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Convergence of the Method of False Position

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ABSTRACT

The method of false position has been applied to calculate the fourth roots of the natural numbers from 1 to 30 in the interval [0, 3] with the stopping tolerance of 0.00001 using C++ computer program. The minimum error 0.000000029282 and minimum percentage error 0.000001251170 have been obtained in the determination of fourth roots of 30. The maximum error 0.000002324581 and maximum percentage error 0.000232458100 have been obtained in the determination of fourth roots of 1. The average value of the error is 0.000000392037 and the average value of percentage error is 0.000027500512. Minimum, maximum and average values of the numerical rate of convergence have been found to be 0.239808153477, 1.851851851852 and 1.197514787730 respectively.

Keywords: Method of false position, rate of convergence, percentage error, trend, algorithm, accuracy, iterations.

INTRODUCTION

The Bisection method does not use values of *f(x)* but only their sign. However, the values could be exploited. One way to use values of *f(x)* is to bias the search according to value of *f(x)* using a weighted average. [1-4]

In order to discuss the method of false position, let us choose c as the intercept of the secant line through *(a, f(a))* and *(b, f(b))*. Let us also assume that *f(x)* is continuous such that $f(a)\, f(b) < 0$ then formula for the secant line is given by-

$$\frac{y-f(b)}{x-b}=\frac{f(a)-f(b)}{a-b}$$

Let y = 0, the intercept then the next approximation is

$$x_1=\frac{af(b)-bf(a)}{a-b}$$

x_1 is first approximation to x^* [5-8]

The algorithm is depicted in Figure-1. [9, 10]

As in Bisection, if $f(x_1) \neq 0 \Rightarrow f(a)\, f(x_1) < 0$ or $f(b)\, f(x_1) < 0$

$\Rightarrow$ there must be a root $x^* \varepsilon [x_1, b]$

Let us suppose $f(b)\, f(x_1) < 0$, then

$$x_2=\frac{x_1 f(b)-bf(x_1)}{f(b)-f(x_1)} \text{ etc.}$$

Similar to the secant method, the false position method also uses a straight line to approximate the function in the local region of interest. The only difference between these two methods is that the secant method keeps the most recent two estimates, while the false position method retains the most recent estimate and the next recent one which has an opposite sign in the function value. [11-15]

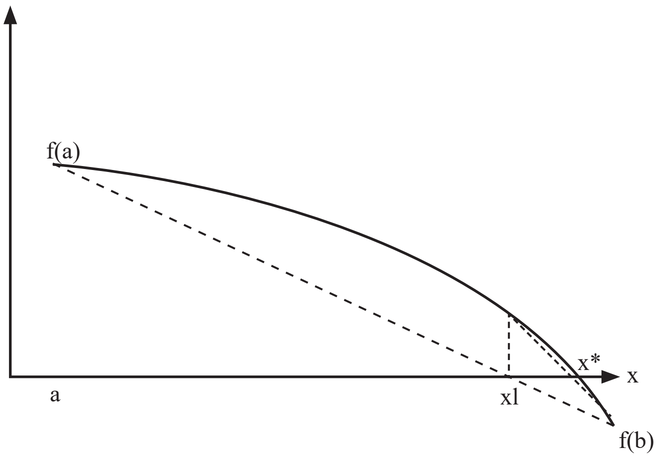


Figure 1: Regula-Falsi Algorithm.

The false position method, which sometimes keeps an older reference point to maintain an opposite sign bracket around the root, has a lower and uncertain convergence rate compared to the secant method. The emphasis on bracketing the root may sometimes restrict the false position method in difficult situations while solving highly nonlinear equations. [16, 17]

MATERIAL AND METHOD

Algorithm of the method of false position is given below:

To find a root of $f(x) = 0$ in the interval $[a_0, b_0]$ with which $f(a_0) f(b_0) < 0$ with tolerance δ

$x_{n+1} = b_n - (b_n - a_n) f(b_n) / [f(b_n) - f(a_n)], n=0, 1, 2, \ldots\ldots$

if ($|f(x_{n+1})| < \delta$) root found, stop iteration

else

if [$f(x_{n+1}) f(b_n) < 0$] $a_{n+1} = x_{n+1}; b_{n+1} = b_n$

else

$a_{n+1} = a_n; b_{n+1} = x_{n+1}$

Computer program developed to calculate fourth roots of natural numbers from 1 to 30 is given below-

```
#include<conio.h>
#include<stdio.h>
#include<math.h>
// method of false position
void main(void)
{
FILE *fpt;
int n;
float a[1000],b[1000],c[1000],delta,rl,ru,d,aa;
double f(float x);
//avr is the variable whose fourth root is to be calculated
double avr =1.0;
clrscr();
//Filename to store result
fpt=fopen(“lavfp1.txt”, “w”);
rl=0; ru=3.0; n=0; a[0]=rl; b[0]=ru; aa=fabs(rl-ru);
//Value of function f(x)
fprintf(fpt,”f(x)=x^4-1\n”);
fprintf(fpt,”rl= %6.2f\n”,rl);
fprintf(fpt,”ru= %6.2f\n”,ru);
//to check existence of root between the interval
d=f(rl)*f(ru);
delta=0.00001;
fprintf(fpt,” n a[n] b[n] c[n] f(c[n])\n”);
printf(“ n a[n] b[n] c[n] f(c[n])\n”);
if (d<0)
{
while(aa > delta)
{
if (a[n]==b[n]) break;
c[n+1]=b[n]-(b[n]-a[n])*f(b[n])/(f(b[n])-f(a[n]));
if (f(c[n+1])*f(b[n])<0)
{
a[n+1]=c[n+1];
b[n+1]=b[n];
}
else
{
b[n+1]=c[n+1];
a[n+1]=a[n];
}
aa=fabs(f(c[n]));
fprintf(fpt,”%3d %15.12f %15.12f %15.12f %18.12f\
n”,n+1,a[n],b[n],c[n],f(c[n]));
printf(“%3d %15.12f %15.12f %15.12f %18.12f\n”,n
+1,a[n],b[n],c[n],f(c[n]));
if (aa > delta) n=n+1;
}
printf(“Root= %20.15f\n”,c[n]);
printf(“Value of function=%20.15f\n”, f(c[n]));
printf(“No. of iterations=%3d\n”,n+1);
printf(“Actual value of root=%15.12f\
n”,pow(avr,0.25));
fprintf(fpt,”Actual value of root=%15.12f\
n”,pow(avr,0.25));
```

```
printf("\n");
getch();
}
else
{
printf("There is no root in the given interval\n");
getch();
}
fclose(fpt);
}
//Function definition
double f(float x)
{
double r;
r=x*x*x*x-avr;
return(r);
}
```

With the help of above computer program, fourth roots of the numbers from 1 to 30 in the interval [0, 3] have been calculated. For this, the following functions have been taken

$f(x) = x^4 - n = 0$ where n = 1, 2, 3, ……. , 30

Numerical accuracy of the method of false position has been has been measured by percentage error and defined as follows–

Percentage error = error in the value of fourth root * 100/actual value of fourth root

Numerical accuracy of the method of false position is inversely proportional to percentage error.

