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RESEARCH ARTICLE

An Integrated GMIR And Lagrangian Optimization Approach For Triangular Fuzzy Inventory Management In Frozen Food Supply Chain

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ABSTRACT

Efficient inventory management in the frozen food supply chain is challenging because of uncertain demand, refrigeration requirements, transportation variability, and backup cooling costs. This study proposes a triangular fuzzy inventory optimization model integrated with the Graded Mean Integration Representation (GMIR) approach to support cost-effective replenishment decisions under uncertain operating conditions. A real-life frozen food application involving products such as frozen shrimp, fish fillets, frozen vegetables, and ready-to-cook items is considered. The uncertain parameters are represented by triangular fuzzy numbers, while deterministic parameters related to refrigeration operations and storage are maintained in crisp form. The fuzzy parameters are defuzzified using the GMIR method, and the resulting inventory model is optimized through a nonlinear cost minimization procedure to determine the optimal order quantity and the corresponding minimum total inventory cost. A numerical example demonstrates the applicability of the proposed framework in balancing replenishment, refrigeration, transportation, spoilage, and backup refrigeration expenses. Furthermore, a sensitivity analysis using a dataset of 200 frozen food products is conducted to evaluate the robustness of the model under varying demand and operational conditions. The results indicate that the proposed GMIR-based triangular fuzzy model provides stable inventory planning in frozen food supply chain systems. The proposed framework offers a practical decisions-support tool for sustainable, energy-efficient, and economically optimized frozen food inventory management.

Keywords: *Triangular fuzzy number, Graded Mean Integration Representation, Frozen food supply chain, Inventory Optimization, Optimal order quantity.*

INTRODUCTION

The frozen food supply chain has become an essential component of modern food distribution systems because it ensures the availability of temperature-sensitive products such as frozen seafood, meat products, poultry, vegetables, dairy products, and ready-to-cook foods throughout extended storage and transportation

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periods. Maintaining product quality and safety in frozen food systems requires efficient refrigeration, storage, transportation, and inventory coordination across the entire cold chain. Taoukis et al. (2016) emphasized that food cold-chain management and optimization are critical for preserving food quality, minimizing spoilage, and improving operational efficiency in refrigerated distribution networks. The increasing global demand for convenient frozen food products has therefore intensified the need for advanced inventory optimization techniques that can effectively manage refrigeration-related operational costs and uncertain supply chain conditions.

Cold-chain inventory optimization has recently received significant attention in the operations research literature. Sebatjane (2025) developed a two-echelon cold-chain inventory optimization framework that jointly considers sustainable lot-sizing, refrigerated transportation, and carbon-emission regulations, emphasizing the significant impact of refrigeration and shipment policies on operational efficiency. Similarly, Yang et al. (2024) proposed an EOQ model for temperature-sensitive deteriorating items in cold-chain operations, showing that deterioration and refrigeration conditions substantially influence replenishment decisions and total inventory cost. In the context of frozen meat operations, Darajat et al. (2018) proposed an inventory simulation model for frozen-meat management to support food-safety programs and reduce operational risks associated with temperature-sensitive products. Furthermore, Hemmati et al. (2023) designed a green-resilient supply chain network for perishable products that incorporates pricing reduction strategies and optimal inventory management, highlighting the growing importance of sustainability and resilience in perishable supply chain systems.

Traditional inventory models generally assume that demand, holding cost, transportation cost, and other operational parameters are known precisely. In real frozen food systems, these assumptions are rarely satisfied because customer demand varies seasonally, transportation expenses fluctuate due to fuel and logistics conditions, and refrigeration-related operating costs may change according to equipment performance and power interruptions. To address such uncertainty, fuzzy set theory has been widely employed in inventory optimization. Hemalatha and Annadurai (2023) presented a fuzzy EOQ model incorporating various fuzzy numbers, while Nand (2025) investigated advanced inventory optimization approaches that integrate fuzzy logic to handle demand variability and uncertain operational conditions. These studies demonstrate that fuzzy-based inventory models can provide more realistic decision-support tools for uncertain supply chain environments.

