The Scientific Temper
OCTOBER 2026 Vol. 17 (10): 7021-7032
ISSN: 0976-8653, E-ISSN: 2231-6396
DOI: 10.58414/SCIENTIFICTEMPER.2026.17.10.2564
A WEB OF SCIENCE JOURNAL

RESEARCH ARTICLE

Explaining Child Death in Bangladesh through the Probability Model

MAHMUDUL HASAN TONMOY[1], TAPAN KUMAR ROY[2], BRIJESH P. SINGH[3]*

[1]*Research Student, Department of Population Science and Human Resource Development, University of Rajshahi, Rajshahi, Bangladesh, E-mail: mhtonmoy1012@gmail.com*

[2]*Professor, Department of Population Science and Human Resource Development, University of Rajshahi, Rajshahi, Bangladesh, E-mail: roy.tapan@gmail.com ORCID ID: https://orcid.org/0000-0003-4314-3290*

[3]*Professor, Department of Statistics, Institute of Science, Banaras Hindu University, Varanasi, India, E-mail: brijesh@bhu.ac.in ORCID ID: https://orcid.org/0000-0002-2429-4758*

Corresponding author: brijesh@bhu.ac.in

ABSTRACT

Child mortality has gained importance, particularly because death rates during the first five years of life are perpetually high in all countries, regardless of whether overall mortality rates are high or low. In order to meet the second target of the third sustainable development goal by 2030, child mortality in Bangladesh must be reduced. For this purpose, the study aimed to develop a suitable probability model for the number of child deaths among women for a fixed parity. The model is developed for analyzing the risk of child death and to use it for comparing such risk of death. The present study utilizes data from Bangladesh Demographic and Health Survey 2017-18. The parameter of the suitable probability model is estimated by method of moments and maximum likelihood method. The result shows that the average risk of child death (p) varies from women to women with parity (p=0.099 for parity 4, p=0.129 for parity 5, p=0.139 for parity 6). It is expected that the study will be helpful for researchers who are interested to work in the field of child mortality which will helpful for policy decision.

Keywords: *Child mortality; probability model; beta binomial; maximum likelihood method*

INTRODUCTION

Infant and early child deaths are referred to as child mortality, sometimes known as under-five mortality. Children are a valuable resource for a country, and a decrease in infant and child mortality is one of the main elements that is taken into account when planning for the development of the country. The degree and quality of medical care as well as the socioeconomic state of the community are both well-indicated by child mortality.

The mortality decrease has been quite rapid in developed countries due to better medical facilities, but the rate of child and infant mortality is still very high in developing countries (Singh et al., 2011). In developing nations, lowering infant and child mortality rates is of utmost importance, and child health is high on the list of issues that should be addressed by public policy. Infant and child mortality rates are frequently regarded as indicators of a country's overall health, and they can also be

Received: 5th October, 2026 **Accepted: 7th October, 2026** **Published: 28th October, 2026**

seen as crucial indicators of a society's development and of the country's overall progress. Different child health improvement programs have been put into place over the years with varied degrees of success in nations like Bangladesh. Over the past few decades, the infant mortality rate has significantly decreased as a result of the aforementioned policy initiatives. Despite this development, Bangladesh still has a higher child death under age five than the global standard (Nilima, 2017).

Various socioeconomic and demographic factors have an impact on infant and child mortality. Mother's education, family income, location of residence, religion, etc., have a significant impact on infant and child mortality among socioeconomic factors, while maternal age, parity, previous birth interval, and previous child's survival status have been identified to be major demographic factors. Evidence suggests in the study conducted by Eberstein et al. (1990) that the first and higher order births suffer significantly higher rate of mortality than births which fall into intermediate range. Similarly, short birth intervals also significantly increase child's mortality (Hossain et al., 2015).

The level of socioeconomic development of the country influences the relative weight of socioeconomic and demographic determinants that are affecting infant and child mortality. According to some studies, demographic factors have a greater impact on infant and child mortality in traditional societies than socioeconomic ones. It is also evident that child mortality has substantially positive impact on fertility. Women frequently continue their pregnancies after an infant die as a way of trying to swiftly replace what was lost, according to research. Thus, research into child mortality is crucial for a variety of reasons, including the fact that high infant mortality rates represent another issue of gross reproductive waste and deplete the physical, financial, and emotional resources of mothers who have such helpless infants.