Numerical rate of convergence of the method of false position = 1 / (1000αβγ)

where α = Total number of iterations

β = Difference between the two guessed values

γ = Stopping tolerance

RESULT AND DISCUSSION

Calculation of fourth root of 1 by the method of false position

Method of false position has been applied to calculate the roots of equation

$$f(x) = x^4 - 1 = 0$$

in the interval [0, 3] using computer program. Initial value of interval, last value of interval, estimated value of root and value of function at estimated value of root in each iteration is given in Table-1. Estimated value of fourth root of 1 by the method of false position after each iteration is shown in Graph-1.

Table-1: Initial value of interval, last value of interval, estimated root and value of function at estimated root in the calculation of fourth root of 1 by the method of false position

Iteration	Initial value of interval	**Last value of interval**	**Estimated root by the method of false position**	**Value of function at estimated root**
1	0.000000000000	3.000000000000	0.000000000000	-1.000000000000
2	0.037037037313	3.000000000000	0.037037037313	-0.999998118324
3	0.073616757989	3.000000000000	0.073616757989	-0.999970629810
4	0.109743900597	3.000000000000	0.109743900597	-0.999854948719
5	0.145420968533	3.000000000000	0.145420968533	-0.999552793475
6	0.180647119880	3.000000000000	0.180647119880	-0.998935062384
7	0.215417340398	3.000000000000	0.215417340398	-0.997846610264
8	0.249721780419	3.000000000000	0.249721780419	-0.996111109718
9	0.283545404673	3.000000000000	0.283545404673	-0.993536162157
10	0.316867768764	3.000000000000	0.316867768764	-0.989918799250
11	0.349662989378	3.000000000000	0.349662989378	-0.985051463897
12	0.381900042295	3.000000000000	0.381900042295	-0.978728465234
13	0.413543015718	3.000000000000	0.413543015718	-0.970752903242
14	0.444551885128	3.000000000000	0.444551885128	-0.960943814493
15	0.474883079529	3.000000000000	0.474883079529	-0.949143463216
16	0.504490554333	3.000000000000	0.504490554333	-0.935224293705
17	0.533326685429	3.000000000000	0.533326685429	-0.919095391996

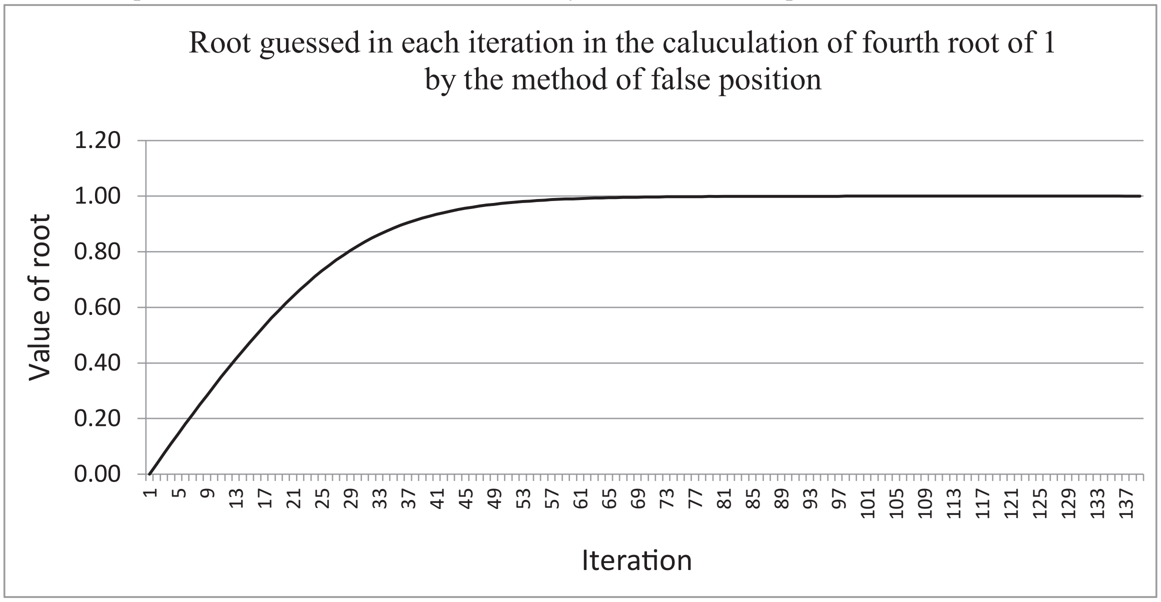
Iteration	Initial value of interval	**Last value of interval**	**Estimated root by the method of false position**	**Value of function at estimated root**
18	0.561343669891	3.000000000000	0.561343669891	-0.900707757704
19	0.588494420052	3.000000000000	0.588494420052	-0.880058521688
20	0.614734113216	3.000000000000	0.614734113216	-0.857193078958
21	0.640021085739	3.000000000000	0.640021085739	-0.832205728907
22	0.664318203926	3.000000000000	0.664318203926	-0.805237826154
23	0.687593638897	3.000000000000	0.687593638897	-0.776474332190
24	0.709821939468	3.000000000000	0.709821939468	-0.746138013411
25	0.730984449387	3.000000000000	0.730984449387	-0.714482618295
26	0.751069724560	3.000000000000	0.751069724560	-0.681784724083
27	0.770073831081	3.000000000000	0.770073831081	-0.648334745307
28	0.788000285625	3.000000000000	0.788000285625	-0.614427989833
29	0.804859757423	3.000000000000	0.804859757423	-0.580356158752
30	0.820669591427	3.000000000000	0.820669591427	-0.546399665038
31	0.835453450680	3.000000000000	0.835453450680	-0.512820474012
32	0.849240362644	3.000000000000	0.849240362644	-0.479857299148
33	0.862064182758	3.000000000000	0.862064182758	-0.447721158462
34	0.873962581158	3.000000000000	0.873962581158	-0.416593377618
35	0.884976387024	3.000000000000	0.884976387024	-0.386624066482
36	0.895148694515	3.000000000000	0.895148694515	-0.357932436373
37	0.904524207115	3.000000000000	0.904524207115	-0.330607601593
38	0.913148343563	3.000000000000	0.913148343563	-0.304711025584
39	0.921066761017	3.000000000000	0.921066761017	-0.280278560408
40	0.928324818611	3.000000000000	0.928324818611	-0.257323220321
41	0.934967100620	3.000000000000	0.934967100620	-0.235838211559
42	0.941036880016	3.000000000000	0.941036880016	-0.215800472144
43	0.946575999260	3.000000000000	0.946575999260	-0.197173028667
44	0.951624572277	3.000000000000	0.951624572277	-0.179907971618
45	0.956220746040	3.000000000000	0.956220746040	-0.163949237245
46	0.960400640965	3.000000000000	0.960400640965	-0.149234706242
47	0.964198291302	3.000000000000	0.964198291302	-0.135698174933
48	0.967645645142	3.000000000000	0.967645645142	-0.123270972673
49	0.970772445202	3.000000000000	0.970772445202	-0.111883860243
50	0.973606467247	3.000000000000	0.973606467247	-0.101467479208
51	0.976173400879	3.000000000000	0.976173400879	-0.091953939331
52	0.978496968746	3.000000000000	0.978496968746	-0.083277399418
53	0.980599105358	3.000000000000	0.980599105358	-0.075374278194
54	0.982499957085	3.000000000000	0.982499957085	-0.068184006517
55	0.984218001366	3.000000000000	0.984218001366	-0.061649226998
56	0.985770225525	3.000000000000	0.985770225525	-0.055715663358
57	0.987172067165	3.000000000000	0.987172067165	-0.050332812741
58	0.988437652588	3.000000000000	0.988437652588	-0.045453427493
59	0.989579916000	3.000000000000	0.989579916000	-0.041033380880
60	0.990610539913	3.000000000000	0.990610539913	-0.037032171983