Among different fuzzy representations, triangular fuzzy numbers are particularly attractive because of their simplicity, computational efficiency, and ability to represent lower, most probable and upper estimates of uncertain parameters. Recent advances have combined fuzzy inventory modelling with machine learning and mathematical programming techniques to improve forecasting accuracy and supply chain efficiency. Kalaiarasi and Swathi (2024) proposed a hybrid approach that integrates fuzzy inventory management with deep learning algorithms to enhance demand forecasting and supply chain performance. Dhandapani et al. (2024) developed a hybrid fuzzy mathematical programming approach for manufacturing inventory models with partial trade credit policy and reliability considerations, illustrating the effectiveness of fuzzy optimization in complex inventory environments. Similarly, Kalaichelvan et al. (2024) applied fuzzy theory and machine learning to optimize the economic order quantity in a pharmaceutical inventory framework, showing that fuzzy-based optimization techniques can significantly improve inventory decision-making under uncertainty.

Effective utilization of fuzzy information requires an appropriate defuzzification mechanism to convert fuzzy parameters into representative crisp values for optimization. The Graded Mean Integration Representation (GMIR) method has emerged as a reliable and computationally efficient technique because it assigns greater importance to the most probable values while still considering the lower and upper bounds of uncertainty. Sundhari et al. (2025) demonstrated the effectiveness of graded mean integration methods for fuzzy modelling and optimization applications, confirming their suitability for practical inventory decision-support systems involving uncertain operational data.

In addition to uncertainty modelling, determining the optimal replenishment quantity requires an efficient nonlinear optimization procedure capable of minimizing the overall inventory cost. The Lagrangian optimization method provides a systematic framework for solving constrained inventory problems involving multiple cost components such as ordering cost, refrigeration operating cost, holding cost, transportation cost, spoilage-related cost, and backup refrigeration cost. Hossain et al. (2024) proposed a profit-cost ratio maximization approach for a manufacturing inventory model with stock-dependent production and price-

dependent demand, while Guo and Zhang (2025) investigated optimal order quantity determination under unknown parameters and variable holding cost rates. Mouly and Thangam (2026) further developed an optimal decision policy for fuzzy inventory systems incorporating advance payment, price for fuzzy inventory systems incorporating advance payment, price-sensitive demand, fluctuating holding costs, and different fuzzy number representations, providing additional evidence of the growing importance of fuzzy nonlinear optimization in modern inventory management.

Motivated by these challenges, this study proposes a GMIR-based triangular fuzzy inventory optimization model for the frozen food supply chain using Lagrangian optimization under uncertain operating conditions. The proposed framework represents uncertain demand, transportation cost, and backup refrigeration cost by triangular fuzzy numbers and employs the GMIR method to obtain representative crisp values. The resulting inventory model is optimized through the Lagrangian approach to determine the optimal order quantity and the corresponding minimum total inventory cost. A numerical example involving frozen food products is presented to demonstrate the applicability of the proposed method, and a sensitivity analysis using 200 frozen food products is conducted to evaluate the robustness of the model under varying operational conditions. The proposed study contributes to the literature by integrating triangular fuzzy uncertainty modelling, GMIR-based defuzzification, and Lagrangian optimization within a unified framework specifically designed for cost-effective, energy-efficient, and sustainable frozen food supply chain inventory management.

PRELIMINARIES

Triangular Fuzzy Number

A Triangular Fuzzy Number is a special type of fuzzy number used to represent uncertain parameters in inventory and supply chain systems. A TFN is denoted by

$\tilde{A} = (a_1, a_2, a_3), a_1 \le a_2 \le a_3$

Where a_1, a_2, a_3 represent the lower, most probable, and upper values, respectively. The membership function of the triangular fuzzy number is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{(x - a_1)}{(a_2 - a_1)}, & a_1 \le x \le a_2 \\ \dfrac{(a_3 - x)}{(a_3 - a_2)}, & a_2 \le x \le a_3 \\ 0, & x > a_3 \end{cases}$$

Triangular fuzzy numbers are widely used because of their simplicity, computational efficiency, and ability to represent imprecise information with limited data.

