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Effect of Suspended Particles on a Couple-Stress Rivlin-Ericksen Ferromagnetic Fluid Heated from Below in a Porous Medium, with Varying Gravity and Magnetic Field.

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ABSTRACT

In this paper we have investigated the effect of suspended particles on a couple-stress Rivlin-Ericksen ferromagnetic fluid heated from below in saturating a porous medium, varying gravity and magnetic field. The dispersion relation has been obtained and clarified analytically using the normal mode principle and linear stability theory. For stationary conventions, it is found that suspended particles always have a destabilizing effect for $\lambda > 0$ and a stabilizing effect for $\lambda < 0$. In the absence of a magnetic field, oscillatory modes are not allowed and the principle of exchange of stability is satisfied. Oscillatory modes and stability in the system are introduced due to the presence of a magnetic field.

Keywords: Thermal instability, Couple-stress fluid, Rivlin-Ericksen fluid, Ferromagnetic fluid, Magnetic field, Suspended particles.

INTRODUCTION

The concept of couple-stress fluid has been identified and developed mathematically (Stokes, 1966, 1984). Convection on a bottom heating fluid layer in different hydrodyanamic's conditions theoretically as well as experimentally have been explained (Chandrasekhar, 1981). A bottom heating couple-stress fluid in porous medium has been examined by (Sharma and Sharma, 2001) . On applying magnetic field and rotation on a bottom heating couple stress fluid, (Kumar and Kumar, 2010) have investigated the effect of dust particle and noted that dust particles have a destabilizing effect whereas there is stabilizing effect due to rotation. Thermal convection in a couple stress fluid on applying horizontal magnetic field and Hall currents has been studied by (Jaimala vishnoi and vikrant jawla, 2013) and found magnetic field has stabilizing effect whereas Hall currents are found to have destabilizing effect on the system. A dusty couple-stress fluid in presence of rotation and magnetic field has been studied (Stanly and Vasanthakumari, 2017) and noted the effects on thermal instability of the system.

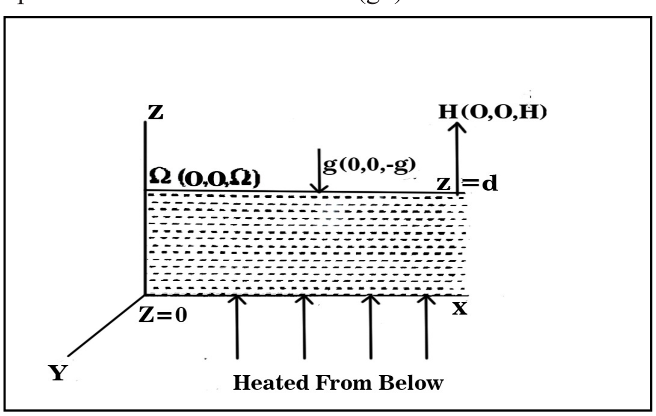
A thorough discussion on stability of ferrofluid has been provided (Rosenweig, 1985) in his monograph, and emphasised the relevance of magnetic fluid research. The thermal instability of ferrofluids in a uniform perpendicular magnetic field was studied (Finlayson, 1970). A horizontal ferrofluid layer has been considered (Stiles and Kagan, 1990) and analysed thermo convective instability in magnetic field. Effect of magnetic field and dust particle on convection of ferromagnetic fluid has been discussed (Sunil et al., 2005, 2008) respectively . The influence of magnetic field and suspended particles on thermal convection in a ferromagnetic fluid through a porous material with varying gravity field has been studied (Pant et al., 2016) and found magnetic field has stabilizing effect for $\lambda > 0$ and has destabilizing effect for $\lambda < 0$. The influence of rotation with a magnetic field on the occurrence of thermal convection in ferromagnetic liquids in a porous material has been discussed (Bhagat et al., 2016) and noted that rotation has stabilizing effect on the system for $\lambda > 0$ and destabilizing effect for $\lambda < 0$ and magnetic field has stabilizing effect under certain conditions.

The effects of dust particles on thermal convection of couple-stress ferromagnetic fluids in rotation and horizontal magnetic field have been examined (Pulkit et al., 2020, 2020). In presence of different fields like rotation, magnetic field etc. (Rahul and Naveen Sharma, 2021, 2021, 2021) studied and noted the behavior of thermal instability of couple stress Rivlin-Ericksen ferromagnetic fluid. On studying bottom heating couple stress rivlin ericksen ferromagnetic fluid with unstable gravity and suspended particles in porous material (Rahul et al., 2021) viewed that no oscillatory modes are exist and the system is not stable. A bottom heating rotating couple stress rivlin ericksen ferromagnetic fluid with varying gravity and suspended particles through porous material has been examined (Rahul and Naveen Sharma, 2021) and found that due to applying rotation field the system is stable and oscillatory modes are exist.