Iteration	Initial value of interval	**Last value of interval**	**Estimated root by the method of false position**	**Value of function at estimated root**
61	0.991540253162	3.000000000000	0.991540253162	-0.033412000095
62	0.992378711700	3.000000000000	0.992378711700	-0.030138416313
63	0.993134737015	3.000000000000	0.993134737015	-0.027179552994
64	0.993816316128	3.000000000000	0.993816316128	-0.024506252155
65	0.994430661201	3.000000000000	0.994430661201	-0.022091940013
66	0.994984328747	3.000000000000	0.994984328747	-0.019912247347
67	0.995483279228	3.000000000000	0.995483279228	-0.017944846650
68	0.995932817459	3.000000000000	0.995932817459	-0.016169747164
69	0.996337831020	3.000000000000	0.996337831020	-0.014568403309
70	0.996702671051	3.000000000000	0.996702671051	-0.013124224808
71	0.997031271458	3.000000000000	0.997031271458	-0.011822138654
72	0.997327208519	3.000000000000	0.997327208519	-0.010648379363
73	0.997593760490	3.000000000000	0.997593760490	-0.009590273802
74	0.997833788395	3.000000000000	0.997833788395	-0.008636732222
75	0.998049914837	3.000000000000	0.998049914837	-0.007777553309
76	0.998244524002	3.000000000000	0.998244524002	-0.007003435446
77	0.998419761658	3.000000000000	0.998419761658	-0.006305986228
78	0.998577535152	3.000000000000	0.998577535152	-0.005677730462
79	0.998719573021	3.000000000000	0.998719573021	-0.005111879351
80	0.998847424984	3.000000000000	0.998847424984	-0.004602335612
81	0.998962521553	3.000000000000	0.998962521553	-0.004143460084
82	0.999066174030	3.000000000000	0.999066174030	-0.003730074950
83	0.999159455299	3.000000000000	0.999159455299	-0.003357942085
84	0.999243438244	3.000000000000	0.999243438244	-0.003022814442
85	0.999319016933	3.000000000000	0.999319016933	-0.002721151102
86	0.999387085438	3.000000000000	0.999387085438	-0.002449405184
87	0.999448359013	3.000000000000	0.999448359013	-0.002204738774
88	0.999503493309	3.000000000000	0.999503493309	-0.001984548140
89	0.999553143978	3.000000000000	0.999553143978	-0.001786226363
90	0.999597787857	3.000000000000	0.999597787857	-0.001607878184
91	0.999638020992	3.000000000000	0.999638020992	-0.001447130048
92	0.999674201012	3.000000000000	0.999674201012	-0.001302559222
93	0.999706745148	3.000000000000	0.999706745148	-0.001172503520
94	0.999736070633	3.000000000000	0.999736070633	-0.001055299590
95	0.999762475491	3.000000000000	0.999762475491	-0.000949759584
96	0.999786198139	3.000000000000	0.999786198139	-0.000854933215
97	0.999807596207	3.000000000000	0.999807596207	-0.000769393087
98	0.999826848507	3.000000000000	0.999826848507	-0.000692426104
99	0.999844133854	3.000000000000	0.999844133854	-0.000623318834
100	0.999859690666	3.000000000000	0.999859690666	-0.000561119226
101	0.999873697758	3.000000000000	0.999873697758	-0.000505113264
102	0.999886333942	3.000000000000	0.999886333942	-0.000454586716
103	0.999897718430	3.000000000000	0.999897718430	-0.000409063517

Iteration	Initial value of interval	**Last value of interval**	**Estimated root by the method of false position**	**Value of function at estimated root**
104	0.999907970428	3.000000000000	0.999907970428	-0.000368067473
105	0.999917149544	3.000000000000	0.999917149544	-0.000331360642
106	0.999925434589	3.000000000000	0.999925434589	-0.000298228284
107	0.999932885170	3.000000000000	0.999932885170	-0.000268432295
108	0.999939620495	3.000000000000	0.999939620495	-0.000241496147
109	0.999945640564	3.000000000000	0.999945640564	-0.000217420015
110	0.999951064587	3.000000000000	0.999951064587	-0.000195727286
111	0.999955952168	3.000000000000	0.999955952168	-0.000176179689
112	0.999960362911	3.000000000000	0.999960362911	-0.000158538929
113	0.999964296818	3.000000000000	0.999964296818	-0.000142805081
114	0.999967873096	3.000000000000	0.999967873096	-0.000128501421
115	0.999971091747	3.000000000000	0.999971091747	-0.000115627997
116	0.999973952770	3.000000000000	0.999973952770	-0.000104184848
117	0.999976575375	3.000000000000	0.999976575375	-0.000093695209
118	0.999978899956	3.000000000000	0.999978899956	-0.000084397506
119	0.999980986118	3.000000000000	0.999980986118	-0.000076053358
120	0.999982893467	3.000000000000	0.999982893467	-0.000068424376
121	0.999984622002	3.000000000000	0.999984622002	-0.000061510575
122	0.999986171722	3.000000000000	0.999986171722	-0.000055311963
123	0.999987542629	3.000000000000	0.999987542629	-0.000049828552
124	0.999988794327	3.000000000000	0.999988794327	-0.000044821939
125	0.999989926815	3.000000000000	0.999989926815	-0.000040292131
126	0.999990940094	3.000000000000	0.999990940094	-0.000036239132
127	0.999991834164	3.000000000000	0.999991834164	-0.000032662945
128	0.999992668629	3.000000000000	0.999992668629	-0.000029325163
129	0.999993383884	3.000000000000	0.999993383884	-0.000026464200
130	0.999994039536	3.000000000000	0.999994039536	-0.000023841645
131	0.999994635582	3.000000000000	0.999994635582	-0.000021457499
132	0.999995172024	3.000000000000	0.999995172024	-0.000019311765
133	0.999995648861	3.000000000000	0.999995648861	-0.000017404443
134	0.999996066093	3.000000000000	0.999996066093	-0.000015735533
135	0.999996483326	3.000000000000	0.999996483326	-0.000014066622
136	0.999996840954	3.000000000000	0.999996840954	-0.000012636125
137	0.999997138977	3.000000000000	0.999997138977	-0.000011444043
138	0.999997437000	3.000000000000	0.999997437000	-0.000010251959
139	0.999997675419	3.000000000000	0.999997675419	-0.000009298292