Arithmetic Operations of Triangular Fuzzy Numbers

Let $\tilde{A} = (a_1, a_2, a_3), \tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. Their arithmetic operations are defined as follows.

Addition $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

Subtraction $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

Multiplication $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3)$

Division Provided that $0 \notin (b_1, b_2, b_3), \tilde{A} \div \tilde{B} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$

These arithmetic principles enable uncertain inventory parameters to be manipulated directly within the fuzzy optimization framework.

Graded Mean Integration Representation (GMIR)

To convert a triangular fuzzy number into a crisp value, the Graded Mean Integration Representation method is employed. For a triangular fuzzy number$\tilde{A} = (a_1, a_2, a_3)$, the GMIR values is defined as

$$GMIR(\tilde{A}) = \frac{(a_1 + 4a_2 + a_3)}{6}$$

The GMIR method assigns a higher weight to the most probable value a_2, thereby producing a balanced and practically meaningful crisp representation of the fuzzy parameter. In the proposed frozen food supply chain model, uncertain parameters such as demand, transportation cost, and backup refrigeration cost are represented by triangular fuzzy numbers and transformed into crisp values using the GMIR technique before performing inventory optimization.

ASSUMPTIONS

1. The inventory system considers a single frozen food product during the planning horizon.
2. The annual demand is uncertain and represented by a triangular fuzzy number.

3. Replenishment is instantaneous, and the ordered quantity becomes available immediately when inventory is replenished.
4. Frozen food products are subject to a constant spoilage or deterioration rate, and shortages are completely backordered.

These assumptions simplify the analysis while capturing the essential uncertainty and operational characteristics of the frozen food supply chain.

NOTATIONS

To facilitate the mathematical formulation of the proposed frozen food supply chain inventory model, the principal decision variables and cost parameters used in this study are summarized below.

TC – Total annual inventory cost

Q – Inventory quantity of frozen food products.

D – Annual demand for frozen food products

O – Ordering and procurement cost per replenishment cycle

C – Refrigeration operating cost parameter

E – Energy efficiency parameter of the cold-storage system

F- Cold-storage holding cost parameter

T – Transportation and refrigerated distribution cost parameter

α - Refrigeration maintenance coefficient

β - Spoilage and shortage influence coefficient

B – Backup refrigeration and emergency cooling cost parameter

FORMULATION OF THE TOTAL COST:

To optimize inventory decisions in the frozen food supply chain under uncertain operating conditions, the total inventory cost is formulated by adapting the inventory cost structure proposed by Sindhuja et al. (2024). The model consists of a replenishment-related cost component that decreases with the inventory quantity and a holding-related operational cost component that increases with the inventory quantity. The total inventory cost function is expressed as

$$TC(Q) = \frac{DO(C-E-O)}{Q(C-E)} + \frac{Q(F+\alpha\beta+T+B)}{2} \quad (1)$$

Differentiate the equation (1) partially with respect to Q, we get

$$\frac{\partial TC(Q)}{\partial Q} = -\frac{DO(C-E-O)}{Q^2(C-E)} + \frac{(F+\alpha\beta+T+B)}{2}$$

Set $\frac{\partial TC(Q)}{\partial Q} = 0$, we get,

$$Q = \sqrt{\frac{2DO(C-E-O)}{(F+\alpha\beta+T+B)(C-E)}}$$

Where Q* represent the optimal replenishment quantity that minimizes the combined ordering, refrigeration, holding, transportation, spoilage, maintenance, and backup refrigeration costs in the frozen food distribution system.