As for growing use of couple-stress Rivlin-Ericksen ferromagnetic fluid in magnetic field and dust in the environment, the present paper attempts to study the effect of suspended particles on thermal instability of couple-stress Rivlin-Ericksen ferromagnetic fluid with magnetic field and variable gravity in a porous medium.

MATHEMATICAL FORMULATION

Consider a thin layer of infinite in compressible, electrically non conducting couple-stress Rivlin-Ericksen ferromagnetic fluid bounded between two infinite horizontal planes situated at $z = 0$ & $z = d$ permitted with suspended particles. A uniform magnetic field H (0, 0, H), variable gravity field g (0, 0, $-g$) applied along the vertical direction which is taken as Z axis. where $g = \lambda$ go, go is the value of g at $z = 0$, which is always positive & λ can be positive or negative as gravity increases or decreases upwards from its value at $z = 0$ (go).



The considered fluid layer is heated from below so that the uniform temperature gradient $\beta = \left(\left|\frac{dT}{dz}\right|\right)$ is maintained.

A ferromagnetic fluid reacts so quickly to a magnetic torque that we can simulate the following conditions to maintain,

$$M \times H = 0 \quad \ldots(1)$$

Where M represents magnetization and H represents the magnetic field intensity.

The Maxwell's equation is also satisfied by ferromagnetic fluid. Here we examine electrically non conducting fluid, therefore the displacement current is negligible. Maxwell's equation becomes,

$$\nabla \cdot I = 0, \nabla \times H = 0 \quad \ldots(2)$$

The magnetic field intensity H, magnetization M and magnetic induction I are correlated in Chu formulation of electro dynamics such that,

$$I = \mu_0 (H + M) \quad \ldots(3)$$

Let us assume that magnetization is depend on the amplitude of magnetic field and temperature. Let us also assume that magnetization is aligned with magnetic field so that,

$$m = \frac{H}{H} M(H,T) \quad \ldots(4)$$

Let us take Pressure, Density, Temperature, Coefficient of thermal expansion, Kinematic viscosity, kinematic viscous elasticity, couple-stress viscosity, medium porosity, medium permeability, magnetic permeability, thermal diffusivity, resistivity, velocity of fluid, velocity of suspended particles, and number density of suspended particles are denoted by p, ρ, T, α, v, v', μ', ε, K_l, μ_e, κ_T, η, $q = (u1,u2,u3)$, $q_s(x,t)$ and $N_s(x,t)$ respectively.

$K = 6\pi\mu\eta'$ is the Stokes drag coefficient. η' is the radius of suspended particle.

In presence of magnetic field and suspended particles the equation of motion, continuity and heat conduction of couple-stress Rivlin-Ericksen ferromagnetic fluid through porous medium are

$$\frac{1}{\varepsilon}\left[\frac{\partial q}{\partial t}+\frac{1}{\varepsilon}(q.\nabla)q\right] = -\frac{1}{\rho_0}\nabla p + g\left(1+\frac{\delta\rho}{\rho_0}\right)+\frac{1}{\rho_0}M\nabla H$$

$$+\left(v-\frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2 q-\frac{1}{K_1}\left(v+v'\frac{\partial q}{\partial t}\right)+\frac{KN_S}{\varepsilon\rho_0}(q_S-q)$$

$$+\frac{\mu_e}{4\pi\rho_0}\left[(\nabla\times H)\times H\right] \quad \ldots(5)$$

$$\nabla \cdot q = 0 \quad \ldots(6)$$

$$\frac{\partial T}{\partial t}+\left(q\cdot\nabla\right)T=\kappa_T\nabla^2 T \qquad \ldots(7)$$

The Maxwell's equation of electromagnetism is,

$$\varepsilon\frac{\partial H}{\partial t}=\left(H\cdot\nabla\right)q+\varepsilon\eta\nabla^2 H \qquad \ldots(8)$$

$$\nabla\cdot H=0 \qquad \ldots(9)$$

The equation of state is

$$\rho=\rho_0\left[1-\alpha\left(T-T_0\right)\right] \qquad \ldots(10)$$

Where T_0 and ρ_0 represents the value of temperature and density at $z = 0$ respectively.

∇ H denotes the magnetic field gradient.

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

$$H = |H|,\ I = |I|,\ M = |M|$$

Due to suspended particles in fluid there is a force get generated which is proportional to the difference of velocity of fluid and velocity of suspended particles. Since the force applied by the suspended particles on fluid and the force applied by fluid on suspended particles are equal. So there must be an another force, equal in magnitude but opposite in sign in equation of motion for the suspended particles. The distance between particles are taken to be relatively large with their diameter so inter particles reactions are not counted.