It has been noted that data on infant and child deaths in developing nations frequently contains significant inaccuracies. Usually, these errors occur due to recall lapse, ignorance and digit preference of the respondents which results in omission of events, misplacement of dates (Singh et al., 2015). Due to this, it is not possible to have proper data or actual reporting on this important vital event. To overcome such limitations and due to lack of reliable observed data on childhood mortality makes it necessary to develop mathematical models because model smoothes the data and provides a reasonable distribution of the event under study. By a model we mean any abstraction of a real situation. Often, the abstraction is presented in the form of probability distribution; therefore, we say that model is a stochastic formulation of the phenomena. It provides concise representation of extensive data set and also provide stimulus to develop further work as well or a model systematize comparison across time as well as over space (Sheldon and Moore, 1968).

Demographers and statisticians have been interested in the representation of mortality data using a parametric model for more than a century. Gompertz (1825) provided the most widely used model, which is still employed by demographers today, to investigate mortality. In spite of wide use of Gompertz model, many more models have been suggested to describe mathematically the survival and mortality trend (Thatcher et al., 1998). Generalized linear and nonlinear model approaches to mortality graduation were given and supported by arguments by Renshaw (1991). The aforementioned models have already been applied to a variety of mortality data. Since the force of mortality is higher in the early stages of life than in adolescence, many people have tried independently to graduate the survival function at these young ages. Perhaps the first person to attempt a study on infant mortality using the hyperbolic function was Keyfitz (1977). For the purpose of analyzing the distribution of infant deaths, Krishnan and Jin (1993) fitted the Pareto distribution using national data from Canada. Chauhan (1997) have graduated the distribution of infant mortality using a model developed by Mukherjee and Islam (1983) in the context of reliability analysis.The models discussed above are based on life expectancy or surviving time. Using the number of child deaths in the family to explain the pattern of child mortality, Bhuyan and Deograties (1999) proposed using the Polya-Aeppli model to examine the pattern of child mortality.

A useful tool for analyzing population changes and variance is the study of infant and child mortality distribution. An attempt has been made in this study to derive the probability model for the number of child death among women for a fixed parity. By child mortality we mean death of child below age 5 years. The model to be proposed and developed aims to search a suitable model for analyzing the risk of child death in the population and to use it for comparing the risk of child deaths with respect to time and space. The application of these proposed models are illustrated through real data taken from BDHS 2017-18.

CONSTRUCTION OF THE MODEL

Consider the women who have given n births (women with parity n). Let us denote p be the probability of death under age five of a child born to a woman and X be the total number of child deaths out of n. Thus, X is a random variable which indicates the number of child death to a woman for fixed parity n. An attempt is made to develop a probability model for the random variable X.

Model-I

The following model is first developed, and it is based on the assumption that every child born to a woman will either die before age five or survive. Let us define for $i^{th}(i = 1, 2, \ldots . n)$ child born to a woman, a random variable u_i taking value 1 if the child dies before age five and 0 otherwise. u_i's are hence Bernoulli variables. Now if we assume that death of children (born to a woman) below age five are independent of each other having same probability p then total number of child death X to the woman is nothing but sum of independent Bernoulli variables and hence follows a binomial distribution. Therefore, the distribution of X may be given by:

$$P[X = x] = \binom{n}{x} p^x (1-p)^{n-x} \; ; \; 0 \le p \le 1 \qquad (1)$$

Where, $x = 0, 1, 2, \ldots \ldots \ldots \ldots, n$.

Model-II

In Model-I, the death of children below age five years to a woman having fixed probability 'p' for all but as discussed in the introduction, p is effected by a number of factors and therefore assumption of p being a constant for all women in the population can easily be questioned. Moreover, neither p can be directly observed nor data on p can be obtained. In such a situation, it seems more logical to consider that p is a random variable following some distribution. We are assuming that probability p of death under age five years for all the children born to a woman are constant but unknown. It varies from woman to woman and follows a probability distribution $g(p)$. Here we are assuming that $g(p)$ is beta-distribution of first kind with parameters (α, β). The p.d.f. of p is defined as $g(p)$:

$$g(p) = \frac{1}{B(\alpha,\beta)} p^{(\alpha-1)} (1-p)^{(\beta-1)} \; ; \; 0 \le p \le 1, \quad \alpha > 0, \quad \beta > 0 \qquad (2)$$

Due to the fact that 'p' the risk of child deaths varies from 0 to 1 and beta distribution enjoys the property of flexibility, and capability of accommodating wide range of variability, this model seems to be most convenient model for p.

