THE SCIENTIFIC TEMPER (2026) Vol. 17 (4): 6206-6212
E-ISSN: 2231-6396, ISSN: 0976-8653
Doi: 10.58414/SCIENTIFICTEMPER.2026.17.4.31
https://scientifictemper.com/

RESEARCH ARTICLE

Bianchi Type-I Bulk Viscous String Cosmological Model with a Dynamical Cosmological Term

Archana Gajanan Ingle[1*], Shailendra D. Deo[2]

Abstract

The Einstein field equations with a cosmological constant are analyzed for a Bianchi type-I universe in the presence of bulk viscosity. The analysis assumes that: the bulk viscosity coefficient is directly proportional to the expansion scalar $(\varsigma \propto \theta)$; the expansion scalar is directly proportional to the shear scalar $(\theta \propto \sigma)$ and the cosmological constant $\wedge$ is directly proportional to the Hubble parameter $(\Lambda \propto H)$. The physical and cosmological implications of these solutions are presented along with a discussion.

Keywords: Bianchi type-I cosmology, Bulk viscosity, String cosmological model, Dynamical cosmological constant, Einstein field equations, Anisotropic universe.

Introduction

The cosmological constant problem remains one of the most eminent and undetermined problems in modern cosmology, as the origin of the universe remains among the greatest unsolved secrets. The pinpoint physical conditions during the nascent universe are still unknown. One possible way to find solutions for the field equations by the law of variation of scale factor, which is given by Pavon (1991). During the past few decades, there has been a growing interest in the progress of string cosmological models to explain the early universe. Cosmic strings are proposed as a possible source of density perturbations, which are necessary for the production of large-scale cosmic structures, and string theory has been developed to explain phenomena that take place in this primordial epoch. As a result, there has been a lot of interest in studying cosmic strings in the context of general relativity. Letelier (1979) and Stachel (1980) devised the general relativistic description of cosmic strings, while Gott III (1985) and Letelier (1983) thoroughly investigated their gravitational effects. In the presence of a magnetic field, Chakraborty (1991) investigated string cosmological theories. Xing (2006) investigated the Bianchi type-III string cosmological model involving bulk viscosity and magnetic fields, whereas Kibble (1976) examined the topology of cosmic domains and strings.

In the context of general relativity, Bali and Upadhaya (2003) recently investigated the LRS Bianchi type-I string cosmological model with a bulk viscous fluid. Bali and Pradhan (2007) investigated Bianchi type-III string cosmological models with time-dependent bulk viscosity in a different study. Additionally, the non-existence of Bianchi type-V string cosmological models incorporating a bulk viscous fluid in general relativity was examined Adhav et al. (2007).

Friedmann cosmology on a codimension-2 brane with time-dependent tension was also investigated by Zhang et al. (2006). The brane tension's absolute value has no bearing on the effective cosmological constant in this paradigm. Numerous scholars have also conducted more investigations in this area. References Yadav et al. (2007); Bali and Anjali (2004); Tiwari (2008); Xing (2003); Xing (2004); Xing (2005); JP et al. (2007); Vishwakarma (2005); Beesham (1993); Tiwari (2011); Pradhan et al. (2007); Pradhan et al. (2006) are cited.

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[1]PGTD Mathematics, Gondwana University, Gadchiroli, Maharashtra, India

[2]Mahatma Gandhi College of Science, Gadchandur, Chandrapur, Maharashtra, India

***Corresponding Author:** Archana Gajanan Ingle, PGTD Mathematics, Gondwana University, Gadchiroli, Maharashtra, India, E-Mail: archanaingle939@gmail.com

How to cite this article: Ingle, A.G., Deo, S.D. (2026). Bianchi Type-I Bulk Viscous String Cosmological Model with a Dynamical Cosmological Term. The Scientific Temper, **17**(4):6206-6212.

Doi: 10.58414/SCIENTIFICTEMPER.2026.17.4.31

Source of support: Nil

Conflict of interest: None.