The equations of motion, continuity for suspended particles for above assumptions, are

$$mN_s\left[\frac{\partial q_s}{\partial t}+\frac{1}{\varepsilon}\left(q_s\cdot\nabla\right)q_s\right]=KN_S\left(q-q_S\right) \qquad \ldots(11)$$

$$\varepsilon\frac{\partial N_S}{\partial t}+\nabla\cdot\left(N_S\cdot q_S\right)=0 \qquad \ldots(12)$$

Where $m\,N_s$ is density of suspended particles.

Let C_V, C_S, ρ_p and C_P denote the capacity of the fluid at constant volume, the heat capacity of suspended particles, density of porous material and heat capacity of porous material respectively. If there is thermal equilibrium, then the equation of heat conduction is,

$$E\cdot\frac{\partial T}{\partial t}+\left(q\cdot\nabla\right)T+\frac{mN_SC_S}{\rho_0C_V}\left(\varepsilon\frac{\partial}{\partial t}+q_S\cdot\nabla\right)T=\kappa_T\nabla^2 T \qquad \ldots(13)$$

Where $E=\varepsilon+\left(1+\varepsilon\right)\frac{\rho_PC_P}{\rho_0C_V}$ is a constant.

$\kappa_T=\frac{q'}{\rho_0C_V}$ where q' is effective thermal conductivity of pure fluids.

In general, to complete the system a state equation is needed, which satisfy M in two thermodynamic variables H and T. In present paper we consider magnetization is independent of magnetic field and depend on temperature only. Therefore,

$$M = M(T)$$

As the first approximation, we consider that

$$M = M_0\left[1-\gamma(T-T_0)\right] \qquad \ldots(14)$$

Where M_0 is the magnetization at $T = T_0$ *and*

$$\gamma=\frac{1}{M_0}\left(\frac{\partial M}{\partial T}\right)_H$$

BASIC STATE PERTURBATION EQUATIONS AND DISPERSION RELATION

In the basic state is supposed to be a rest state of fluid and is given by

$$q = (0,0,0),\ p = p(z),\ \rho = \rho(z) = \rho_0(1 + \alpha\beta z),\ M = M(z) = M_0(1 + \gamma\beta z)$$

$$T = T(z) = T_0 - \beta z, H = (0,0,H),\ q_S = (0,0,0),\ N_S = N_0 \qquad \ldots(15)$$

To study the character of equilibrium let us apply a little perturbation on the layer of fluid due to which some disturbances takes place in the system. Now we consider that these small disturbances are the function of time and space variable. The perturbed flow may be represented as

$$q = (0,0,0) + (u_1, u_2, u_3),\ T = T(z) + \theta,\ \rho = \rho(z) + \delta\rho,\ p = p(z) + \delta p,\ H = (0,0,H) + (h_x, h_y, h_z),\ q_S = (0,0,0) + (l, r, s)\ \ M = M(z) + M \qquad \ldots(16)$$

Where the perturbation in fluid velocity q, velocity of suspended particles q_S, density ρ, pressure p, magnetic field, temperature T and magnetization M are $q\,(u_1, u_2, u_3)$, $q_S(l,r,s)$ $\delta\rho$, δp, (h_x, h_y, h_z), θ, δM respectively.

To decompose the disturbances we use Normal Mode method. Let us take perturbation is in the following form,

$$(u3, \theta, \zeta, \xi, hz) = [W(z), \Theta(z), Z(z), X(z), V(z)]\cdot exp\,(i\,k_x x + i\,k_y y + nt) \qquad \ldots(17)$$

Where k_x and k_y are wave number in X and Y direction and k is the resultant disturbances wave number such that,

$$K=\sqrt{\left[\left(k_x\right)^2+\left(k_y\right)^2\right]}$$

n is the frequency of any arbitrary disturbances which is a complex constant.

Now by using relation (17) we obtain

$$\left(D^2-a^2\right)\left[\begin{matrix}\left(\frac{\sigma}{\varepsilon}+\frac{M\sigma}{\varepsilon\left(1+\tau_1\sigma\right)}+\frac{\left(1+\sigma F_1\right)}{P_t}\right)\\ +F\left(D^2-a^2\right)^2\\ -\left(D^2-a^2\right)\end{matrix}\right]W+\frac{\alpha a^2\lambda d^2}{v}$$

$$\left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)\theta-\frac{\mu_e Hd}{4\pi\rho_0 v}\left(D^2-a^2\right)DV=0 \qquad \ldots(18)$$

$$\left[\begin{matrix}\left(\frac{\sigma}{\varepsilon}+\frac{M\sigma}{\varepsilon\left(1+\tau_1\sigma\right)}+\frac{\left(1+\sigma F_1\right)}{P_t}\right)\\ +F\left(D^2-a^a\right)^2\\ -\left(D^2-a^2\right)\end{matrix}\right]Z=\frac{\mu_e Hd}{4\pi\rho_0 v}DX \qquad \ldots(19)$$

$$\varepsilon\left[\left(D^2-a^2\right)-\sigma P_2\right]X=-\frac{Hd}{\eta}DZ \qquad \ldots(20)$$

$$\varepsilon\left[\left(D^2-a^2\right)-\sigma P_2\right]V=-\frac{Hd}{\eta}DW \qquad \ldots(21)$$

$$\left[\left(D^2-a^2\right)-\sigma E_1P_1\right]\Theta=-\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\frac{\beta d^2}{\kappa_T}W \qquad \ldots(22)$$