Because of this, for a given woman, the distribution of number of child death x out n birth follows a binomial distribution, and the probability of death 'p' follows beta distribution of first kind with parameter α and β, the joint distribution of x and p is given by

$$P[X = x \cap P = p] = P\{X = x|p\} \times g(p) = \binom{n}{x} p^x (1-p)^{n-x} \frac{1}{B(\alpha,\beta)} p^{(\alpha-1)} (1-p)^{(\beta-1)} \qquad (3)$$

and the marginal distribution of x is given as

$$P[X = x] = \binom{n}{x} \frac{B(\alpha+x, \; \beta+n-x)}{B(\alpha,\beta)} \; ; \quad \alpha, \beta > 0 \qquad (4)$$

where, $x = 0, 1, 2 \ldots \ldots \ldots \ldots \ldots . . n$

The distribution as shown in (4) is referred to as a beta-binomial distribution and is a logical extension of the binomial model taking into account the random nature of 'p' in the population. The parameters α and β are its shape parameter. It may be worthwhile to mention here that the parameters α and β can be estimated (say $\hat{\alpha}$ and $\hat{\beta}$) by analyzing data through (4) and hence an estimate of the distribution of p in the population can also be obtained. If someone is interested in getting a single value (like p of the model-I) for comparing the child death of two places, one may take mean, $\frac{\hat{\alpha}}{\hat{\alpha}+\hat{\beta}}$ as an estimate of average child death of the region.

ESTIMATION OF PARAMETERS OF MODELS

The estimation procedures of the parameters of those two models are given. The estimation procedure for the model under consideration for the number of death of children under age five years for fixed parity. For this purpose, we consider the maximum likelihood estimation procedure to estimate the parameters involved in the Model-I and II. But for model-II, the normal equations do not have nice close form solution therefore we propose the use of Newton-Raphson method but it needs a starting value. Estimate obtained by method of moment can be taken as the starting value.

Maximum Likelihood Estimates of Model-I

For Model-I, the maximum likelihood estimate of risk of death of children under age five years that is 'p' is same as estimate obtained by method of moment which is give as

$$\hat{p} = \frac{\bar{x}}{n} \qquad (5)$$

Where $\bar{x}$ is the mean number of death of children under age five years per female and n is the number of parity.

Simply, it can be shown in the following

We know that mean and variance of the Binomial Distribution can be expressed as

$$Mean = E(X) = np$$

$$Variance = Var(X) = npq$$

Thus, once n is known parameter p can be estimated as

$$\hat{p} = \frac{E(X)}{n} = \frac{\bar{x}}{n} \quad (6)$$

Method of Maximum Likelihood Estimation also provide same statistic.

Method of Moment (MM) Estimates of Model-II

Estimates of the parameters of Model-II by method of moments can be obtained as follows:

The first two raw moments of Beta-Binomial distribution can be written as

$$E(X) = \frac{n\alpha}{(\alpha+\beta)} \quad (7)$$

$$E(X^2) = \frac{n\alpha[n(1+\alpha)+\beta]}{(\alpha+\beta)(\alpha+\beta+1)} \quad (8)$$

Let us denotes μ_1' and μ_2' be the first two raw moments. Then by replacing $E(X)$ and $E(X^2)$ by μ_1' and μ_2' respectively we get two equations with two unknowns α and β, as follows;

$$\mu_1' = \frac{n\alpha}{(\alpha+\beta)} \quad (9)$$

and

$$\mu_2' = \frac{n\alpha[n(1+\alpha)+\beta]}{(\alpha+\beta)(\alpha+\beta+1)} \quad (10)$$

From equation (9) we get,

$$\beta = \left(\frac{n-\mu_1'}{\mu_1'}\right)\alpha \quad (11)$$

Substituting this value of β in equation (10) and after further simplification we get,

$$\alpha = \frac{n\mu_1'^2 - \mu_1'\mu_2'}{n(\mu_1'-\mu_2') - n\mu_1'^2 + \mu_1'^2} \quad (12)$$

Using equation (12) and (11) moment estimates of the parameters α and β can be obtained.

Method of Maximum Likelihood Estimation (MLE) of Model-II

Estimates of the parameters of model-II through maximum likelihood estimation can be obtained as follows:

The probability density function of Beta-binomial model can be written as,

$$P[X = x] = \binom{n}{x}\frac{B(\alpha+x,\ \beta+n-x)}{B(\alpha,\beta)};\quad \alpha,\beta > 0 \quad (13)$$

$$P[X = x] = \binom{n}{x}\frac{\Gamma(\alpha+x)\Gamma(\beta+n-x)\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+n)\Gamma(\alpha)\Gamma(\beta)};\quad \alpha,\beta > 0 \quad (14)$$

Where, $x = 0, 1, 2, \ldots\ldots\ldots, n$

More naturally it can be written as,

$$f(x_i) = \binom{n}{x_i}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \cdot \frac{\Gamma(\alpha+x_i)\Gamma(\beta+n-x_i)}{\Gamma(\alpha+\beta+n)} \quad (15)$$