Received: 06/04/2026 **Accepted:** 21/04/2026 **Published:** 25/04/2026

Over the frame of General Relativity (GR), a number of modified theories of gravity and exotic fluid models have been proposed in an effort to understand the universe's accelerating expansion. Among these, modified gravity frameworks $\int(R,T)$, $\int(Q,T)$ and $\int(G)$ gravities, higher - and lower-dimensional space-time models, and Chaplygin gas have collected a lot of attention because of their plausible to test the thermodynamic consistency of cosmic evolution, chronology for dark energy and dark matter phenomena, and unite early- and late-time cosmic acceleration.

Modified Chaplygin Gas (MCG) models, which expertly introduce between dust-like matter at early epochs and a cosmological constant at later ages, were held into account as dark energy candidates, laying the foundation for this line of study. Khadekar, Kumar, and Islam (2019) investigated MCG cosmology in the sight of bulk viscosity in an innovative (2+1)-dimensional framework, showing that viscous influences greatly affect the cosmic dynamics in the Friedmann Robertson Walker (FRW) space-time Khadekar et al., 2019). Islam et al. (2019) expanded this work in $\int(R,T)$ gravity by generating (2+1)-dimensional cosmological models with an adaptable cosmological constant $\Lambda(R,T)$, showing how the junction between geometry and matter drives rapid expansion.

Georgiev et al. (2020) established that a strict analysis of integral inequalities on time scales, which offer a unified path to discrete and continuous cosmological evolution equations, robustly formalizes the structure of these cosmological frameworks mathematically. With this process, Kumar and Khadekar (2021) examined holographic dark energy (HDE) models with scalar fields and MCG in (2+1)-dimensional space-time, showing the relation between modified fluid cosmologies and holographic principles Praveen and Khadekar, 2021). These works show that low-dimensional cosmological models are useful for examining the nature of dark energy after they are both mathematically and physically tractable.

Cosmic models have been beyond advanced by the addition of viscosity, scalar fields, and electromagnetic effects. Islam and Kumar (2021), for example, examined Reissner NordstrÃ˝um type stellar configuration in $\int(R,T)$ gravity, emphasizing the influence of matter-geometry coupling on compact star structure (Islam and Praveen (2021)). Analogous to this, Kumar, Khadekar, and Dagwal (2022) investigated scalar field dynamics as a driver of cosmic expansion by presenting a twofluid model within the Saez Ballester scalar tensor theory in (2+1)-dimensional space-time Praveen et al., (2022). As noted by Kumbhar et al. (2022) Kumbhar et al. (2022), the preface of string clouds and quark matter in higher-dimensional and viscous fluid cosmologies significantly expanded the scope. At cosmological scales, these theories offer a miniature illustration of matter interactions.

Evolutions in modified gravity theories, like Weyl-type $\int(Q,T)$ gravity, where Gadbail et al. (2022) examined the interaction of divergence-free deceleration parameters and produced a geometrically consistent model of cosmic acceleration Gadbail et al., (2022), have been accompanied by theoretical advancements. In a similar vein, viscous modified ghost scalar field models with variable gravitational coupling G were developed by Talole, Kumar, and Islam (2023), denoting that dark energy evolution might be mimicked by time variation in gravitational constants Talole et al., 2023). A viscous Modified Cosmic Chaplygin Gas (MCCG) cosmology with a changeable Λ was built by Ramtekkar, Kumar, and Khadekar (2023), connecting fluid viscosity in FRW models to the dynamical cosmological constant Ramtekkar et al. (2023).

A united depiction of anisotropic cosmic sources was realized by beyond advancements that added string clouds with quark matter and viscous fluids admitting conformal motion Islamet al. (2024). Contrarily Panda et al. (2024) investigated the thermodynamics of non-canonical $\int\left(\overline{R},\overline{T}\right)$ gravity, examining thermodynamic stability and entropy production within the dark universe framework Panda et al. (2024). Simultaneously, Khadekar et al. (2025) prompted a viscous mimetic gravity model motivated by the holographic principle, providing a cohesive explanation of cosmic acceleration in both early and late times Khadekar et al. (2025). In a similar manner, Dhankar et al. (2025) used several cosmological data sets to supervise a combined observational analysis of $\int(R)$ parametrizations, constraining model parameters using statistical methods like Markov Chain Monte Carlo (MCMC) analysis Kumar et al. (2025).