Where we have represent the coordinates *x*, *y* and *z* in new units of length d, time t and (d^2/κ_T). Let $a=kd, \sigma=nd^2/v$,

$$F=\frac{\mu'}{\rho_0 d^2 v}, F_1=\frac{v'}{d^2}, P_t=K_1/d^2, P_1=v/\kappa_T, P_2=v/\eta,$$

$$\tau=\frac{m}{\kappa}, \tau_1=\frac{\tau v}{d^2}E_1=\left(E+b\varepsilon\right), B=\left(1+b\right)$$ $x^*=x/d$, $y^*=y/d$, $z^*=z/d$, $D^*=dD$ and dropping * for convenience.

Where P1 and Pt denotes prandlt number and dimensionless medium permeability.

Now eliminate Θ, X, V and Z from (18) with the help of (19), (20), (21) and (22), then we get Stability governing equation

$$\left(D^2-a^2\right)\left[\begin{matrix}\left(\frac{\sigma}{\varepsilon}+\frac{M\sigma}{\varepsilon\left(1+\tau_1\sigma\right)}+\frac{\left(1+\sigma F_1\right)}{P_t}\right)\\ +F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\end{matrix}\right]W$$

$$-\lambda a^2R_f\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\left(\frac{1}{\left(D^2-a^2\right)-\sigma E_1P_1}\right)W$$

$$+\frac{Q\left(D^2-a^2\right)}{\varepsilon\left(\left(D^2-a^2\right)-\sigma P_2\right)}D^2W=0 \qquad \ldots(23)$$

Where

$$R_f=\frac{\alpha\beta d^4}{v\kappa_T}\left[g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right]=\frac{\alpha\beta d^4 g_0}{v\kappa_T}\left[1-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda g_0}\right]$$

$$=R\left[1-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda g_0}\right]$$

R_f is the Rayleigh number for ferromagnetic fluid. R is the Rayleigh number for fluid.

$T_A=\left(\frac{2\Omega d^2}{v}\right)^2$ is the modified Taylor number.

$Q=\frac{\mu_e H^2 d^2}{4\pi\rho_0 v\eta}$ is the Chandrasekhar number

The perturbation in the temperature on boundaries is zero, because the boundaries are kept at constant temperature. So the appropriate condition on boundary is $W=0, Z=0, \Theta=0$ at $z=0$ & $z=1$, $DZ=D^2W=D^4W=0$ at $z=0$ & $z=1$. …(24)

From (24), it is clear that all even order derivative of W vanish on boundaries. Therefore proper solution of (23) characterizing the lowest mode is,

$W=W_0\ \sin\pi z$, W_0 is a constant ...(25)

By using (25) with (23), we get,

$$R_1=\frac{1}{\lambda x}\left(1+x\right)\left(1+x+i\sigma_1E_1P_1\right)\left(\frac{1+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right)$$

$$\left[\left(\frac{i\sigma_1}{\varepsilon}+\frac{Mi\sigma_i}{\varepsilon\left(1+i\sigma_1\pi^2\tau_1\right)}\right)+\frac{\left(1+i\sigma_1F_3\right)}{P}+F_2\left(1+x\right)^2+\left(1+x\right)\right]$$

$$+\frac{1}{\lambda x}\frac{Q_1\left(1+x\right)\left(1+x+i\sigma_1E_1P_1\right)}{\varepsilon\left(1+x+i\sigma_1P_2\right)}\left(\frac{1+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right)$$

…(26)

Where

$$x=\frac{a^2}{\pi^2},i\sigma_1=\frac{\sigma}{\pi^2},F_2=\pi^2F,P=\pi^2P_t,$$

$$F_3=\pi^2F_1,R_1=\frac{R_f}{\pi^4},T_{A1}=\frac{T_A}{\pi^4},Q_1=\frac{Q}{\pi^2}$$

STATIONARY CONVECTION

At stationary convection, when stability sets, the marginal state will be characterized by Put in (26), we get,

$$R_1=\frac{(1+x)^2}{B\lambda x}\left[\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon(1+x)}\right] \quad \dots(27)$$

Clearly (27) shows the modified Rayleigh number R_1 as a function of B, F_2, P, Q_1 and x parameters. Clearly viscoelastic parameter F_3 disappears with σ_1. It shows that for stationary convection Rivlin-Ericksen fluid behaves like an ordinary Newtonian Fluid.