Now the joint likelihood,

$$f(\tilde{x}) = f(x|\alpha,\beta) = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta).\Gamma(\alpha+\beta+n)}\right]^n \left[\prod_{i=0}^{n}\left\{\binom{n}{x_i}\Gamma(\alpha+x_i)\Gamma(\beta+n-x_i)\right\}\right] \quad (16)$$

Now taking logarithm on both side,

$$\ln f(\tilde{x}) = n\ln\left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta).\Gamma(\alpha+\beta+n)}\right] + \sum_{i=0}^{n}\ln\left\{\binom{n}{x_i}\Gamma(\alpha+x_i)\Gamma(\beta+n-x_i)\right\}$$

$$= n[\ln\Gamma(\alpha+\beta) - \ln\Gamma(\alpha) - \ln\Gamma(\beta) - \ln\Gamma(\alpha+\beta+n)] + \sum_{i=0}^{n}\ln\left\{\binom{n}{x_i}\Gamma(\alpha+x_i)\Gamma(\beta+n-x_i)\right\}$$

$$= n[\ln\Gamma(\alpha+\beta) - \ln\Gamma(\alpha) - \ln\Gamma(\beta) - \ln\Gamma(\alpha+\beta+n)] + \sum_{i=0}^{n}\ln\binom{n}{x_i} + \sum_{i=0}^{n}\ln\Gamma(\alpha+x_i) + \sum_{i=0}^{n}\Gamma(\beta+n-x_i) \quad (17)$$

Now applying partial differentiation on equation (17),

$$\frac{\partial}{\partial\alpha}\ln\{f(\tilde{x})\} = n[\Psi_0(\alpha+\beta) - \Psi_0(\alpha) - \Psi_0(\alpha+\beta+n)] + \sum_{i=0}^{n}\Psi_0(\alpha+x_i)$$

$$\frac{\partial}{\partial\beta}\ln\{f(\tilde{x})\} = n[\Psi_0(\alpha+\beta) - \Psi_0(b) - \Psi_0(\alpha+\beta+n)] + \sum_{i=0}^{n}\Psi_0(\beta+n+x_i)$$

Now to apply Newton- Raphson method

$$\frac{\partial}{\partial\alpha^2}\ln f(\tilde{x}) = n[\Psi_1(\alpha+\beta) - \Psi_1(\alpha) - \Psi_1(\alpha+\beta+n)] + \sum_{i=0}^{n}\Psi_1(\alpha+x_i)$$

$$\frac{\partial}{\partial\beta^2}\ln f(\tilde{x}) = n[\Psi_1(\alpha+\beta) - \Psi_1(b) - \Psi_1(\alpha+\beta+n)] + \sum_{i=0}^{n}\Psi_1(\beta+n+x_i)$$

$$\frac{\partial}{\partial\alpha\,\partial\beta}\ln f(\tilde{x}) = n[\Psi_1(\alpha+\beta) - \Psi_1(\alpha+\beta+n)]$$

$$\frac{\partial}{\partial\beta\,\partial\alpha}\ln f(\tilde{x}) = n[\Psi_1(\alpha+\beta) - \Psi_1(\alpha+\beta+n)]$$

Now,

$$j = \begin{vmatrix} \frac{\partial}{\partial\alpha^2}\ln f(\tilde{x}) & \frac{\partial}{\partial\alpha\,\partial\beta}\ln f(\tilde{x}) \\ \frac{\partial}{\partial\beta\,\partial\alpha}\ln f(\tilde{x}) & \frac{\partial}{\partial\beta^2}\ln f(\tilde{x}) \end{vmatrix}$$

Now we apply Newton-Raphson iterative technique as

$$\begin{bmatrix}\alpha\\ \beta\end{bmatrix}_{n+1} = \begin{bmatrix}\alpha\\ \beta\end{bmatrix}_{n} - j^{-1}\left(\begin{bmatrix}\alpha\\ \beta\end{bmatrix}_{n}\right)\nabla f\left(\begin{bmatrix}\alpha\\ \beta\end{bmatrix}_{n}\right) \quad (18)$$

As we need a starting value of α and β, estimates obtained by moments can be used as starting value. Following those principles and using statistical software named R, maximum likelihood estimates of parameters α and β can be easily obtained.

RESULTS AND DISCUSSION

Data from the BDHS 2017–18 have been used to demonstrate how the model outlined above is applied. The survey gathered data from women who had ever been married in the age range of 15 to 49 on topics such as fertility, mortality, family planning, preferred methods of conception, and significant elements of reproductive health. Information has also been gathered on all live births to mothers and their survival status (date of birth of child, date of death of children, age at death). Age at death has been noted in cases of death. Women who haven't given birth in the five years from the survey's start date are assumed to be passed their reproductive period, hence only these women were taken into account for this study. We selected the women who haven't given birth in the past five years from the survey date in order to avoid censored cases. Thus, 10572 ever-married women who had no births in the five years before to the survey date are included in our dataset. Among them 3894 are from urban area and 6678 from rural area. These women are divided among different parities, and then women of fixed parity are distributed according to the number of children dying under age five. Then we have tried to find out if the proposed probability models fit the distribution well.