INTEGRATED INVENTORY MODEL FOR TFN WITH CRISP PRODUCTION QUANTITY:

Considering the triangular fuzzy parameters for demand, refrigerated transportation cost, and backup refrigeration cost as follows:

$\tilde{D} = (D_1, D_2, D_3), \tilde{T} = (T_1, T_2, T_3), \tilde{B} = (B_1, B_2, B_3)$

And by applying Graded Mean Integration Representation we get:

$$TC(Q) = \frac{1}{6}\left\{\begin{array}{l} \frac{D_1O(C-E-O)}{Q(C-E)} + \frac{Q(F+\alpha\beta+T_1+B_1)}{2} \\ +4\left[\frac{D_2O(C-E-O)}{Q(C-E)} + \frac{Q(F+\alpha\beta+T_2+B_2)}{2}\right] \\ +\left[\frac{D_3O(C-E-O)}{Q(C-E)} + \frac{Q(F+\alpha\beta+T_3+B_3)}{2}\right] \end{array}\right\} \quad (2)$$

Now differentiate the equation (2) partially with respect to Q and equate it to 0 we get:

$$Q = \sqrt{\frac{2(D_1O+4D_2O+D_3O)(C-E-O)}{[(F+\alpha\beta+T_1+B_1)+4(F+\alpha\beta+T_2+B_2)+(F+\alpha\beta+T_3+B_3)](C-E)}}$$

The obtained value of Q* represents the optimal order quantity for the frozen food inventory system. This quantity provides a balance between replenishment-related costs and the combined holding, refrigeration, transportation, and backup refrigeration costs.

INTEGRATED INVENTORY MODELS FOR TFN WITH FUZZY PRODUCTION QUANTITY:

To enhance inventory management in the frozen food supply chain under uncertain demand, transportation, and refrigeration operating conditions, an integrated triangular fuzzy inventory model is developed. The corresponding total inventory cost function for the proposed system is expressed as follows.

$$TC(Q) = \frac{1}{6}\left\{\begin{array}{l} \frac{D_1O(C-E-O)}{Q_3(C-E)} + \frac{Q_1(F+\alpha\beta+T_1+B_1)}{2} \\ +4\left[\frac{D_2O(C-E-O)}{Q_2(C-E)} + \frac{Q_2(F+\alpha\beta+T_2+B_2)}{2}\right] \\ +\left[\frac{D_3O(C-E-O)}{Q_1(C-E)} + \frac{Q_3(F+\alpha\beta+T_3+B_3)}{2}\right] \end{array}\right\}$$

With $Q_1 \le Q_2 \le Q_3$. We replace inequality conditions $Q_1 \le Q_2 \le Q_3$ into the following inequality: $Q_2 - Q_1 \ge 0, Q_3 - Q_2 \ge 0$. We use the Lagrange method to find the minimum total cost $TC(Q)$.

Step 1: To find the min $P[TC(Q)]$, put

$$\frac{\partial TC}{\partial Q_1} = 0, Q_1 = \sqrt{\frac{2D_3O(C-E-O)}{(F+\alpha\beta+T_1+B_1)(C-E)}}$$

$$\frac{\partial TC}{\partial Q_2} = 0, Q_2 = \sqrt{\frac{2(4D_2O)(C-E-O)}{(F+\alpha\beta+T_2+B_2)(C-E)}}$$

$$\frac{\partial TC}{\partial Q_3} = 0, Q_3 = \sqrt{\frac{2D_1O(C-E-O)}{(F+\alpha\beta+T_3+B_3)(C-E)}}$$

From the above, $Q_1 > Q_2 > Q_3$. It is does not satisfy the constraint $Q_1 \le Q_2 \le Q_3$, so go to the next step.

