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RESEARCH ARTICLE

# Optimization of Retail Inventory Systems Through Promotional Efforts and Preservation Technology Within a Green Investment Framework

**F. S. Josephine, W. Ritha***

## Abstract

In today's competitive and dynamic market, retail management must strike a balance between environmental responsibility and operational efficiency. This study creates a framework for sustainable retail inventory that combines environmental cost considerations with conventional decision-making elements like pricing, customer demand, and inventory control. The model includes savings from resource optimization and energy efficiency as well as costs associated with carbon taxes, emission penalties, and regulatory compliance. Potential revenue streams from carbon credit mechanisms are also taken into consideration. Cycle length and investment in preservation technologies are two important decision factors that are examined. Finding the best values that minimize overall costs per cycle while preserving product quality and lessening the impact on the environment is the goal. The findings show that the ideal cycle time and investment levels vary slightly, with total costs per cycle ranging from USD 48,000 to USD 49,000. There is potential for long-term cost savings and enhanced sustainability performance with increased investment in preservation technology. The results demonstrate how environmentally friendly retail practices can be profitable. This integrated strategy helps decision-makers achieve profitability while supporting sustainability objectives.

**Keywords:** Retail Inventory Systems, Preservation Technology, Green Investment, Environmental Cost Modelling, Carbon Emission Management.

## Introduction

Retail inventory systems have significantly changed in response to increased market competition, erratic demand, and the growing importance of sustainability. In addition to more traditional factors like pricing and demand, promotional strategies and product deterioration have an impact on inventory decisions in modern retail settings, particularly for perishable goods. Due to substantial financial losses from spoiling, decreased product quality, and increased waste disposal costs, deterioration has an impact on both profitability and customer satisfaction. In order to address these problems, recent research emphasizes the role of preservation technology (PT), which helps extend product shelf life and stabilize inventory levels (Mashud et al., 2021; Tiwari et al., 2022).Concurrently, promotional activities—like price reductions, advertising, and online campaigns—are essential for influencing demand trends and speeding up inventory turnover, especially for goods that must be sold quickly (Barman et al., 2021). In order to achieve operational efficiency in retail systems, it is now crucial to incorporate preservation investment and promotional strategies into inventory models.

Environmental sustainability has become a crucial factor in retail decision-making, along with economic considerations. Green investment frameworks are necessary because retail operations contribute to environmental degradation through energy consumption, packaging waste, and carbon emissions. Businesses can match profitability with sustainability goals by incorporating environmental costs into inventory models, such as carbon

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taxes, emission penalties, and investments in pollution control (Sarkar et al., 2020; De & Mahata, 2019). Additionally, even though they come at an extra cost, green branding and eco-friendly initiatives improve a company's reputation and draw in environmentally conscious customers. According to recent studies, investments in pollution control and energy-efficient technologies not only lessen their negative effects on the environment but also produce long-term financial gains through cost savings and regulatory compliance (Ghosh & Shah, 2023). Therefore, striking a balance between economic performance and environmental responsibility requires creating an integrated inventory model that concurrently takes into account deterioration, promotional efforts, preservation technology, and green investment.

### *Literature Review*

Taleizadeh et al. (2021) created an inventory model that takes price-dependent demand and environmental factors into account, showing that the best pricing strategies have a big impact on both ecological performance and profitability. Similar to this, Wu et al. (2021) looked at deteriorating inventory systems under carbon emission restrictions and demonstrated how important regulatory policies are in influencing inventory choices and emission reduction tactics. In their analysis of the effects of preservation investments on perishable inventory systems, Khan et al. (2022) discovered that these investments greatly improve system stability and decrease product deterioration. In a related study, Yang et al. (2023) improved product quality and decreased waste by integrating cutting-edge storage and monitoring technologies into inventory models. Coordinated promotional strategies can effectively accelerate inventory turnover, according to Choi et al.'s (2022) investigation into the impact of dynamic pricing and advertising on consumer demand for perishable goods. Similarly, Li et al. (2023) highlighted that, especially in competitive retail environments, demand is extremely sensitive to both price fluctuations and marketing intensity. In order to demonstrate the financial advantages of ecologically conscious behavior, Hang and Liu (2022) created a green inventory model that included carbon taxation and emission reduction techniques. In a similar vein, Ahmed et al. (2024) looked at green investment in sustainable supply chain systems and showed that proactive environmental policies increase resilience and long-term cost effectiveness.