To examine the effect of suspended particles, couple-stress, permeability and magnetic field we have to study the behaviour of dR_1/dB, dR_1/dF_2, dR_1/dP and dR_1/dQ_1.

$$\frac{dR_1}{dB}=-\frac{(1+x)^2}{B^2\lambda x}\left[\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon(1+x)}\right] \quad \dots(28)$$

(28) shows that suspended particles has a stabilizing effect on thermal instability of couple-stress Rivlin-Ericksen ferromagnetic fluid in presence of gravity field and magnetic field through porous medium if $\lambda < 0$ (when gravity decreases upwards from its value at z = 0) and a destabilizing effect if $\lambda > 0$ (when gravity increases upwards from its value at z = 0).

$$\frac{dR_1}{dF_2}=\frac{(1+x)^4}{B\lambda x} \quad \dots(29)$$

(29) shows that couple-stress has stabilizing effect on thermal instability of couple-stress Rivlin-Ericksen ferromagnetic fluid in presence of gravity field, suspended particles and magnetic field through porous medium if $\lambda > 0$, (when gravity increases upwards from its value at z = 0) and destabilizing effect if $\lambda < 0$ (when gravity decreases upwards from its value at z = 0)

$$\frac{dR_1}{dP}=-\frac{(1+x)^2}{BP^2\lambda x} \quad \dots(30)$$

(30) shows that permeability has a stabilizing effect on thermal instability of couple-stress Rivlin-Ericksen ferromagnetic fluid in presence of gravity field, suspended particles and magnetic field through porous medium if $\lambda < 0$, (when gravity decreases upwards from its value at z = 0) and a destabilizing effect if $\lambda > 0$ (when gravity increases upwards from its value at z = 0).

$$\frac{dR_1}{dQ_1}=\frac{(1+x)}{B\varepsilon\lambda x} \quad \dots(31)$$

(31) shows that magnetic field has stabilizing effect on thermal instability of couple-stress Rivlin-Ericksen ferromagnetic fluid in presence of gravity field and suspended particles through porous medium if $\lambda > 0$, (when gravity increases upwards from its value at z = 0) and destabilizing effect if $\lambda < 0$ (when gravity decreases upwards from its value at z = 0).

Replace R_1 by R_f/π^4 and R_f by $R\left[1-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda g_0}\right]$

$$R=\frac{\frac{\pi^2(1+x)^2}{B\lambda x}\left[\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}+\frac{Q_1}{\varepsilon(1+x)}\right]}{\left[1-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda g_0}\right]} \quad \dots(32)$$

From (32)

$$\frac{dR}{dM_0}=\frac{\frac{\pi^4(1+x)^2}{B\lambda x}\left[\begin{array}{c}\left\{F_2(1+x)^2+(1+x)+\frac{1}{P}\right\}\\+\frac{Q1}{\varepsilon(1+x)}\end{array}\right]\left(\frac{\gamma\nabla H}{\rho_0\alpha\lambda g_0}\right)}{\left[1-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda g_0}\right]^2} \quad \dots(33)$$

(33) shows that magnetization has stabilizing effect on thermal instability of couple-stress Rivlin-Ericksen ferromagnetic fluid in presence of gravity field, suspended particles and magnetic field through porous medium for each value of λ either $\lambda > 0$, or $\lambda < 0$ (either gravity increases upwards from its value at z = 0 or gravity decreases upwards from its value at z = 0).

STABILITY OF THE SYSTEM & OSCILLATORY MODES

Multiplying (18) by the conjugate of W i.e. W* and integrate over the range of z and making use of (19) - (22)

together with boundary condition (24), we obtain,

$$\left[\left(\frac{\sigma}{\varepsilon}+\frac{M\sigma}{\varepsilon(1+\tau_1\sigma)}+\frac{(1+\sigma F_1)}{P_t}\right)\right]I_1+I_2+FI_3$$
$$+\frac{\mu_e\eta\in}{4\pi\rho_0 v}\left(I_4+P_2\sigma^* I_5\right)-\frac{\alpha a^2\lambda\kappa_T}{\beta v}\left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)$$
$$\left(\frac{1+\sigma^*\tau_1}{B+\sigma^*\tau_1}\right)\left[I_6+\sigma^* E_1 P_1 I_7\right]=0 \qquad \ldots(34)$$