Step 2: Convert the inequality $Q_2 - Q_1 > 0$ into equality constraint $Q_2 - Q_1 = 0$ and the Lagrangian function as $L(Q_1, Q_2, Q_3, \lambda) = P[TC(Q)] - \lambda(Q_2 - Q_1)$

$$\frac{\partial L}{\partial Q_1} = \frac{1}{6}\left[-\frac{D_3O(C-E-O)}{Q_1^2(C-E)} + \frac{(F+\alpha\beta+T_1+B_1)}{2}\right] + \lambda = 0$$

$$\frac{\partial L}{\partial Q_2} = \frac{1}{6}\left[-\frac{4D_2O(C-E-O)}{Q_2^2(C-E)} + \frac{4(F+\alpha\beta+T_2+B_2)}{2}\right] - \lambda = 0$$

$$\frac{\partial L}{\partial Q_3} = \frac{1}{6}\left[-\frac{D_1O(C-E-O)}{Q_3^2(C-E)} + \frac{(F+\alpha\beta+T_3+B_3)}{2}\right] = 0$$

$$\frac{\partial L}{\partial \lambda} = -(Q_2 - Q_1)$$

$$Q_1 = Q_2 = \sqrt{\frac{2(D_3O+4D_2O)(C-E-O)}{[(F+\alpha\beta+T_1+B_1)+4(F+\alpha\beta+T_2+B_2)](C-E)}}$$

$$Q_3 = \sqrt{\frac{2D_1O(C-E-O)}{(F+\alpha\beta+T_3+B_3)(C-E)}}$$

From the above, $Q_1 > Q_2 > Q_3$. It is does not satisfy the constraint $Q_1 \le Q_2 \le Q_3$, so go to the next step.

Step 3: Convert the inequality $Q_2 - Q_1 \ge 0, Q_3 - Q_2 \ge 0$ into equality constraints $Q_2 - Q_1 = 0$ and $Q_3 - Q_2 = 0$. The Lagrangian function is

$$L(Q_1, Q_2, Q_3, \lambda_1, \lambda_2) = P[TC(Q)] - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2)$$

$$\frac{\partial L}{\partial Q_1} = \frac{1}{6}\left[-\frac{D_3O(C-E-O)}{Q_1^2(C-E)} + \frac{(F+\alpha\beta+T_1+B_1)}{2}\right] + \lambda_1 = 0$$

$$\frac{\partial L}{\partial Q_2} = \frac{1}{6}\left[-\frac{4D_2O(C-E-O)}{Q_2^2(C-E)} + \frac{4(F+\alpha\beta+T_2+B_2)}{2}\right] - \lambda_1 + \lambda_2 = 0$$

$$\frac{\partial L}{\partial Q_3} = \frac{1}{6}\left[-\frac{D_1O(C-E-O)}{Q_3^2(C-E)} + \frac{(F+\alpha\beta+T_3+B_3)}{2}\right] - \lambda_2 = 0$$

$$\frac{\partial L}{\partial \lambda_1} = -(Q_2 - Q_1)$$

$$\frac{\partial L}{\partial \lambda_2} = -(Q_3 - Q_2)$$

$$Q_1 = Q_2 = Q_3 = \sqrt{\frac{2(D_3O+4D_2O+D_1O)(C-E-O)}{[(F+\alpha\beta+T_1+B_1)+4(F+\alpha\beta+T_2+B_2)+(F+\alpha\beta+T_3+B_3)](C-E)}}$$

Therefore, the obtained solution satisfies all the specified inequality constraints, and the corresponding total cost is determined using the nonlinear Lagrangian method for the proposed fuzzy inventory model.

NUMERICAL ILLUSTRATION FOR FROZEN SHRIMP SUPPLY CHAIN

To demonstrate the applicability of the proposed inventory model, a real-life inspired numerical illustration is presented for a frozen shrimp supply chain. Frozen shrimp is selected because it is a highly perishable seafood product that requires continuous refrigeration, temperature-controlled transportation, and strict inventory management throughout the cold-chain network.

Product Description

The study considers a distribution system that supplies frozen shrimp packs to supermarkets, restaurants, and retail outlets. The objective is to determine an economical replenishment quantity that minimize the overall inventory cost associated with procurement, refrigeration, storage, transportation, spoilage control, and emergency backup refrigeration operations.

Crisp Parameter Values

The deterministic operating parameters considered for the frozen shrimp supply chain are summarized below.