Actual value of fourth root of 1	1.000000000000
Calculated value of fourth root of 1 by the method of false position	0.999997675419
Error in the calculated value of fourth root of 1 by the method of false position	0.000002324581
Percentage error in the value of fourth root of 1 calculated by the method of false position	0.000232458100
Numerical rate of convergence of the method of false position in the calculation of fourth root of 1	0.239808153477

Graph-1: Estimated value of fourth root of 1 by the method of false position after each iteration



Calculation of fourth root of 2 by the method of false position

Method of false position has been applied to calculate the roots of equation

$f(x) = x^4 - 2 = 0$

in the interval [0, 3] using computer program. Initial value of interval, last value of interval, estimated value of root and value of function at estimated value of root in each iteration is included in Table-2. Estimated value of fourth root of 2 by the method of false position after each iteration is shown in Graph-2.

Table-2: Initial value of interval, last value of interval, estimated root and value of function at estimated root in the calculation of fourth root of 2 by the method of false position

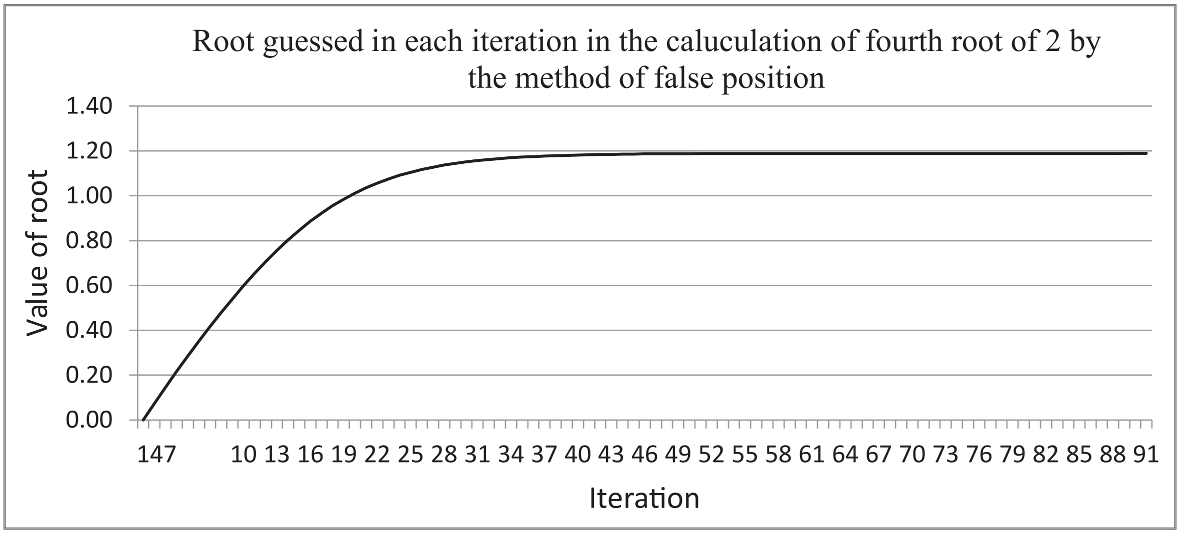
Iteration	Initial value of interval	Last value of interval	Estimated root by the method of false position	Value of function at estimated root
1	0.000000000000	3.000000000000	0.000000000000	-2.000000000000
2	0.074074074626	3.000000000000	0.074074074626	-1.999969893176
3	0.146318092942	3.000000000000	0.146318092942	-1.999541655390
4	0.216763630509	3.000000000000	0.216763630509	-1.997792271495
5	0.285411536694	3.000000000000	0.285411536694	-1.993364309899
6	0.352221488953	3.000000000000	0.352221488953	-1.984609122037
7	0.417107969522	3.000000000000	0.417107969522	-1.969731287294
8	0.479941368103	3.000000000000	0.479941368103	-1.946941772123
9	0.540553987026	3.000000000000	0.540553987026	-1.914619970625
10	0.598749935627	3.000000000000	0.598749935627	-1.871476684956
11	0.654318153858	3.000000000000	0.654318153858	-1.816702779534
12	0.707047462463	3.000000000000	0.707047462463	-1.750083878787
13	0.756742238998	3.000000000000	0.756742238998	-1.672061880132
14	0.803237438202	3.000000000000	0.803237438202	-1.583729370808
15	0.846410870552	3.000000000000	0.846410870552	-1.486754760700
16	0.886192083359	3.000000000000	0.886192083359	-1.383246725315
17	0.922566771507	3.000000000000	0.922566771507	-1.275578662936

Iteration	Initial value of interval	Last value of interval	Estimated root by the method of false position	Value of function at estimated root
18	0.955577194691	3.000000000000	0.955577194691	-1.166197666500
19	0.985317945480	3.000000000000	0.985317945480	-1.057447454877
20	1.011929035187	3.000000000000	1.011929035187	-0.951423237624
21	1.035587072372	3.000000000000	1.035587072372	-0.849871192793
22	1.056495070457	3.000000000000	1.056495070457	-0.754138113670
23	1.074872493744	3.000000000000	1.074872493744	-0.665164349150
24	1.090946316719	3.000000000000	1.090946316719	-0.583509972156
25	1.104943633080	3.000000000000	1.104943633080	-0.509402134451
26	1.117084860802	3.000000000000	1.117084860802	-0.442799034023
27	1.127579927444	3.000000000000	1.127579927444	-0.383449244838
28	1.136624336243	3.000000000000	1.136624336243	-0.330955938341
29	1.144398093224	3.000000000000	1.144398093224	-0.284824746461
30	1.151064157486	3.000000000000	1.151064157486	-0.244510956548
31	1.156769037247	3.000000000000	1.156769037247	-0.209449302460
32	1.161643028259	3.000000000000	1.161643028259	-0.179080439440
33	1.165800809860	3.000000000000	1.165800809860	-0.152870217783
34	1.169343233109	3.000000000000	1.169343233109	-0.130316794101
35	1.172358036041	3.000000000000	1.172358036041	-0.110960414648
36	1.174921512604	3.000000000000	1.174921512604	-0.094383858156
37	1.177099347115	3.000000000000	1.177099347115	-0.080215525026
38	1.178948402405	3.000000000000	1.178948402405	-0.068124240712
39	1.180517435074	3.000000000000	1.180517435074	-0.057819353010
40	1.181848168373	3.000000000000	1.181848168373	-0.049047274734
41	1.182976245880	3.000000000000	1.182976245880	-0.041587842996
42	1.183932304382	3.000000000000	1.183932304382	-0.035249160789
43	1.184742212296	3.000000000000	1.184742212296	-0.029867431346
44	1.185428261757	3.000000000000	1.185428261757	-0.025300082122
45	1.186009168625	3.000000000000	1.186009168625	-0.021426510836
46	1.186501026154	3.000000000000	1.186501026154	-0.018142280501
47	1.186917424202	3.000000000000	1.186917424202	-0.015358713717
48	1.187269806862	3.000000000000	1.187269806862	-0.013000791697
49	1.187568068504	3.000000000000	1.187568068504	-0.011003372055
50	1.187820434570	3.000000000000	1.187820434570	-0.009312133318
51	1.188034057617	3.000000000000	1.188034057617	-0.007879689393
52	1.188214778900	3.000000000000	1.188214778900	-0.006667263899
53	1.188367724419	3.000000000000	1.188367724419	-0.005640748548
54	1.188497066498	3.000000000000	1.188497066498	-0.004772341627
55	1.188606500626	3.000000000000	1.188606500626	-0.004037375884
56	1.188699126244	3.000000000000	1.188699126244	-0.003415138387
57	1.188777446747	3.000000000000	1.188777446747	-0.002888885838