Additionally, recent research by Chen et al. (2025) highlighted the incorporation of carbon control mechanisms and renewable energy usage in inventory systems, demonstrating that such strategies greatly improve sustainability performance. The majority of these studies address preservation technology, promotional tactics, and environmental considerations in isolation or in small combinations, despite the fact that they make a substantial contribution to the literature. Relatively little research has been done on a comprehensive framework that concurrently incorporates green investment, promotional efforts, preservation technology, and deterioration control. By creating an integrated model that captures the combined effects of these crucial elements within a single retail inventory system, the current study seeks to close this gap.

### *Mathematical Model*

*Notations*

$S$ : Selling cost (USD/ unit)
$O_c$ : Ordering cost (USD/ order)
$P_c$ : Purchasing cost (USD/ unit)
$\Psi$ : Rate of deterioration in the absence of preservation technologies
$h_c$ : Holding cost (USD/ unit/unit time)
$P_{ce}$ : Cost of carbon emission due to preservation (USD/ unit)
$S_{ce}$ : Cost of carbon emission due to storage (USD/ unit)
$T_{ce}$ : Cost of carbon emission due to transportation (USD/ unit)
$T_\beta$ : Fixed cost of transportation (USD)
$d_C$ : Cost of deterioration (USD/ unit)
$R$ : The cost of transportation variables (USD/ unit)
$\delta$ : The distance in the kilometre from the supplier to the retailer
$\xi$ : Period for the fixed cost of transportation
$\eta$ : Percentage of revenue utilized for advertising
$\mu$ : Elasticity of advertisement
$\kappa$ : Effectiveness of technology for preservation
$N$ : Permitted delay time
$P_e$ : Interest received
$g_t$ : Percentage of interest received that is invested in the green inventory.
$G_c$ : Green branding cost
$E$ : Pollution prevention cost
$TE$ : Total expenses (USD/ cycle)
Decision variables
$L$ : Cycle length (time unit)
$p_t$ : Investment in preservation technologies (USD)

### *Assumptions*

- For Products that are deteriorating naturally, a retail model is taken into consideration. Promotional strategies are employed to increase sales.
- Demand is defined as $D(\mu, S) = x + y\mu + f(S)$ where $f(S)$ is the decreasing function of S and μ is the advertisement elasticity.
- This Study has taken into account the total proportion of this earned interest has investing in green technology since it is though that the permissible delay is bigger than the cycle length.
- There is no shortage.
- Negligible lead time.

### Solution Methodology

The following differential equation shows the inventory level.

$$\frac{dI(l)}{l} + \beta\,(1 - e^{-\kappa P_t})\, I(l) = -\,(x + y\mu + f(S)) \tag{1}$$

Using a boundary condition, where I (l) = 0. The solution of the above equation is,

$$I(l) = \frac{x + y\mu + f(S)}{\beta(1 - e^{-\kappa P_t})}\left[e^{\beta(1-e^{-\kappa P_t})(L-l)} - 1\right] \tag{2}$$

The initial inventory level is determined by

$$Z = I(0) = \frac{x + y\mu + f(S)}{\beta(1 - e^{-\kappa P_t})}\left[e^{\beta(1-e^{-\kappa P_t})L} - 1\right] \tag{3}$$

The total expenses includes ordering cost, holding cost, purchasing cost, transportation cost, carbon emission cost, deterioration cost, promotion cost. Green technology cost, green branding cost, pollution prevention cost.

### Ordering Cost

It describes the expenses involved with the each order placed for inventory replenishment. It involves the several costs related with the purchasing process. It may include costs for the order procedure, communication, and documentation. The ordering cost is determined by

$$OC = \frac{O_c}{L} \tag{4}$$

### Holding Cost

Holding or carrying costs are crucial. It refers to the costs of preserving and storing products for a set of period of time.