Parameter	Description	Value
D	Annual demand	12,000 packs
O	Ordering cost parameter	25
C	Refrigeration operating parameter	60
E	Energy efficiency parameter	10
F	Cold-storage holding parameter	8
α	Refrigeration maintenance coefficient	2
β	Spoilage influence coefficient	3
T	Refrigeration maintenance coefficient	12
B	Backup refrigeration cost parameter	5

Using these parameters, the proposed crisp inventory model determines the optimal replenishment quantity for frozen shrimp and the corresponding minimum total inventory cost.

Crisp Model Results

The optimized crisp inventory policy yields the following results.

Performance measure	Value
Optimal replenishment quantity	98 packs
Minimum total inventory cost	3,049.61 cost units

These results indicate that replenishing approximately 98 packs per cycle provides the most economical balance between ordering-related expenses and cold-storage operational costs for the frozen shrimp distribution system.

Trapezoidal Fuzzy Representation

In practical frozen shrimp operations, certain parameters are uncertain due to seasonal demand variation, fluctuating transportation conditions, and possible power interruptions affecting backup refrigeration systems. To capture this uncertainty, the following parameters are represented by trapezoidal fuzzy numbers.

Parameter	Trapezoidal fuzzy representation
$\tilde{D}$	(11,500, 11,800, 12,200, 12,500)
$\tilde{T}$	(10,11,13,14)
$\tilde{B}$	(4,4.5,5.5,6)

The remaining parameters are assumed to remain deterministic because they are relatively stable under normal operating conditions.

Defuzzified Parameter Values

Applying the Graded Mean Integration Representation method converts the trapezoidal fuzzy parameters into crisp values suitable for inventory optimization.

$$\tilde{D} = 12{,}000, \tilde{T} = 12, \tilde{B} = 5$$

After incorporating the defuzzified values into the proposed inventory model, the optimized fuzzy inventory policy for frozen shrimp is obtained.

Performance measure	Value
Optimal replenishment quantity	98 packs
Minimum total inventory cost	3,049.61 cost units

DISCUSSION

The numerical illustration demonstrates that the proposed trapezoidal fuzzy inventory model can effectively represent uncertainty in the frozen shrimp supply chain while preserving the computational structure of the original crisp inventory model. The fuzzy representation provides greater flexibility for handling demand fluctuations, transportation variability, and backup refrigeration uncertainty, which are common in real cold-chain seafood distribution systems.

The obtained results show that the proposed model supports:

- Efficient replenishment planning for frozen shrimp products,
- Better control of refrigeration and storage expenses,
- Reduced spoilage-related operational losses,
- Improved transportation and backup refrigeration management, and
- More reliable decision-making under uncertain cold-chain conditions.

SENSITIVITY ANALYSIS OF THE PROPOSED FROZEN FOOD SUPPLY CHAIN MODEL

Sensitive analysis is performed to examine the influence of triangular fuzzy parameters on the optimal inventory decisions and the total inventory cost of the frozen food supply chain. A dataset consisting of 200 frozen food products was used to evaluate the robustness of the proposed model under varying demand, refrigeration, transportation, and energy-related conditions. The analysis compares both crisp and fuzzy inventory solutions obtained through the GMIR-based defuzzification approach.