In this section, we've discussed regarding the estimation of parameters and fitting the proposed models. Two estimating techniques are utilized in this work to estimate the parameters for comparison purposes, as was stated in the sections above. Since two models for fixed parity are provided, it is to determine their frequencies after estimating the parameters for different parities. The corresponding tables likewise include estimates for the parameters, values of chi-square with the degree of freedom, and the *P*-value. In case of model-II (Beta-Binomial Model), average risk of child death has been also given in the table. The value of χ^2 and *P*-value shown in the tables clearly indicate that model-I (Binomial Model) is not suitable model for distribution of child death to a woman for fixed parity, whereas model-II (Beta-Binomial Model) fits the data and describes the data excellently.

Tables 1 to 3 represents expected frequencies along with observed frequencies for parity 4, 5, and 6 respectively. Parameters value, chi-square, degrees of freedom p-value and average risk of children death also given in these tables. Values of chi-square and p-values in table 1 to 3 and in summary in table 4 clearly indicate that Beta-Binomial model fits well with the data than Binomial model. As the p-values obtained in case of Beta-Binomial model is always greater than 0.05, so we do not have enough evidence to reject the null hypothesis. Thus we may accept the null hypothesis that is the proposed Beta-Binomial Model fits well with the distribution of women of fixed parity according to their number of child death. Further, expected frequencies, obtained using parameters that are estimated using the maximum likelihood method, are closer to the observed frequencies than when using the method of moment estimates. Thus MLE provides better fitting than moment estimator. Sample mean and variance are very close to the mean and variance obtained through the proposed model, shown in table 12, which is also an indication of model fitting.

Table 1. Observed and expected distributions of child deaths to the women with parity 4

Number of death children	***Observed***	***Frequency of Women***		
		Binomial	***Beta-Binomial***	
			MM	***MLE***
0	1057	1042.762	1060.919	1059.715
1	441	462.477	430.756	432.786
2	79	76.918	86.222	85.694
3	9	5.843	10.104	9.805
4	2			
Total	1588	1588	1588	1588

	Parameters	P=0.099	α=3.602 β= 32.490	α=3.865 β=34.855
	$\chi^2_{0.5}$(after pooling)	5.799	0.942	0.831
	D.F.	2	1	1
	***p*-value**	0.055	0.332	0.392
	Average Risk of Child Death (p)		0.098	0.099

Note: MM=Method of Moment Estimate, MLE=Maximum Likelihood Estimate

Table 2. Observed and expected distributions of child deaths to the women with parity 5

Number of death children	***Observed***	***Frequency of Women***		
		Binomial	***Beta-Binomial***	
			MM	***MLE***
0	440	426.521	439.076	439.334
1	294	314.789	295.598	295.218
2	94	92.930	94.659	94.680
3	20	14.759	19.667	19.768
4	1			
5	0			
Total	849	849	849	849
	Parameters	P=0.129	α=6.133 β= 41.552	α=6.006 β=40.693
	$\chi^2_{0.5}$(after pooling)	4.449	0.105	0.088
	D.F.	2	1	1
	***p*-value**	0.108	0.745	0.767
	Average Risk of Child Death (p)		0.129	0.129

Note: MM=Method of Moment Estimate, MLE=Maximum Likelihood Estimate

Table 3. Observed and expected distributions of child deaths to the women with parity 6

Number of death children	***Observed***	***Frequency of Women***		
		Binomial	***Beta-Binomial***	
			MM	***MLE***
0	182	166.018	184.458	183.875
1	144	160.571	136.781	137.466
2	54	64.710	61.067	61.220
3	22	15.701	19.480	19.348
4	4		5.214	5.091
5	1			
6	0			
Total	407	407	407	407
	Parameters	P=0.139	α=2.649 β=16.430	α=2.742 β=17.005
	$\chi^2_{0.5}$(after pooling)	13.152	1.566	1.546
	D.F.	2	2	2
	***p*-value**	0.001	0.457	0.462
	Average Risk of Child Death (p)		0.139	0.139

Note: MM=Method of Moment Estimate, MLE=Maximum Likelihood Estimate

Figure 1. Simulated beta distribution of risk of children death for women with parity 4, 5 and 6 in Bangladesh