This cost may include warehousing, insurance and protection, deterioration and replacement tax and licenses, and handling labour expenses. In the retailing scenario and the holding cost is determined by

$$HC = \frac{h_c\,(x + y\mu + f(S))}{L\,(\beta(1 - e^{-\kappa P_t}))^2}\left[e^{\beta\,(1 - e^{-\kappa P_t})\,L} - L\beta(1 - e^{-\kappa P_t}) - 1\right] \tag{5}$$

### Purchasing Cost

Purchasing cost is often known as acquisition cost, is a key factor for inventory management. It indicates the costs related to acquiring or purchasing inventory products for a business. This cost may contain a number of component costs, such as the cost of products, shipment, and freight. Optimizing supply chain expenses, particularly purchasing prices, is an important aspect of cost control for firms, as it may greatly affect profitability. Strategies to minimize purchasing expenses may include bulk purchase, contracting with suppliers, improving supply chain procedures, and establishing cost effective procurement strategies

$$HC = \frac{h_c\,(x + y\mu + f(S))}{L\,(\beta(1 - e^{-\kappa P_t}))^2}\left[e^{\beta\,(1 - e^{-\kappa P_t})\,L} - L\beta(1 - e^{-\kappa P_t}) - 1\right] \tag{6}$$

### Transportation Cost

Transportation costs include all expenses related to moving products, such as gasoline, labour, vehicle purchasing, maintenance, and related taxes. These expenses, which can be classified as either fixed or variable, have a major impact on the economic efficiency and profitability of business. Despite the volume of shipments, fixed costs like car purchases or warehouse rents don't change. Variable expenses such as fuel, tolls, and driver payment vary in direct proportion to transport activity, distance travelled, and other operational parameters.

$$TC = \frac{1}{L}\left(T_\beta \xi + RZ\delta\right) \tag{7}$$

### Carbon Emission Cost

This term refers to the economic cost of emitting one ton of carbon dioxide into the atmosphere. The environment impact of releasing greenhouse gases is valued economically. Carbon emission from transportation, storage, and preservation processes are the cause of this expense. It is determined by

$$CEC = \frac{1}{L}(S_{ce} + P_{ce} + T_{ce})\left[\frac{(x + y\mu + f(S))}{(\beta\,(1 - e^{-\kappa P_t}))^2}\left[e^{\beta(1-e^{-\kappa P_t})L} - L\beta\,(1 - e^{-\kappa P_t}) - 1\right]\right] \tag{8}$$

### Deterioration Cost

The amount spent on the deterioration or spoiling of the product kept in stock over time is known as the deterioration cost. Perishable goods, medicine, food products, chemicals, and any other product with a short shelf life are particularly affected by this expense. The type of goods, shelf life, and storage circumstances may all play a role. It is determined by

$$DC = \frac{d_c}{L}\left(\psi - \beta\,(1 - e^{-\kappa P_t})\right)\left[\frac{(x + y\mu + f(S))}{(\beta(1 - e^{-\kappa P_t}))^2}\left[e^{\beta(1-e^{-\kappa P_t})L} - L\beta\,(1 - e^{-\kappa P_t}) - 1\right]\right] \tag{9}$$

### Promotion Cost

This refers to marketing and promotional operation aiming at enhancing products visibility, brand awareness, and customer demand. Effective advertising can indirectly affect inventory levels by influencing demand and sales volume.in a wider economic environment; strategically executed advertising can significantly improve revenue. It is thought that a portion of profit is invested in advertising. It is determined by

$$PC = \frac{1}{L}\left(\eta(S - P_c)\frac{(x + y\mu + f(S))}{\beta\,(1 - e^{-\kappa P_t})}\left[e^{\,\beta(1-e^{-\kappa P_t})\,L} - 1\right]\right) \tag{10}$$

### Green Technology Cost

This is referred to as the cost of sustainable or clean technology. It is the expenditure related to the creation, application, and usage of technologies with a minimal impact on environment. These devices are made to encourage sustainable practice, cut emission, and use fewer resources. It may include costs for development, manufacturing and production, implementation and integration, as well as operational and maintenance costs. Because it is expected that the allowable delay period N is higher than the cycle time L, some proportion of generated interest is invested in green technology.