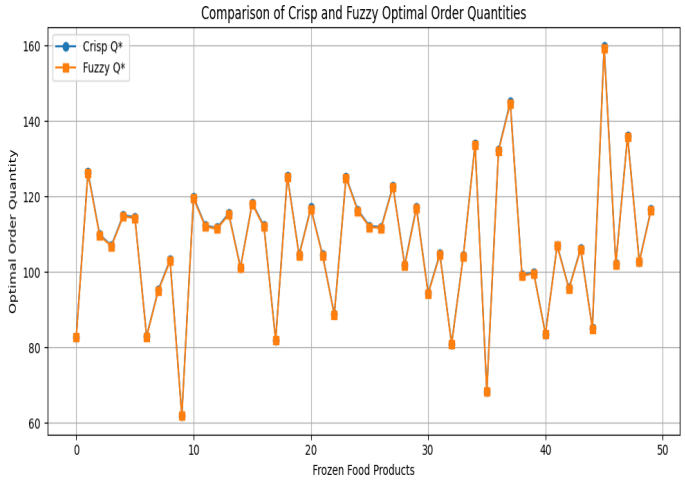


Figure 1 Comparison of Crisp and Fuzzy Optimal Order Quantity

Figure 1 presents the comparison between the crisp optimal order quantity and the fuzzy optimal order quantity for the frozen food products. The two curves exhibit very similar trends, indicating that the GMIR-based fuzzy model preserves the stability of the deterministic solution while incorporating uncertainty in demand, transportation, and backup refrigeration costs. Slight deviations observed for several products demonstrate the ability of the fuzzy model to adjust replenishment decisions under uncertain operating conditions.

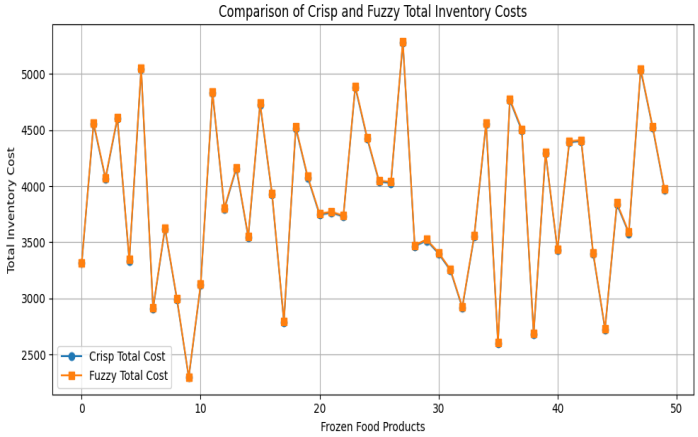


Figure 2 Comparison of Crisp and Fuzzy Total Inventory Costs

Figure 2 illustrates the variation in total inventory cost under crisp and fuzzy environments. The fuzzy total cost follows the same overall pattern as the crisp cost, but moderate fluctuations are observed because of the uncertainty incorporated through triangular fuzzy parameters. The results confirm that the proposed fuzzy model can represent realistic variations in refrigeration, transportation, and spoilage-related expenses without producing unstable inventory policies.

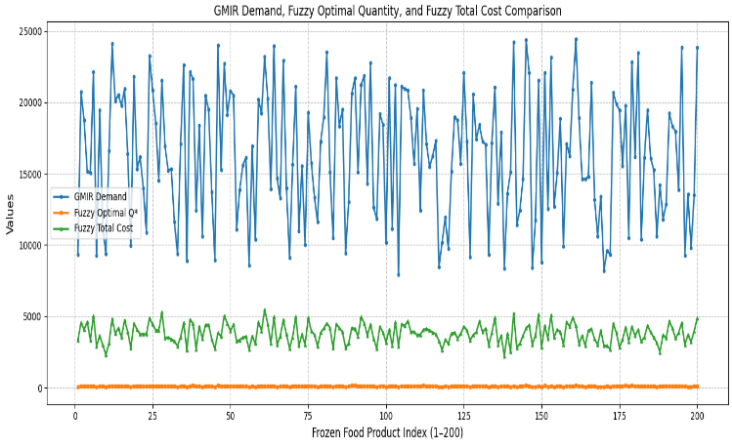


Figure 3 Influence of GMIR Demand on Inventory Performance

Figure 3 shows the relationship between GMIR demand, fuzzy optimal quantity, and fuzzy total inventory cost for all 200 frozen food products. The figure demonstrates that products with higher defuzzified demand generally require larger replenishment quantities, which consequently increases the total inventory cost. This confirms the expected positive relationship between demand intensity and operational cost in frozen food supply chain systems. The GMIR-based demand representation effectively captures uncertain demand behaviour while maintaining computational consistency in the optimization process.