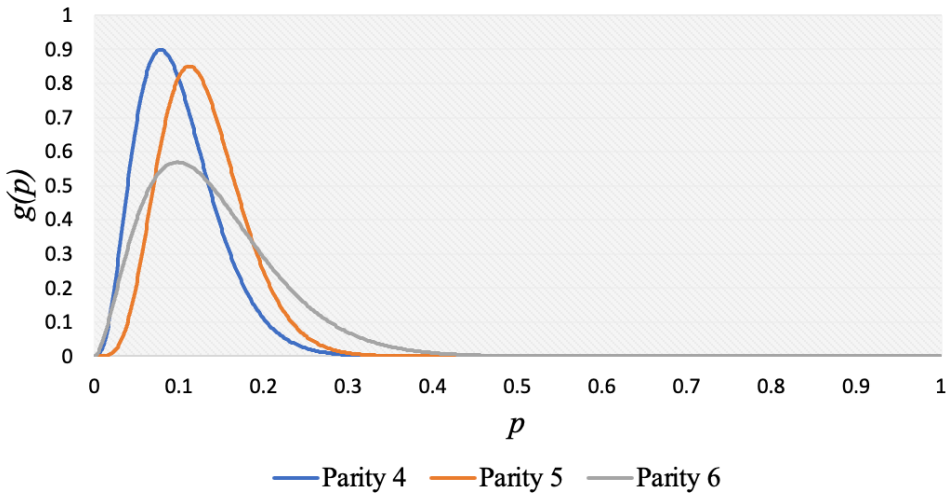


Table 4. Comparison of model fitting based on p-value

Parity	***p-value***		
	Binomial	***Beta-Binomial***	
		MM	***MLE***
4	0.055	0.332	0.392
5	0.108	0.745	0.767
6	0.001	0.457	0.462

From table 1 to 3 and table 4 it is evident that for parity 4, 5 and 6, Beta-Binomial model fits better than Binomial Model especially when parameters of Beta-Binomial model estimated using MLE.

Thus, we apply model-II (Beta-Binomial Model) for rural and urban women of parity 4, 5, and 6 to their distribution according to the number of child death, to get an estimate of the average risk of child death. Expected frequencies along with observed frequencies, parameter values, chi-square and p-values also presented in tables 5 to 10. Estimates of average risk of children death for both rural and urban women of parity 4, 5 and 6 are also presented as summary in table 11. Table 11 manifested that risk of children death higher for women of parity 6 and relatively lower in case of women of parity 4.

Table 5. Observed and expected distributions of child deaths to the rural women with parity 4

Number of death children	***Observed***	***Frequency of Women***
		Beta-Binomial (MLE)
0	726	728.784
1	300	291.483
2	51	57.950
3	6	6.784
4	2	
Total	1085	1085
	Parameters	α=3.479 β=31.793
	$\chi^2_{0.5}$(after pooling)	1.311
	D.F.	1
	p-value	0.252
	Average Risk of Child Death(p)	0.099

Note: MLE=Maximum Likelihood Estimate

Table 6. Observed and expected distributions of child deaths to the urban women with parity 4

Number of death children	***Observed***	***Frequency of Women***
		Beta-Binomial (MLE)

0	331	330.828
1	141	141.471
2	28	30.700
3	3	
4	0	
Total	503	503
	Parameters	α=5.112 β=44.822
	$\chi^2_{0.5}$(after pooling)	0.005
	D.F.	-
	***p*-value**	-
	Average Risk of Child Death (p)	0.102

Note: MLE= Maximum Likelihood Estimate

Figure 2. Simulated beta distribution of risk of child death for women with parity 4 in rural and urban area in Bangladesh

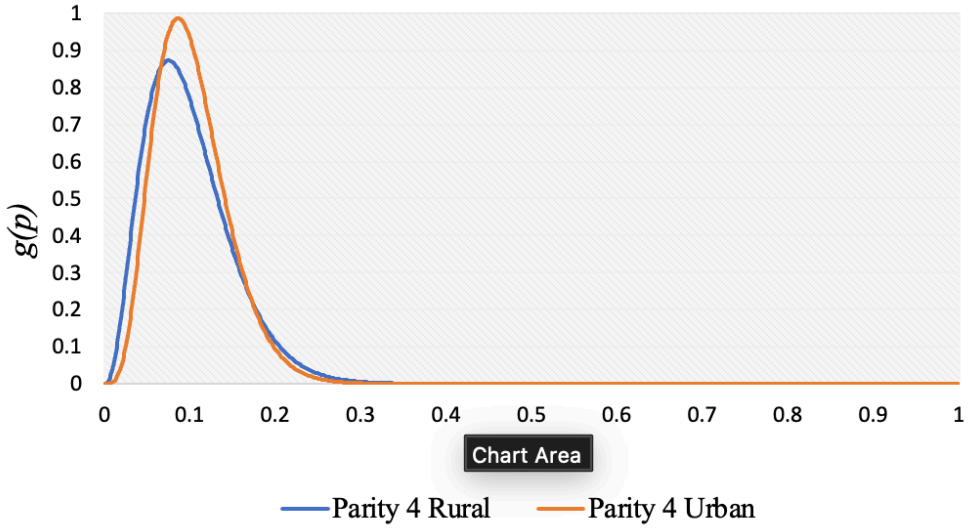


Table 7. Observed and expected distributions of child deaths to the rural women with parity 5