$$GTC = \frac{g_t}{L}\left[Sp_e\left(\int_0^L I(l)\,dl\right) + Sp_e(N-L)\int_0^L (\mathrm{x}+\mathrm{y}\mu+\mathrm{f(S)})\right] \tag{11}$$

### Green Branding Cost

The expense of green branding includes advertising products as environmentally friendly in order to attract consumers.

$$GBC = G_c * d(\mu, S) * L \tag{12}$$

### Pollution Prevention Cost

Pollution prevention costs include investments in clean technology to mitigate environmental damage.

$$PPC = \frac{E}{L} \tag{13}$$

Total expenses is the sum of all the above costs and determined by
TE = OC+HC+ PC+TC+CEC+DC+PC+GTC+GBC+PPC

$$\begin{aligned} TE = {} & \frac{O_c}{L} + \frac{h_c\,(x+y\mu+f(s))}{\mathrm{L}\,(\beta(1-e^{-\kappa P_t}))^2}\left[e^{\beta(1-e^{-\kappa P_t})L} - L\beta(1-e^{-\kappa P_t}) - 1\right] + \frac{p_c\,(\mathrm{x}+\mathrm{y}\mu+\mathrm{f(S)})}{\mathrm{L}\,(\beta(1-e^{-\kappa P_t}))}\left[e^{\beta(1-e^{-\kappa P_t})L} - 1\right] \\ & + \frac{1}{L}(T_\beta\xi + \mathrm{RZ}\delta) + \frac{1}{L}(S_{ce}+P_{ce}+T_{ce})\left[\frac{(\mathrm{x}+\mathrm{y}\mu+\mathrm{f(S)})}{(\beta(1-e^{-\kappa P_t}))^2}\left[e^{\beta(1-e^{-\kappa P_t})L} - L\beta(1-e^{-\kappa P_t}) - 1\right]\right] \\ & + \frac{d_c}{L}\left(\psi - \beta(1-e^{-\kappa P_t})\right)\left[\frac{(\mathrm{x}+\mathrm{y}\mu+\mathrm{f(S)})}{(\beta(1-e^{-\kappa P_t}))^2}\left[e^{\beta(1-e^{-\kappa P_t})L} - L\beta(1-e^{-\kappa P_t}) - 1\right]\right] \\ & + \frac{1}{L}\left(\eta(S-P_c)\frac{(\mathrm{x}+\mathrm{y}\mu+\mathrm{f(S)})}{\beta(1-e^{-\kappa P_t})}\left[e^{\beta(1-e^{-\kappa P_t})L} - 1\right]\right) \\ & + \frac{g_t}{L}\left[Sp_e\left(\int_0^L I(l)dl\right) + Sp_e(N-L)\int_0^L(\mathrm{x}+\mathrm{y}\mu+\mathrm{f(S)})\right] + G_c * d(\mu,S) * L + \frac{E}{L} \end{aligned} \tag{14}$$

By the Taylor series, the equation becomes

$$\begin{aligned} \mathrm{TE}(\mathrm{L}, T_p) & \\ = {} & \left[\mathrm{h_c} + S_{ce} + P_{ce} + T_{ce} + Sp_e g_t + d_c(\psi - \beta(1-e^{-p_t\kappa}))\right]\left(\frac{(\mathrm{x}+\mathrm{y}\mu+\mathrm{f(S)})\mathrm{L}}{2}\right) \\ & + \frac{1}{L}(O_c + T_\beta\xi + \mathrm{E}) + (P_c + R + \eta(S-P_c))\,(\mathrm{x}+\mathrm{y}\mu+\mathrm{f(S)})(1 \\ & + \frac{\beta(1-e^{-p_t\kappa})L}{2}) + Sp_e\,g_t(N-L)(\mathrm{x}+\mathrm{y}\mu+\mathrm{f(S)}) + T_p + G_c * (\mathrm{x}+\mathrm{y}\mu+\mathrm{f(S)}) \\ & * L \end{aligned} \tag{15}$$