Iteration	Initial value of interval	Last value of interval	Estimated root by the method of false position	Value of function at estimated root
58	1.188843727112	3.000000000000	1.188843727112	-0.002443452339
59	1.188899755478	3.000000000000	1.188899755478	-0.002066858716
60	1.188947081566	3.000000000000	1.188947081566	-0.001748715810
61	1.188987135887	3.000000000000	1.188987135887	-0.001479426639
62	1.189020991325	3.000000000000	1.189020991325	-0.001251791948
63	1.189049720764	3.000000000000	1.189049720764	-0.001058607821
64	1.189074039459	3.000000000000	1.189074039459	-0.000895071724
65	1.189094543457	3.000000000000	1.189094543457	-0.000757180552
66	1.189111948013	3.000000000000	1.189111948013	-0.000640127797
67	1.189126610756	3.000000000000	1.189126610756	-0.000541510870
68	1.189139008522	3.000000000000	1.189139008522	-0.000458124606
69	1.189149498940	3.000000000000	1.189149498940	-0.000387564961
70	1.189158439636	3.000000000000	1.189158439636	-0.000327427426
71	1.189165949821	3.000000000000	1.189165949821	-0.000276910848
72	1.189172267914	3.000000000000	1.189172267914	-0.000234412033
73	1.189177632332	3.000000000000	1.189177632332	-0.000198327602
74	1.189182162285	3.000000000000	1.189182162285	-0.000167855925
75	1.189185976982	3.000000000000	1.189185976982	-0.000142195295
76	1.189189195633	3.000000000000	1.189189195633	-0.000120543946
77	1.189191937447	3.000000000000	1.189191937447	-0.000102100066
78	1.189194321632	3.000000000000	1.189194321632	-0.000086061805
79	1.189196348190	3.000000000000	1.189196348190	-0.000072429208
80	1.189198017120	3.000000000000	1.189198017120	-0.000061202311
81	1.189199447632	3.000000000000	1.189199447632	-0.000051579219
82	1.189200639725	3.000000000000	1.189200639725	-0.000043559949
83	1.189201593399	3.000000000000	1.189201593399	-0.000037144515
84	1.189202427864	3.000000000000	1.189202427864	-0.000031530998
85	1.189203143120	3.000000000000	1.189203143120	-0.000026719403
86	1.189203739166	3.000000000000	1.189203739166	-0.000022709734
87	1.189204216003	3.000000000000	1.189204216003	-0.000019501994
88	1.189204692841	3.000000000000	1.189204692841	-0.000016294250
89	1.189205050468	3.000000000000	1.189205050468	-0.000013888440
90	1.189205408096	3.000000000000	1.189205408096	-0.000011482627
91	1.189205646515	3.000000000000	1.189205646515	-0.000009878751

Actual value of fourth root of 2	1.189207115003
Calculated value of fourth root of 2 by the method of false position	1.189205646515
Error in the calculated value of fourth root of 2 by the method of false position	0.000001468488
Percentage error in the value of fourth root of 2 calculated by Bisection method	0.000123484606
Numerical rate of convergence of the method of false position in the calculation of fourth root of 2	0.366300366300

Graph-2: Estimated value of fourth root of 2 by the method of false position after each iteration



Calculation of fourth root of 3 by the method of false position

Method of false position has been applied to calculate the roots of equation

$f(x) = x^4 - 3 = 0$

in the interval [0, 3] using computer program. Initial value of interval, last value of interval, estimated value of root and value of function at estimated value of root in each iteration is included in Table-3. Estimated value of fourth root of 3 by the method of false position after each iteration is shown in Graph-3. In the similar manner, the fourth roots of natural numbers from 4 to 30 have been calculated with the help of computer program.

Table-3: Initial value of interval, last value of interval, estimated root and value of function at estimated root in the calculation of fourth root of 3 by the method of false position

Iteration	Initial value of interval	Last value of interval	Estimated root by the method of false position	Value of function at estimated root
1	0.000000000000	3.000000000000	0.000000000000	-3.000000000000
2	0.111111111939	3.000000000000	0.111111111939	-2.999847584205
3	0.218101754785	3.000000000000	0.218101754785	-2.997737249658
4	0.321060180664	3.000000000000	0.321060180664	-2.989374587898
5	0.419941723347	3.000000000000	0.419941723347	-2.968900306809
6	0.514545142651	3.000000000000	0.514545142651	-2.929903887780
7	0.604526042938	3.000000000000	0.604526042938	-2.866445028223
8	0.689437568188	3.000000000000	0.689437568188	-2.774066942513
9	0.768790423870	3.000000000000	0.768790423870	-2.650673236385
10	0.842121601105	3.000000000000	0.842121601105	-2.497079625805
11	0.909060597420	3.000000000000	0.909060597420	-2.317077634508
12	0.969382345676	3.000000000000	0.969382345676	-2.116959902910
13	1.023038148880	3.000000000000	1.023038148880	-1.904613674384
14	1.070161223412	3.000000000000	1.070161223412	-1.688413788979
15	1.111050009727	3.000000000000	1.111050009727	-1.476177330952
16	1.146135091782	3.000000000000	1.146135091782	-1.274387655305
17	1.175937175751	3.000000000000	1.175937175751	-1.087786055028
18	1.201025605202	3.000000000000	1.201025605202	-0.919301923544
19	1.221981167793	3.000000000000	1.221981167793	-0.770240351686