Number of death children	***Observed***	***Frequency of Women Beta-Binomial (MLE)***
0	313	313.953
1	219	215.371
2	65	69.966
3	16	14.710
4	1	
5	0	
Total	614	614
	Parameters	α=6.321 β=42.069
	$\chi^2_{0.5}$(after pooling)	0.773
	D.F.	1
	***p*-value**	0.379
	Average Risk of Child Death (p)	0.131

Note: MLE= Maximum Likelihood Estimate

Table 8. Observed and expected distributions of child deaths to the urban women with parity 5

Number of death children	***Observed***	***Frequency of Women Beta-Binomial (MLE)***

0	127	125.431
1	75	79.766
2	29	24.731
3	4	5.072
4	0	
5	0	
Total	235	235
	Parameters	α=5.277 β=37.492
	$\chi^2_{0.5}$(after pooling)	1.268
	D.F.	1
	***p*-value**	0.260
	Average Risk of Child Death (p)	0.123

Note: MLE= Maximum Likelihood Estimate

Figure 3. Simulated beta distribution of risk of child death for women with parity 5 in rural and urban area in Bangladesh

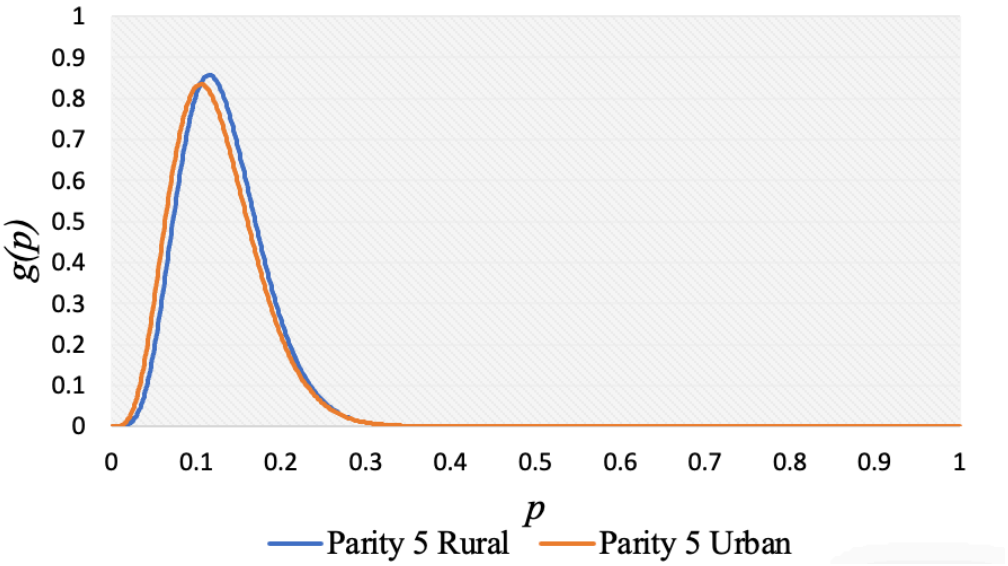


Table 9. Observed and expected distributions of child deaths to the rural women with parity 6

Number of death children	***Observed***	***Frequency of Women Beta-Binomial (MLE)***
0	132	133.491
1	108	103.041
2	42	47.116
3	17	19.353
4	3	
5	1	
6	0	
Total	303	303
	Parameters	α=2.805 β=16.804
	$\chi^2_{0.5}$(after pooling)	0.951
	D.F.	1
	***p*-value**	0.329
	Average Risk of Child Death (p)	0.143

Note: MLE= Maximum Likelihood Estimate

Table 10. Observed and expected distributions of child deaths to the urban women with parity 6

Number of death children	***Observed***	***Frequency of Women Beta-Binomial (MLE)***
0	50	50.388
1	36	34.425
2	12	14.097
3	5	5.089
4	1	
5	0	
6	0	
Total	104	104
	Parameters	α=2.654 β=18.306
	$\chi^2_{0.5}$(after pooling)	0.549
	D.F.	1
	***p*-value**	0.458
	Average Risk of Child Death (p)	0.127

