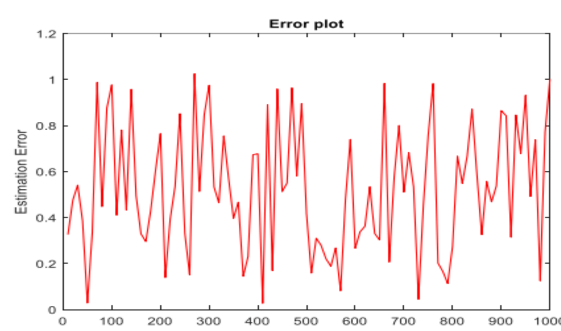

Figure 5: Estimation error plot using EKF

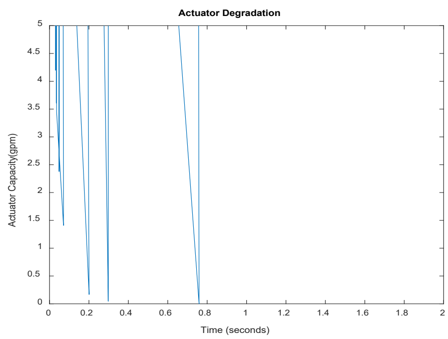

Figure 6: Response of Actuator Capacity

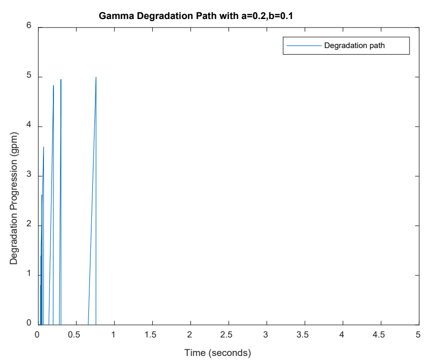

Figure 7: Gamma Degradation of Faulty System

Conclusion

A detailed study was made in modeling two tank non-interacting level control system with actuator degradation. Actuator degradation is expressed in terms of actuator capacity which is initialized around 5 gpm and the degradation path is generated using random gamma numbers.

The Gamma process modeling of two tank system explains the stochasticdegradation of theactuator resulting in a loss of efficiency. This leads to failure in the plant. In this work, gamma process is used to describe the continuous actuator degradation generated by the frequent changes in set-point of the system to meet the production demands. For this a non-linear state estimator, EKF is used to predict the level in the tank which is subjected to actuator degradation.

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