Iteration	Initial value of interval	Last value of interval	Estimated root by the method of false position	Value of function at estimated root
20	1.239367246628	3.000000000000	1.239367246628	-0.640608238602
21	1.253709435463	3.000000000000	1.253709435463	-0.529484530879
22	1.265483736992	3.000000000000	1.265483736992	-0.435360816651
23	1.275111317635	3.000000000000	1.275111317635	-0.356420338782
24	1.282957315445	3.000000000000	1.282957315445	-0.290751573627
25	1.289333939552	3.000000000000	1.289333939552	-0.236486060537
26	1.294504761696	3.000000000000	1.294504761696	-0.191886812099
27	1.298690080643	3.000000000000	1.298690080643	-0.155394183885
28	1.302072763443	3.000000000000	1.302072763443	-0.125640943565
29	1.304803371429	3.000000000000	1.304803371429	-0.101453441151
30	1.307005405426	3.000000000000	1.307005405426	-0.081837079322
31	1.308779835701	3.000000000000	1.308779835701	-0.065957631446
32	1.310208797455	3.000000000000	1.310208797455	-0.053122758342
33	1.311358809471	3.000000000000	1.311358809471	-0.042762856216
34	1.312284111977	3.000000000000	1.312284111977	-0.034407442132
35	1.313028216362	3.000000000000	1.313028216362	-0.027675399815
36	1.313626527786	3.000000000000	1.313626527786	-0.022254064162
37	1.314107537270	3.000000000000	1.314107537270	-0.017890233484
38	1.314494132996	3.000000000000	1.314494132996	-0.014379471389
39	1.314804792404	3.000000000000	1.314804792404	-0.011556058368
40	1.315054416656	3.000000000000	1.315054416656	-0.009285909790
41	1.315254926682	3.000000000000	1.315254926682	-0.007461482411
42	1.315416097641	3.000000000000	1.315416097641	-0.005994393545
43	1.315545558929	3.000000000000	1.315545558929	-0.004815557283
44	1.315649509430	3.000000000000	1.315649509430	-0.003868762777
45	1.315733075142	3.000000000000	1.315733075142	-0.003107472803
46	1.315800189972	3.000000000000	1.315800189972	-0.002495946569
47	1.315854072571	3.000000000000	1.315854072571	-0.002004919950
48	1.315897345543	3.000000000000	1.315897345543	-0.001610534098
49	1.315932154655	3.000000000000	1.315932154655	-0.001293258839
50	1.315960049629	3.000000000000	1.315960049629	-0.001038985835
51	1.315982460976	3.000000000000	1.315982460976	-0.000834686407
52	1.316000461578	3.000000000000	1.316000461578	-0.000670587287
53	1.316014885902	3.000000000000	1.316014885902	-0.000539085781
54	1.316026568413	3.000000000000	1.316026568413	-0.000432577260
55	1.316035866737	3.000000000000	1.316035866737	-0.000347803144
56	1.316043376923	3.000000000000	1.316043376923	-0.000279330432
57	1.316049456596	3.000000000000	1.316049456596	-0.000223899282
58	1.316054344177	3.000000000000	1.316054344177	-0.000179336428
59	1.316058158875	3.000000000000	1.316058158875	-0.000144555318
60	1.316061258316	3.000000000000	1.316061258316	-0.000116295445
61	1.316063761711	3.000000000000	1.316063761711	-0.000093470016
62	1.316065788269	3.000000000000	1.316065788269	-0.000074992193

Iteration	Initial value of interval	Last value of interval	Estimated root by the method of false position	Value of function at estimated root
63	1.316067457199	3.000000000000	1.316067457199	-0.000059775098
64	1.316068768501	3.000000000000	1.316068768501	-0.000047818768
65	1.316069841385	3.000000000000	1.316069841385	-0.000038036290
66	1.316070675850	3.000000000000	1.316070675850	-0.000030427680
67	1.316071391106	3.000000000000	1.316071391106	-0.000023906002
68	1.316071867943	3.000000000000	1.316071867943	-0.000019558211
69	1.316072344780	3.000000000000	1.316072344780	-0.000015210415
70	1.316072702408	3.000000000000	1.316072702408	-0.000011949565
71	1.316072940826	3.000000000000	1.316072940826	-0.000009775664

Actual value of fourth root of 3	1.316074012952
Calculated value of fourth root of 3 by the method of false position	1.316072940826
Error in the calculated value of fourth root of 3 by the method of false position	0.000001072126
Percentage error in the value of fourth root of 3 calculated by Bisection method	0.000081463997
Numerical rate of convergence of the method of false position in the calculation of fourth root of 3	0.469483568075

Graph-3: Estimated value of fourth root of 3 by the method of false position after each iteration

Consolidated analysis of the fourth roots of numbers from 1 to 30 calculated by the method of false position

The value of fourth root, error in the determination of fourth root, percentage error and numerical rate of convergence in the method of false position are shown in Table-4 (a) and Table-4(b). The actual value of fourth root and the value of fourth root calculated by the method of false position are shown in Graph-4. Error in the value of fourth root calculated by the method of false position is given in Graph-5. Percentage error in the values of fourth root calculated by the method of false position is given in Graph-6. Numerical rate of convergence in the determination of the fourth roots by the method of false position are given in Graph-7.

Table-4(a): Actual value of fourth root, value of fourth root calculated by the method of false position and error in the determination of fourth root by the method of false position in finding the roots of equations

$f(x) = x^4 - n = 0; n = 1, 2, \ldots, 30$

S. No.	Function	No. of Iterations	Actual value of fourth root	Value of fourth root calculated by the method of false position	Error in the fourth root calculated by the method of false position
1	$f(x)=x^4-1$	139	1.000000000000	0.999997675419	0.000002324581
2	$f(x)=x^4-2$	91	1.189207115003	1.189205646515	0.000001468488
3	$f(x)=x^3-3$	71	1.316074012952	1.316072940826	0.000001072126
4	$f(x)=x^3-4$	60	1.414213562373	1.414212703705	0.000000858668
5	$f(x)=x^3-5$	53	1.495348781221	1.495348215103	0.000000566118
6	$f(x)=x^3-6$	48	1.565084580073	1.565084099770	0.000000480303
7	$f(x)=x^3-7$	43	1.626576561698	1.626576066017	0.000000495681
8	$f(x)=x^3-8$	40	1.681792830507	1.681792378426	0.000000452081
9	$f(x)=x^3-9$	38	1.732050807569	1.732050538063	0.000000269506