Where

$$I_1=\int\left(|DW|^2+a^2|W|^2\right)dz$$

$$I_2=\int\left(|D^2W|^2+a^4|W|^2+2a^2|DW|^2\right)dz$$

$$I_3=\int\left(|D^3W|^2+3a^2|D^2W|^2+3a^4|DW|^2+a6|W|^2\right)dz$$

$$I_4=\int\left(|D^2V|^2+a^4|V|^2+2a^2|DV|^2\right)dz$$

$$I_5=\int\left(|DV|^2+a^2|V|^2\right)dz$$

$$I_6=\int\left(|D\Theta|^2+a^2|\Theta|^2\right)dz$$

$$I_7=\int\left(|\Theta|^2\right)dz$$

All above specified integrals I1- I7 are +ve definite. Put σ = i σ_i in (34), equating imaginary part, we obtain,

$$\sigma_i\begin{bmatrix}\left(\frac{1}{\varepsilon}+\frac{M}{\varepsilon\left(1+\tau_1^2\sigma_i^2\right)}+\frac{F_1}{P_t}\right)I_1\\ -\frac{\mu_e\eta\in}{4\pi\rho_0 v}P_2I_5+\frac{\alpha a^2\lambda\kappa_T}{\beta v}\left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)\\ \left\{\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)I_6+\left(\frac{B+\tau_1^2\sigma_1^2}{B^2+\tau_1^2\sigma_i^2}\right)E_iP_1I_7\right\}\end{bmatrix}=0 \qquad \ldots(35)$$

In absence magnetic field (35) becomes,

$$\sigma_i\begin{bmatrix}\left(\frac{1}{\varepsilon}+\frac{M}{\varepsilon\left(1+\tau_1^2\sigma_i^2\right)}+\frac{F_1}{P_t}\right)I_1+\frac{\alpha a^2\lambda\kappa_T}{\beta v}\\ \left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)\begin{Bmatrix}\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_1^2}\right)I_6+\\ \left(\frac{B+\tau_1^2\sigma_i^2}{B^2+\tau_1^2\sigma_i^2}\right)E_1P_1I_7\end{Bmatrix}\end{bmatrix}=0 \qquad \ldots(36)$$

From (36) the quantity is in bracket will be positive definite if $g>\frac{\gamma M_0\nabla H}{\rho_0\alpha}$ and B > 1.

It means $\sigma_i=0$, modes are non oscillatory or oscillatory modes are not allowed and principle of exchange of stability is satisfied if $g_0>\frac{\gamma M_0\nabla H}{\rho_0\alpha}$ and B > 1.

Therefore the system is stable in the presence of viscoelastic parameter F1 and magnetic field. The presence of viscoelastic parameter F1 and magnetic field introduces oscillatory modes, stability in the system and principle of exchange of stability is not satisfied.

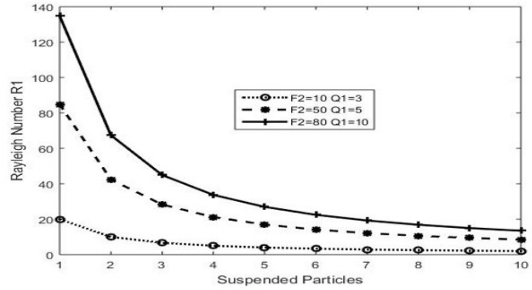
NUMERICAL COMPUTATIONS

Dispersion equation (27) is analysed also. The critical Rayleigh number R_1 & Rayleigh number for ferromagnetic fluid is calculated for different values of couple-stress F_2, suspended particles B, permeability P, magnetic field Q1 and Magnetization M_0.

In fig. 2, a graph has been plotted between the critical Rayleigh number R_1 and the suspended particles parameter B for different values for F2 = [10 50 80] , Q1=[3 5 10] and λ =10 > 0. Which shows R_1 decreases with increase in B for λ > 0. So suspended particles have a destabilizing effect on the system for λ >0.

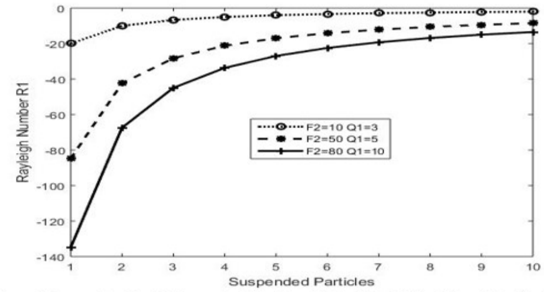
In fig. 3, a graph has been plotted between the critical Rayleigh number R_1 and the suspended particles parameter B for different values for F2=[10 50 80] , Q1=[3 5 10] and λ= –10< 0. Which shows R_1 increases with increase in B for λ < 0. So suspended particles have a stabilizing effect on the system for λ <0.

In fig. 4, a graph has been plotted between critical Rayleigh number R_1 and couple-stress parameter F_2 for *B* = [10, 20, 30], Q1 = [20, 30, 60] and λ = 10 > 0, which shows R_1 increases with increases in F_2. So couple-stress has stabilizing effect on the system for λ >0.