The study optimizes the cost function using traditional optimization techniques. The cost function is differentiable with regard to cycle length and preservation technology the first order partial derivative are described below and equating to zero we can obtain $\frac{\partial TE}{\partial L} = 0$ and $\frac{\partial TE}{\partial T_p} = 0$ and the expressions are given below.

$$\frac{\partial TE}{\partial L} = 0$$

$$\begin{aligned} \frac{\partial TE}{\partial L} = {} & \\ & \frac{1}{2}\left[h_c + S_{ce} + P_{ce} + T_{ce} + SP_e g_t + d_c\left(\psi - \beta(1-e^{-\kappa P_t})\right)\right] - \\ & \frac{O_c + T_\beta\xi}{L^2} + \frac{1}{2}(C + R + \eta(S-P_c)d\beta(1-e^{-\kappa P_t}) - \frac{E}{L^2} + \\ & G_c d - SP_e g_t d \end{aligned}$$

$$\begin{aligned} & \frac{1}{2}\left[h_c + S_{ce} + P_{ce} + T_{ce} + SP_e g_t + d_c\left(\psi - \beta(1-e^{-\kappa P_t})\right)\right] - \\ & \frac{O_c + T_\beta\xi}{L^2} + \frac{1}{2}(C + R + \eta(S-P_c)d\beta(1-e^{-\kappa P_t}) - \frac{E}{L^2} + \\ & G_c d - SP_e g_t d = 0 \end{aligned}$$

$$\begin{aligned} & \frac{1}{2}\left[h_c + S_{ce} + P_{ce} + T_{ce} + SP_e g_t + d_c\left(\psi - \beta(1-e^{-\kappa P_t})\right)\right] d + \\ & \frac{1}{2}(C + R + \eta(S-P_c)d\beta(1-e^{-\kappa P_t}) + G_c d - \\ & SP_e g_t d = \frac{O_c + T_\beta\xi + E}{L^2} \end{aligned}$$

$$\begin{aligned} & \frac{1}{2}\left[h_c + S_{ce} + P_{ce} + T_{ce} + SP_e g_t + d_c\psi + (C + R + \eta(S-P_c) - d_c)\beta(1-e^{-\kappa P_t})\right] d + \\ & G_c d - SP_e g_t d = \frac{O_c + T_\beta\xi + E}{L^2} \end{aligned}$$

$$\begin{aligned} & \frac{d}{2}\left[\left[h_c + S_{ce} + P_{ce} + T_{ce} + SP_e g_t + d_c\psi + (C + R + \eta(S-P_c) - d_c)\beta(1-e^{-\kappa P_t})\right] + \right. \\ & \left. 2G_c - 2\,SP_e g_t\right] = \frac{O_c + T_\beta\xi + E}{L^2} \end{aligned}$$

$$\begin{aligned} & L^2 \\ & = \frac{2(O_c + T_\beta\xi + E)}{d\begin{bmatrix} h_c + S_{ce} + P_{ce} + T_{ce} + SP_e g_t + d_c\psi + 2G_c + \\ (C + R + \eta(S-P_c) - d_c)\beta(1-e^{-\kappa P_t}) \end{bmatrix} - 2\,SP_e g_t} \end{aligned}$$

$$\mathbf{L}^* = \sqrt{\frac{2(O_c + T_\beta\,\boldsymbol{\xi} + E)}{d\begin{bmatrix} h_c + S_{ce} + P_{ce} + T_{ce} + SP_e g_t + d_c\psi + 2G_c + \\ (C + R + \eta(S-P_c) - d_c)\beta(1-e^{-\kappa P_t}) \end{bmatrix} - 2\,SP_e g_t}} \tag{16}$$

$$\begin{aligned} \frac{\partial TE}{\partial T_p} = {} & -\frac{1}{2}d_c\,dL\beta\alpha e^{-\kappa P_t} + \frac{1}{2}\,(C + R + \\ & \eta(S-P_c))dL\beta\alpha e^{-\kappa P_t} + 1 \\ & -\frac{1}{2}d_c\,dL\beta\alpha e^{-\kappa P_t} \\ & \quad + \frac{1}{2}\,(C + R \\ & \quad + \eta(S-P_c))dL\beta\alpha e^{-\kappa P_t} + 1 = 0 \end{aligned}$$