Note: MLE= Maximum Likelihood Estimate

Figure 4. Simulated beta distribution of risk of child death for women with parity 6 in rural and urban area in Bangladesh

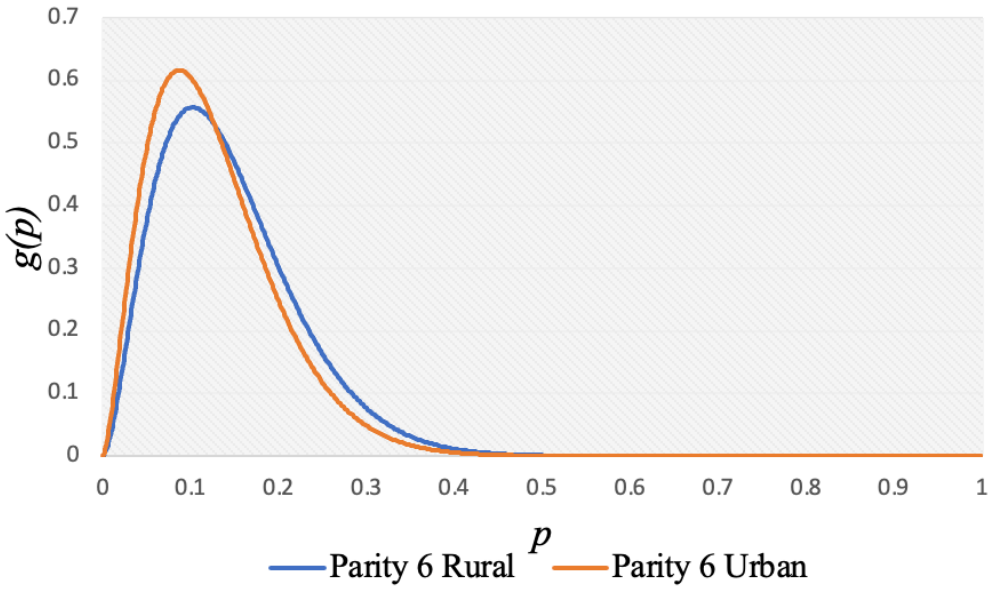


Table 11. Comparison of average risk of children death among rural and urban women of Bangladesh

Parity	***Average Risk of Child Death***		
	Total	***Rural***	***Urban***
4	0.099	0.098	0.102
5	0.129	0.131	0.123
6	0.139	0.143	0.127

Table 12. Comparison of sample mean and variance with mean and variance obtained by Beta-Binomial model

Parity	***Sample Mean***	***Sample Variance***	***Beta Binomial Model***	
			Mean	***Variance***
4	0.399	0.389	0.399	0.387
5	0.643	0.606	0.643	0.607

6	0.833	0.896	0.833	0.890
Rural				
4	0.395	0.388	0.395	0.385
5	0.653	0.614	0.653	0.614
6	0.858	0.920	0.858	0.914
Urban				
4	0.410	0.389	0.410	0.389
5	0.617	0.585	0.617	0.590
6	0.760	0.817	0.760	0.815

Besides, the estimated distribution of the risk of child death for women with parity 4, 5, and 6 is shown in figure 1, and figures 2 to 4 show parity-wise estimated distribution of the risk of child death for rural and urban women in Bangladesh. Figure 1 shows that the distribution of p(risk of child death) is of different pattern for parity 4, 5 and 6, and both are positively skewed, which indicates that there is a small group of women contributing most of child death. Figure 3 and 4 shows that mode of the distribution of p for rural area is generally higher than urban area. And from figure 5 it is found that the average risk of children death increases with increasing parity. The average risk of child deaths is mostly higher in the rural area; this may be due to lack of healthcare facility, a low standard of living in a rural area, and other factors affecting child health.

CONCLUSION

In conclusion, we can say that Model-II (Beta-Binomial Model) gives a way to study the distribution of p (risk of child death) which cannot be studied otherwise since p itself is unobservable. To lower the risk of child mortality in Bangladesh, better health care policies and programs should be implemented for women living in rural areas and having higher parity.

Acknowledgements

We are grateful to the MEASURE DHS for providing us with the data set. In addition, we would like to acknowledge all individuals and institutions involved in the implementation of the BDHS 2017-2018.

Data Availability Statement

Data are publicly available as Bangladesh Standard Demographic and Health Survey, BDHS, 2017=18 Dataset and can be accessed through the web sites:

http://dhsprogram.com/data/dataset/Bangladesh_Standard-DHS_2017.cfm?flag=0

Conflict of interest: None

Source of support: None

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