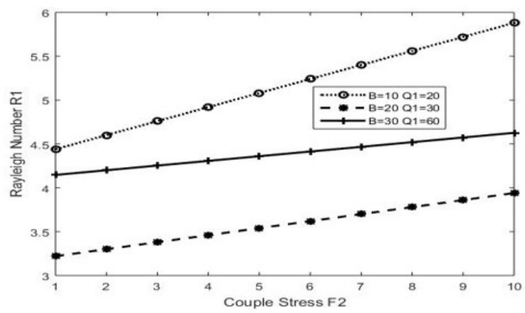
λ=10, x=1, P=0.2, ε=0.5, F2= [10, 50,80], Q1= [3, 5, 10]

Fig. 2



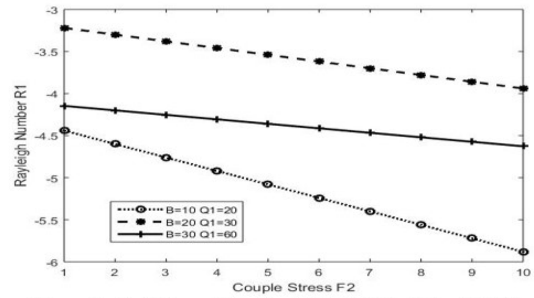
λ=-10, x=1, P=0.2, ε=0.5, F2= [10, 50,80], Q1= [3, 5, 10]

Fig. 3



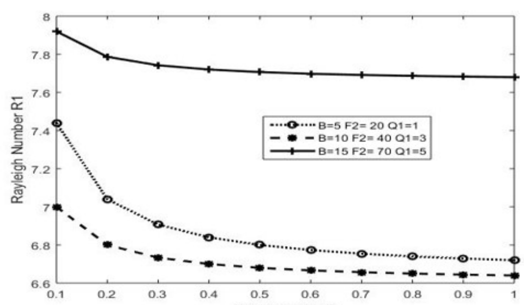
λ=10, x=1, P=0.2, ε=0.1, B= [10,20,30], Q1= [20,30, 60]

Fig. 4



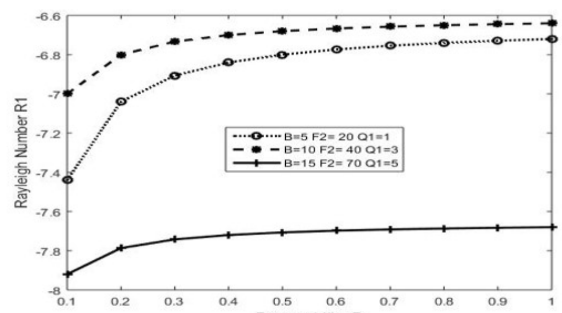
λ =-10, x=1, P=0.2, ε=0.1, B= [10,20,30], Q1= [20,30, 60]

Fig. 5



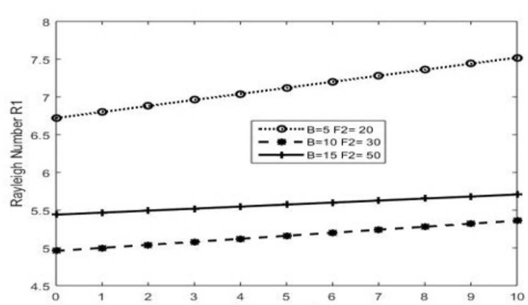
λ=10, x=1, ε=0.5, B= [5,10,15], Q1= [1, 3, 5], F2=[20,40,70]

Fig. 6



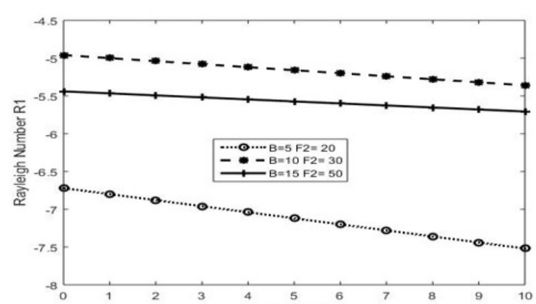
λ=-10, x=1, ε=0.5, B= [5,10,15], Q1= [1, 3, 5], F2=[20,40,70]

Fig. 7



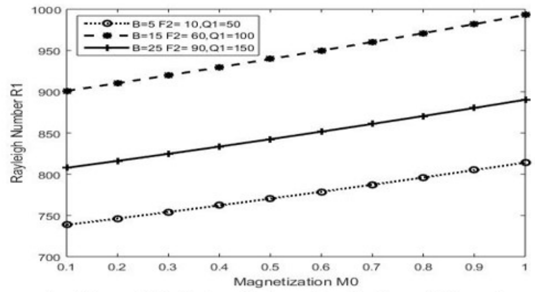
λ=10, x=1, P=0.5, ε=0.5, F_2= [20, 30, 50] B= [5,10,15]