$$\begin{aligned} \frac{1}{2}\,dL\beta\alpha e^{-\kappa P_t}\left[(C + R + \eta(S-P_c)) - d_c\right] & \\ = -1 & \end{aligned}$$

$$\frac{1}{2}\,dL\beta\alpha e^{-\kappa P_t}\left[d_c - (C + R + \eta(S-P_c))\right] = 1$$

$$e^{-\kappa P_t} = \frac{2}{dL\beta\alpha\left[d_c - (C + R + \eta(S-P_c))\right]}$$

Taking log on both sides,

$$-\kappa P_t = \log\left(\frac{2}{dL\beta\alpha\left[d_c - (C + R + \eta(S-P_c))\right]}\right)$$

$$p_t{}^* = \frac{1}{\kappa}\,\log\left(\frac{dL\beta\alpha\left[d_c - (C + R + \eta(S-P_c))\right]}{2}\right) \tag{17}$$

$$\boldsymbol{L}^* = \sqrt{\frac{\mathbf{2}(\boldsymbol{O_c} + \boldsymbol{T_\beta}\boldsymbol{\xi} + \mathbf{E})}{\boldsymbol{x} + \boldsymbol{y\mu} + \boldsymbol{f(S)}[\mathbf{h_c} + \boldsymbol{S_{ce}} + \boldsymbol{P_{ce}} + \boldsymbol{T_{ce}} + \boldsymbol{Sp_e g_t} + \boldsymbol{d_c}\boldsymbol{\psi} + \mathbf{2}\boldsymbol{G_c} + (\boldsymbol{P_c} + R + \boldsymbol{\eta}(\boldsymbol{S} - \boldsymbol{P_c}) - \boldsymbol{d_c})\boldsymbol{\beta}(\mathbf{1} - \boldsymbol{e}^{-\,\boldsymbol{p_t\kappa}})] - \mathbf{2}\boldsymbol{Sp_e g_t}}} \tag{18}$$

$$\boldsymbol{p_t} * = \frac{\mathbf{1}}{\boldsymbol{\kappa}}\,\mathbf{log}\left(\frac{\boldsymbol{\beta\alpha}(\mathbf{x} + \mathbf{y\mu} + \mathbf{f(S)})\mathbf{L}\,[\boldsymbol{d_c} - (\boldsymbol{P_c} + \boldsymbol{R} + \boldsymbol{\eta}(\boldsymbol{S} - \boldsymbol{P_c}))}{\mathbf{2}}\right) \tag{19}$$

### *Numerical Examples*

*Example 1*

S = 14(USD/unit), $P_c$ = 5(USD/unit), $h_c$ = 1(USD/unit/year), $T_{ce}$ = 0.2(USD/unit), $T_\beta$ = 70(USD/order), R = 0.1(USD/unit), ξ = 1, x = 8000, y = 7, N = 0.1, $g_t$ = 0.6, β = 0.1, E = 95, κ = 0.01, $O_c$ = 100(USD/unit), ψ = 0.5(USD/unit), $S_{ce}$ = 0.2(USD/unit), $P_{ce}$ = 0.1(USD/unit), $d_c$ = 10(USD/unit), η = 0.02(USD/unit), μ = 0.4, δ = 1, f(S) = 0.07, $G_C$ = 3

EQUATION OF TE, L*, $T_p^*$

$$\begin{aligned}\mathrm{TE}(\mathrm{L},T_p) &= [\mathrm{h_c} + S_{ce} + P_{ce} + T_{ce} + Sp_e g_t \\ &+ d_c(\psi - \beta(1-e^{-p_t\kappa}))]\left(\frac{(\mathrm{x + y\mu + f(S))L}}{2}\right) \\ &+ \frac{1}{L}\,(O_c + T_\beta\xi + \mathrm{E}) + (P_c + R + \eta(S-P_c))\,(\mathrm{x+y\mu} \\ &+ \mathrm{f(S)})(1 + \frac{\beta(1-e^{-p_t\kappa})L}{2}) + Sp_e\, g_t(N-L)(\mathrm{x+y\mu+f(S)}) \\ &+ T_p + G_c * (\mathrm{x+y\mu+f(S)}) * L\end{aligned}$$