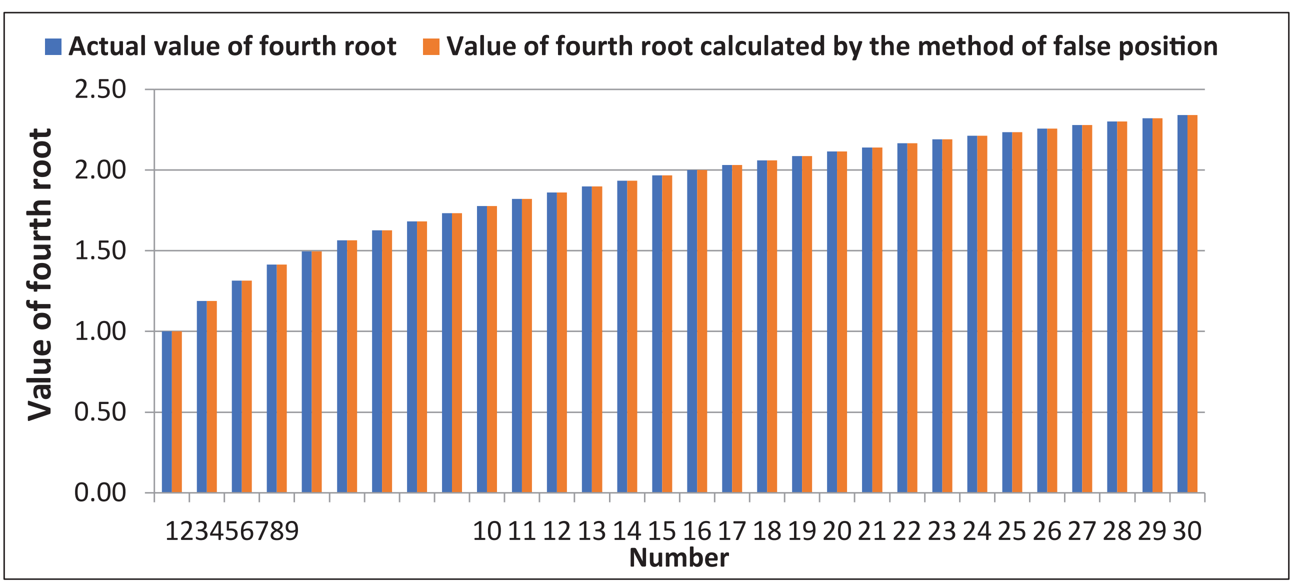
10	*f(x)=x³-10*	35	1.778279410039	1.778279185295	0.000000224744
11	*f(x)=x³-11*	33	1.821160286838	1.821159958839	0.000000327999
12	*f(x)=x³-12*	31	1.861209718204	1.861209392548	0.000000325656
13	*f(x)=x³-13*	30	1.898828922116	1.898828744888	0.000000177228
14	*f(x)=x³-14*	29	1.934336420268	1.934336185455	0.000000234813
15	*f(x)=x³-15*	28	1.967989671265	1.967989444733	0.000000226532
16	*f(x)=x³-16*	26	2.000000000000	1.999999761581	0.000000238419
17	*f(x)=x³-17*	26	2.030543184869	2.030543088913	0.000000095956
18	*f(x)=x³-18*	25	2.059767143907	2.059767007828	0.000000136079
19	*f(x)=x³-19*	23	2.087797629930	2.087797403336	0.000000226594
20	*f(x)=x³-20*	22	2.114742526881	2.114742279053	0.000000247828
21	*f(x)=x³-21*	23	2.140695142928	2.140695095062	0.000000047866
22	*f(x)=x³-22*	22	2.165736770668	2.165736675262	0.000000095406
23	*f(x)=x³-23*	21	2.189938703095	2.189938545227	0.000000157868
24	*f(x)=x³-24*	20	2.213363839401	2.213363647461	0.000000191940
25	*f(x)=x³-25*	19	2.236067977500	2.236067771912	0.000000205588
26	*f(x)=x³-25*	19	2.258100864353	2.258100748062	0.000000116291
27	*f(x)=x³-25*	18	2.279507056955	2.279506921768	0.000000135187
28	*f(x)=x³-25*	18	2.300326633791	2.300326347351	0.000000286440
29	*f(x)=x³-25*	18	2.320595787106	2.320595741272	0.000000045834
30	*f(x)=x³-25*	18	2.340347319321	2.340347290039	0.000000029282
Average value					0.000000392037
Minimum value					0.000000029282
Maximum value					0.000002324581

Table-4(b): Actual value of fourth root, percentage error in the calculation of fourth root and numerical rate of convergence of the method of false position in the determination of roots of equations ***f(x) = x⁴ – n = 0; n = 1, 2, …., 30***

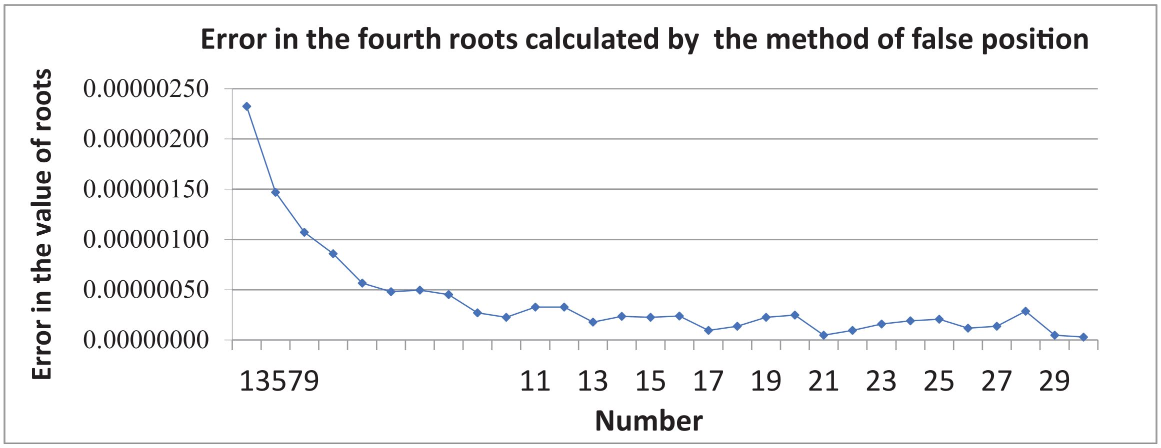
S. No.	**Function**	**No. of Iterations**	**Actual value of fourth root**	**Percentage error in the fourth root calculated by the method of false position**	**Numerical rate of convergence of the method of false position**
1	*f(x)=x⁴-1*	139	1.000000000000	0.000232458100	0.239808153477
2	*f(x)=x⁴-2*	91	1.189207115003	0.000123484606	0.366300366300
3	*f(x)=x³-3*	71	1.316074012952	0.000081463997	0.469483568075
4	*f(x)=x³-4*	60	1.414213562373	0.000060717003	0.555555555556
5	*f(x)=x³-5*	53	1.495348781221	0.000037858607	0.628930817610
6	*f(x)=x³-6*	48	1.565084580073	0.000030688647	0.694444444444
7	*f(x)=x³-7*	43	1.626576561698	0.000030473867	0.775193798450
8	*f(x)=x³-8*	40	1.681792830507	0.000026880923	0.833333333333
9	*f(x)=x³-9*	38	1.732050807569	0.000015559929	0.877192982456