Fig. 8



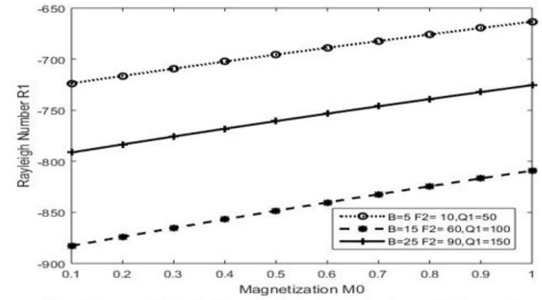
λ=-10, x=1, P=0.5, ε=0.5, F_2= [20, 30, 50] B= [5,10,15]

Fig. 9



$\lambda=10$, $x=1$, $P=0.5$, $\varepsilon=0.5$, $\gamma=1$, $\rho 0=1$, $g=9.8$, $\alpha=1$, $B=[5,15,25]$, $Q1=[50,100,150]$, $F_2=[10,60,90]$

Fig. 10



$\lambda=-10$, $x=1$, $P=0.5$, $\varepsilon=0.5$, $\gamma=1$, $\rho 0=1$, $g=9.8$, $\alpha=1$, $B=[5,15,25]$, $Q1=[50,100,150]$, $F_2=[10,60,90]$

Fig. 11

In fig. 5, a graph has been plotted between critical Rayleigh number R_1 and couple-stress parameter F_2 for B = [10, 20, 30], Q1 = [20, 30, 60] and $\lambda = -10 < 0$,which shows R_1 decreases with increases in F_2. So couple-stress has destabilizing effect on the system for $\lambda < 0$.

In fig. 6, a graph has been plotted between critical Rayleigh number R_1 and permeability of the medium P for B= [5, 10, 15], F2= [20, 40, 70], Q_1= [1, 3, 5] and $\lambda = 10 > 0$, which shows R_1 decreases with increases P. So permeability has destabilizing effect on the system for $\lambda > 0$.

In fig. 7, a graph has been plotted between critical Rayleigh number R_1 and permeability of the medium P for for B= [5, 10, 15], F2= [20, 40, 70], Q_1= [1, 3, 5] and $\lambda = -10 < 0$, which shows R_1 increases with increases P. So permeability has a stabilizing effect on the system for $\lambda < 0$.

In fig. 8, a graph has been plotted between critical Rayleigh number R_1 and Magnetic Field for F_2 = [20, 30, 50], B= [5, 10, 15] and $\lambda = 10 > 0$, which shows R_1 increases with increases in Q_1. So Magnetic Field has stabilizing effect on the system for $\lambda = 10 > 0$.

In fig. 9, a graph has been plotted between critical Rayleigh number R_1 and Magnetic Field for F_2 = [20, 30, 50], B= [5, 10, 15] and $\lambda = -10 < 0$, which shows R_1 decreases with increases in Q_1. So Magnetic Field has destabilizing effect on the system for $\lambda = -10 < 0$.

In fig. 10, a graph has been plotted between Rayleigh number R and magnetization of ferromagnetic fluid M_0 for F_2= [10, 60, 90], B= [5, 15, 25], Q1 = [50, 100, 150] and $\lambda = 10 > 0$, which shows R increases with increases in M_0. So magnetization has stabilizing effect on the system for $\lambda > 0$.

In fig. 11, a graph has been plotted between Rayleigh number R and magnetization of ferromagnetic fluid M_0 for F_2= [10, 60, 90], B= [5, 15, 25], Q1 = [50, 100, 150] and $\lambda = -10 < 0$, which shows R increases with increases in M_0. So magnetization has stabilizing effect on the system for $\lambda < 0$.

CONCLUSION

In the present paper, the effect of different parameters such as suspended particles, couple-stress, viscoelasticity, permeability, magnetic field and magnetization has been examined when a couple-stress Rivlin-Ericksen ferromagnetic fluid heated from below in presence of varying gravity, magnetic field and suspended particles through a porous medium.

Results are as follows:-

1. Suspended particles always have destabilizing effect for $\lambda > 0$ and stabilizing effect for $\lambda < 0$.
2. Couple-stress has a stabilizing effect if $\lambda > 0$ and a destabilizing effect if $\lambda < 0$ on the thermal instability of couple-stress Rivlin-Ericksen ferromagnetic fluid in the presence of varying gravity and magnetic fields through a porous medium.
3. Permeability of porous medium has a stabilizing effect if $\lambda > 0$ and a destabilizing effect if $\lambda < 0$.
4. Magnetic field has a stability effect on considered fluid layer $\lambda > 0$ and a destabilizing effect if $\lambda < 0$.
5. Magnetization always has a stabilizing effect on considered fluid layer.
6. Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid for stationary convection.
7. The presence of viscoelastic parameter, magnetic field introduces oscillatory modes and principle of exchange is not satisfied.

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