$$L^* = \sqrt{\frac{2(O_c + T_\beta\xi + \mathrm{E})}{x + y\mu + f(S)[\mathrm{h_c} + S_{ce} + P_{ce} + T_{ce} + Sp_e g_t + d_c\psi + 2G_c + (P_c + R + \eta(S-P_c) - d_c)\beta(1-e^{-p_t\kappa})] - 2Sp_e g_t}}$$

$$p_t * = \frac{1}{\kappa}\log\left(\frac{\beta\alpha(\mathrm{x+y\mu+f(S)})\mathrm{L}\,[d_c - (P_c + R + \eta(S-P_c))}{2}\right)$$

By substituting all the values in these equations, the optimum solution is

$$L^* = \sqrt{\frac{2(100+7*0.1+95)}{8000+7*0.4+0.07[1+0.2+0.1+0.2+14*0.03*0.6+10*0.5+2*3+(5+0.1+0.02(14-5)-10)0.1((1-e^{-0.01*14})-2*14*0.03*0.6]}}$$

$$L^* = \sqrt{\frac{530}{8002.87[6.752+6-0.2157-0.5040]}}$$
$$= \sqrt{\frac{530}{8002.87[12.752-0.7197]}}$$
$$= \sqrt{\frac{530}{8002.87[12.0323]}}$$
$$= \sqrt{\frac{530}{96292.9327}}$$
$$= \sqrt{0.0055}$$

L* = 0.0742

$$p_t^* = \frac{1}{0.01}\log\left(\frac{0.5938 + 4.72}{2}\right)$$

= 100 log (1.4014)

$p_t^*$ = 14.65

TE = 1.75 + 1336.1 + 3571.43 + 42326.98 + 52 + 14.65 + 1781.44

TE = 49084.35

### *Example 2*

S = 14(USD/unit), $P_c$ = 5(USD/unit), $h_c$ = 0.8(USD/unit/year), $T_{ce}$ = 0.16(USD/unit), $T_\beta$ = 60(USD/order), R = 0.08(USD/unit), ξ = 1,
x = 8000, y = 7, N = 0.1, $g_t$ = 0.6, β = 0.1, E = 80, κ = 0.01, $O_c$ = 80(USD/unit), ψ = 0.4(USD/unit), $S_{ce}$ = 0.16(USD/unit), $P_{ce}$ = 0.08(USD/unit), $d_c$ = 0.4(USD/unit), η = 0.015(USD/unit), μ = 0.4, δ = 1, f(S) = 0.07, $G_C$ = 2.5

EQUATION OF TE, L*, $T_p^*$

$$\begin{aligned}\mathrm{TE}(\mathrm{L},T_p) &= [\mathrm{h_c} + S_{ce} + P_{ce} + T_{ce} + Sp_e g_t \\ &+ d_c(\psi - \beta(1-e^{-p_t\kappa}))]\left(\frac{(\mathrm{x + y\mu + f(S))L}}{2}\right) \\ &+ \frac{1}{L}\,(O_c + T_\beta\xi + \mathrm{E}) + (P_c + R + \eta(S-P_c))\,(\mathrm{x+y\mu} \\ &+ \mathrm{f(S)})(1 + \frac{\beta(1-e^{-p_t\kappa})L}{2}) + Sp_e\, g_t(N-L)(\mathrm{x+y\mu+f(S)}) \\ &+ T_p + G_c * (\mathrm{x+y\mu+f(S)}) * L\end{aligned}$$

$$L^* = \sqrt{\frac{2(O_c + T_\beta\xi + \mathrm{E})}{x + y\mu + f(S)[\mathrm{h_c} + S_{ce} + P_{ce} + T_{ce} + Sp_e g_t + d_c\psi + 2G_c + (P_c + R + \eta(S-P_c) - d_c)\beta(1-e^{-p_t\kappa})] - 2Sp_e g_t}}$$

$$p_t * = \frac{1}{\kappa}\log\left(\frac{\beta\alpha(\mathrm{x+y\mu+f(S)})\mathrm{L}\,[d_c - (P_c + R + \eta(S-P_c))}{2}\right)$$