An important evolution in concurrent cosmology is the manifestation of observational data analysis into theoretical models. By examining bulk viscosity perturbations in MCG cosmology, Munyeshyaka, Dhankar, and Ntahompagaze (2025) settled connections between thermodynamics and structure formation Munyeshyaka et al. (2025). Thereafter, Dhankar et al. (2025) used MCMC statistical inference to explore cosmological constraints on interacting holographic dark energy and extended Chaplygin gas in higher dimensions, verifying that these models are coherent with current data (Thakran et al. (2025)).

Modified Gauss-Bonnet gravity $\left(\int(G)\right)$ has seen latest ascents. In power-law $\int(G)$ gravity, Munyeshyaka et al. (2025) examined the matter power spectrum, bridging the gap between theoretical predictions and large-scale structural observations Munyeshyaka et al. (2025). In a associated research, Samanta et al. (2025) investigated the dynamical stability of the scalar-tensor sector by examining tachyonic field configurations associated with global monopoles in Brans âĂ, SDicke theory Samanta et al. (2025). Dhankar et al. (2025) nurtured the thermodynamic perspective by elongating holographic cosmology under a generalized equation of state framework Dhankar et al.

(2025a) and testing the Generalized Second Law (GSL) in (2+1)-dimensional cosmology using holographic entropy bounds and observational constraints Dhankar et al. (2025b).

Also, a diversity of observational data sets were used to constrain multi-fluid cosmologies in modified Gauss-Bonnet gravity, demonstrating the flexibility of $\int(\mathrm{G})$ models in explaining present cosmic acceleration Dhankar et al. (2025c). The observational feasibility of such strange fluids was confirmed by MCMC-based investigations of bulk viscous MCG cosmologies in lower dimensional environments, which commended these studies Dhankar et al. (2025d). Investigation of the matter power spectrum in $\int(\mathrm{Q}, \mathrm{L}_m)$ gravity was used to address the broad inferences of structure development by establishing a connection between geometric âĂ, Smatter coupling and cosmological observables Dhankar et al. (2025e). Finally, Sanyal et al. (2025) examined cosmic hysteresis in reconstructed $\int(\mathrm{R})$ bounce models, highlighting the thermodynamic implications of cyclic cosmology Sanyal et al. (2025), while Dhankar et al. (2025) tested Gauss âĂ, SBonnet gravity models using the Dark Energy Spectroscopic Instrument (DESI) BAO data, giving one of the first extensive empirical validations of modified gravity at cosmic scales Dhankar et al. (2025f).

Formulation of the Problem

We examine Bianchi type-I space-time with a variable cosmic constant Λ in this letter. We assume that the cosmological constant is proportional to the Hubble parameter $(\Lambda \propto H)$, the expansion scalar is proportional to the shear scalar $(\theta \propto \sigma)$, and the coefficient of bulk viscosity is proportional to the expansion scalar $(\zeta \propto \theta)$ in order to obtain an explicit solution.

Consider the form of Bianchi type-I metric

$$ds^2 = -dt^2 + X_1^2\,dx^2 + X_2^2\,dy^2 + X_3^2\,dz^2, \tag{1}$$

where X_1, X_2, and X_3 are functions of time t.

For a cloud of strings with bulk viscosity, the energy-momentum tensor is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(u_i u_j + g_{ij}), \tag{2}$$

Where $\rho = \rho_p + \lambda$ is the rest energy density of the string cloud with particles attached to it, ρ_p is the rest energy density of the particles, $\theta = u^i{}_{;i}$ is the scalar of expansion, ζ is the bulk viscosity coefficient, and λ is the tension density of string cloud. In this case, u^i represents the cloud four velocity vector, and ζ^i represents the anisotropic directions, or string directions.