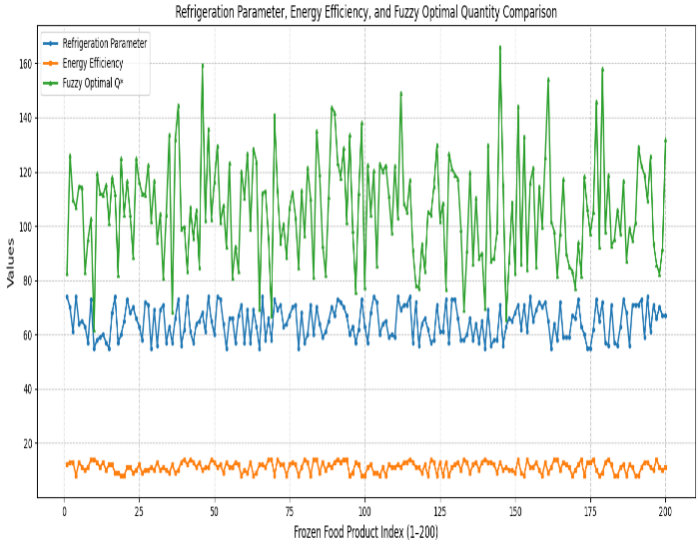


Figure 4 Effect of Refrigeration and Energy Parameters

Figure 4 analyses the effect of the refrigeration parameter and energy efficiency parameter on the fuzzy optimal order quantity. The refrigeration parameter exhibits noticeable variability across the products, reflecting differences in cold-storage requirements. In contrast, the energy efficiency parameter remains relatively stable. The figure indicates that increases in refrigeration-related operational requirements tend to influence the optimal inventory quantity more significantly than variations in energy efficiency, emphasizing the importance of refrigeration management in frozen food inventory systems.

DISCUSSION

The sensitivity analysis reveals several important observations:

- The fuzzy optimal order quantities remain close to the crisp solutions, indicating that the proposed GMIR-based triangular fuzzy model is stable and robust.
- Demand uncertainty has a direct impact on both the optimal replenishment quantity and the total inventory cost.
- Refrigeration-related parameters exert a stronger influence on inventory decisions than energy efficiency variations.
- The fuzzy model successful incorporates operational uncertainty while preserving economically reasonable replenishment policies for all 200 frozen food products.

Overall, the sensitivity analysis demonstrates that the proposed triangular fuzzy inventory optimization framework provides reliable decision support for frozen food supply chain management by effectively balancing demand uncertainty, refrigeration operations, transportation costs, and total inventory expenditure.

CONCLUSION

This study proposed a triangular fuzzy inventory optimization model for the frozen food supply chain by integrating the Graded Mean Integration Representation method with a nonlinear inventory cost framework. The model effectively incorporates uncertainty in demand, transportation cost, and backup refrigeration cost while maintaining the operational simplicity of deterministic inventory parameters. The numerical example demonstrated that the proposed approach can successfully determine the optimal order quantity and the minimum total inventory cost by balancing replenishment, refrigeration, holding, spoilage, transportation, and emergency cooling expenses. The GMIR-based defuzzification procedure provided stable representative values for the fuzzy parameters and enabled efficient inventory optimization under uncertain operating conditions.

The sensitivity analysis conducted using a dataset of 200 frozen food products confirmed the robustness and practical applicability of the proposed model. The results showed that the fuzzy optimal order quantities remain close to the crisp solutions, indicating the stability of the GMIR-based formulation. Furthermore, the analysis revealed that demand variability and refrigeration-related operational parameters have a significant influence on inventory decisions and total cost, emphasizing the importance of cold-storage management in frozen food supply chain operations. Overall, the proposed framework offers a reliable and practical decision-support tool for cost-effective, energy-efficient, and sustainable frozen food inventory management under uncertain environments and provides a useful foundation for future extensions involving multi-product systems, machine learning-based demand forecasting, and real-time cold-chain monitoring technologies.

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