By substituting all the values in the above equations, the optimal solution is

$$L^* = \sqrt{\frac{2(80+60*1+80)}{8000+7*0.4+0.07[0.8+0.16+0.08+0.16+14*0.03*0.6+10*0.4+2*2.5+(5+0.08+0.015(14-5)-10)0.1((1-e^{-0.01*61})-2*14*0.03*0.6]}}$$

$$L^* = \sqrt{\frac{440}{8002.87[5.2+5-0.2187-0.5040]}}$$

$$L^* = \sqrt{\frac{440}{8002.87[5.2+5-0.2187-0.5040]}}$$
$$= \sqrt{\frac{440}{8002.87[10.4520-0.7227]}}$$
$$= \sqrt{\frac{440}{8002.87[9.7293]}}$$
$$= \sqrt{\frac{440}{77862.3231}}$$
$$= \sqrt{0.0057}$$

L* = 0.0755

$$p_t{}^* = \frac{1}{0{\cdot}01}\log(\frac{0.6042 * 5.2}{2})$$

= 100 log (1.5710)

$p_t^* = 19.6$

TE = 1.45 + 1069.46+ 3571.43 + 41846 + 49 + 19.6 + 1511

TE = 48065.10

**THE OPTIMUM SOLUTION IS**

| | | |
|---|---|---|
| L* | 0.0742 years | 0.0755 years |
| $p_{t*}$ | USD 14.65 | USD 19.6 |
| TE | USD 49084.35/cycle | USD 48065.10 /cycle |

The optimal cycle time (L*) reduces total cost per cycle (TE), whereas the optimal preservation technology cost ( $p_t$ *) balances preservation investment and deterioration losses. This yields the best cost-effective inventory strategy under the given conditions

## Discussion

The current study's findings are in line with recent developments in sustainable inventory modelling, especially when it comes to combining preservation technology with environmental factors. Ali et al. (2025), for example, created a green inventory framework in which demand is based on price, stock, and environmental "green level," demonstrating that preservation investment greatly lowers deterioration and enhances system performance. Similar to this, Kumari and Gill (2024) showed that preservation technology stabilizes overall costs and successfully controls deterioration rates, though its direct cost impact may differ based on demand structure. These conclusions are corroborated by the current model, which demonstrates that the best investment in preservation technology results in increased cost effectiveness and decreased waste, especially when paired with marketing tactics that quicken inventory turnover.

Comparatively speaking, this study's inclusion of promotional activities goes beyond many conventional sustainable inventory models, which frequently concentrate mostly on pricing and deterioration. When Priyadharshini and Uthayakumar (2025) integrated time-dependent advertising into a green inventory model, they discovered that the demand for eco-friendly products is greatly increased by promotional intensity. Similarly, Aggarwal and Sharma (2024) highlighted how preservation technology and green marketing work together to optimize inventory choices under sustainability constraints. By explicitly balancing promotional costs with preservation investment and environmental expenses, the current model offers a more integrated framework in contrast. This demonstrates that promotional tactics are essential instruments for lowering deterioration-related losses in addition to being demand drivers.

The study also backs the growing consensus that, in order to achieve long-term sustainability, environmental costs must be taken into account when making inventory decisions. By showing that the best cycle length and preservation investment can be attained while taking environmental costs into consideration, the study's findings support this direction and offer managerial and policy-level insights.

## Conclusion

This study effectively creates an integrated sustainable inventory system for decaying products that maintains a balance between environmental and economic goals. The concept shows how these factors produce conflicting effects on ideal cycle durations by combining proactive pollution control charges for clean technology with strategic green branding investments for market positioning. Green branding encourages more frequent ordering to reduce marketing costs per unit, whereas pollution protection promotes longer cycles for distributing fixed investments. The optimized decisions greatly lower total expenses while advancing sustainability objectives. This study offers a sustainable strategy for retail companies dedicated to environmental conservation by giving management a useful, analytically based tool to adopt greener practices while retaining revenue.

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