10	*f(x)=x³-10*	35	1.778279410039	0.000012638280	0.952380952381
11	*f(x)=x³-11*	33	1.821160286838	0.000018010434	1.010101010101
12	*f(x)=x³-12*	31	1.861209718204	0.000017497018	1.075268817204
13	*f(x)=x³-13*	30	1.898828922116	0.000009333539	1.111111111111
14	*f(x)=x³-14*	29	1.934336420268	0.000012139185	1.149425287356
15	*f(x)=x³-15*	28	1.967989671265	0.000011510855	1.190476190476
16	*f(x)=x³-16*	26	2.000000000000	0.000011920950	1.282051282051
17	*f(x)=x³-17*	26	2.030543184869	0.000004725629	1.282051282051
18	*f(x)=x³-18*	25	2.059767143907	0.000006606529	1.333333333333
19	*f(x)=x³-19*	23	2.087797629930	0.000010853248	1.449275362319
20	*f(x)=x³-20*	22	2.114742526881	0.000011719069	1.515151515152
21	*f(x)=x³-21*	23	2.140695142928	0.000002236006	1.449275362319
22	*f(x)=x³-22*	22	2.165736770668	0.000004405244	1.515151515152
23	*f(x)=x³-23*	21	2.189938703095	0.000007208779	1.587301587302
24	*f(x)=x³-24*	20	2.213363839401	0.000008671852	1.666666666667
25	*f(x)=x³-25*	19	2.236067977500	0.000009194165	1.754385964912
26	*f(x)=x³-25*	19	2.258100864353	0.000005149957	1.754385964912
27	*f(x)=x³-25*	18	2.279507056955	0.000005930527	1.851851851852
28	*f(x)=x³-25*	18	2.300326633791	0.000012452154	1.851851851852
29	*f(x)=x³-25*	18	2.320595787106	0.000001975100	1.851851851852
30	*f(x)=x³-25*	18	2.340347319321	0.000001251170	1.851851851852
Average values				0.000027500512	1.197514787730
Minimum values				0.000001251170	0.239808153477
Maximum values				0.000232458100	1.851851851852

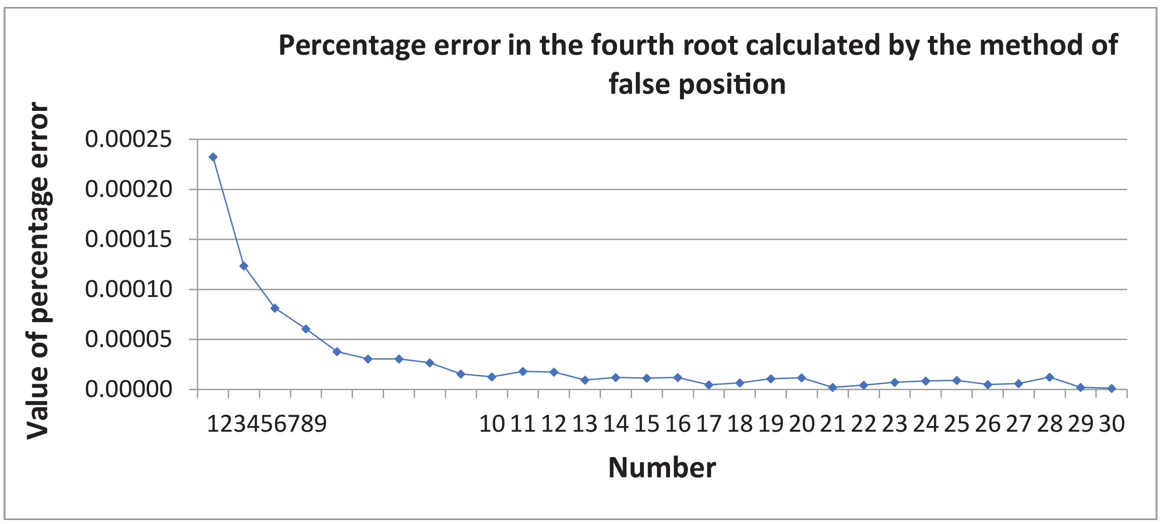
Graph-4: Actual value of fourth root and the value of root calculated by the method of false position in the equations f(x) = x⁴ – n=0; n=1, 2, …., 30



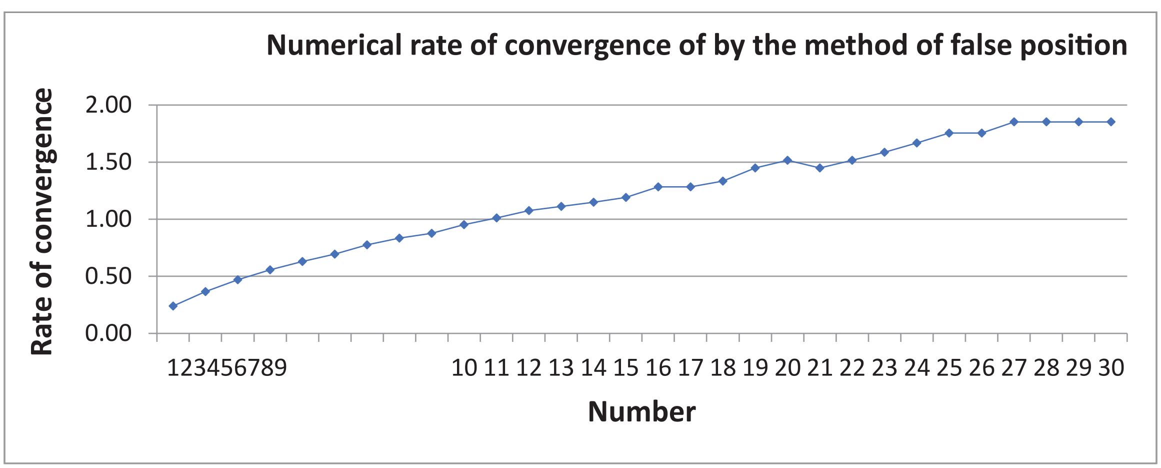
Graph-5: Error in the value of fourth root calculated by the method of false position in the equations $f(x) = x^4 - n = 0$; n=1, 2, …., 30



Graph-6: Percentage error in the value of fourth root calculated by the method of false position in the equations $f(x) = x^4 - n = 0$; n=1, 2, …., 30



Graph-7: Numerical rate of convergence in the determination of the fourth root by the method of false position in the equations $f(x) = x^4 - n = 0$; n=1, 2, …., 30



Declaration: *We also declare that all ethical guidelines have been followed during this work and there is no conflict of interest among authors.*

CONCLUSIONS

Fourth roots of the natural numbers from 1 to 30 have been found by the method of false position and these values have been compared with the actual values. The minimum error 0.000000029282 and minimum percentage error 0.000001251170 has been obtained in the determination of fourth roots of 30. The maximum error 0.000002324581 and maximum percentage error 0.000232458100 has been obtained in the determination of fourth roots of 1. The average value in the error is 0.000000392037 and the average value of percentage error is 0.000027500512. The normal trend in the fourth roots as obtained by the method of false position is that the error and percentage error in the roots increase as the number increase. Generally, numerical rate of convergence in the determination of the fourth root of numbers from 1 to 30 by the method of false position increases as the number increases. Minimum, Maximum and average values of the numerical rate of convergence are 0.239808153477, 1.851851851852 and 1.197514787730 respectively.

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