They satisfy the relation given by Eq. (3)

$$u^i u_i = x^i x_i = -1, \qquad u^i x_i = 0. \tag{3}$$

The shear scalar σ and scalar of expansion θ are

$$\theta = u^i;_i = \frac{\dot{X}_1}{X_1} + \frac{\dot{X}_2}{X_2} + \frac{\dot{X}_3}{X_3}\,, \tag{4}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{\dot{X}_2^2}{X_1^2} + \frac{\dot{X}_3^2}{X_3^2} + \frac{\dot{X}_3^2}{X_3^2} - \frac{\dot{X}_1\dot{X}_2}{X_1X_2} - \frac{\dot{X}_2\dot{X}_3}{X_2X_3} - \frac{\dot{X}_1\dot{X}_3}{X_1X_3}\right) \tag{5}$$

where the Hubble parameter is denoted by H.

The variable cosmological term $\Lambda(t)$ and Einstein's field equations with $8\pi G = 1$ are

$$R_{ij} - \frac{1}{2}Rg_{ij} = T_{ij} - \Lambda(t)g_{ij} \tag{6}$$

Einstein's field equations for the metric(1) can be expressed as

$$\frac{\ddot{X}_2}{X_2} + \frac{\ddot{X}_3}{X_3} + \frac{\dot{X}_2\dot{X}_3}{X_2X_3} = \zeta\theta - \Lambda \tag{7}$$

$$\frac{\ddot{X}_1}{X_1} + \frac{\ddot{X}_3}{X_3} + \frac{\dot{X}_1\dot{X}_3}{X_1X_3} = \zeta\theta - \Lambda \tag{8}$$

$$\frac{\ddot{X}_1}{X_1} + \frac{\ddot{X}_3}{X_3} + \frac{\dot{X}_1\dot{X}_2}{X_1X_2} = \zeta\theta - \Lambda \tag{9}$$

$$\frac{\dot{X}_1\dot{X}_2}{X_1X_2} + \frac{\dot{X}_2\dot{X}_3}{X_2x_3} + \frac{\dot{X}_1\dot{X}_3}{X_1X_3} = \lambda - \rho \tag{10}$$

$$\frac{\dot{X}_1}{X_1} - \frac{\dot{X}_2}{X_2} = 0 \tag{11}$$

where the ordinary differentiation with regard to t is shown by dots on X_1, X_2 and X_3. Equation (11) gives us

$$X_1 = n_1 X_2, \tag{12}$$

where n_1 is an integration constant.

We assume that the coefficient of bulk viscosity is proportional to the expansion scalar Xing (2006); Tiwari and Dwivedi (2009); Tiwari and Sharma (2011), in order to get more generic solution.,

i.e.,

$$\zeta = n\theta \tag{13}$$

The six unknowns X2, X3, ζ, Λ, ρ and λ are now involved in four separate equations (7), (10), (12), and (13) respectively. Therefore, in order to solve the system, two more relationships are required.

Solution Methodology

We assume that the shear scalar and the scalar of expansion are proportional. When one parameter evolves, it is thought that the other parameters should follow Tiwari and Dwivedi (2009); Roy et al. (1985); Collins et al. (1980). In general, the above condition implies to

$$B = X_3^m \tag{14}$$

where m is a constant, and the second condition is

$$\Lambda = aH \tag{15}$$

When Λ gradually diminishes with cosmic time rather than being a pure constant. Additionally, Λ fluctuates in a way that conserves energy Arbabl (1997). Using Eq. (12) and substituting Eq. (15) into Eq. (4), we get

$$\theta = (2m+1)\frac{\dot{X}_3}{X_3}, \tag{16}$$

$$\zeta = n(2m+1)\frac{\dot{X}_3}{X_3}. \tag{17}$$

Eq. (7) can be written by using Eqs. (4), (15), (16), and (17) as

$$\left(\frac{\dot{X}_3}{X_3}\right) + \left\{1 + \frac{m^2 - n(2m-1)}{m+1}\right\}\frac{\dot{X}_3^2}{X_3^2} = \frac{-a(2m+1)}{3(m+1)}\frac{\dot{X}_3}{X_3}. \tag{18}$$

Integration yields

$$\frac{\dot{X}_3}{X_3} = \left[\frac{(m^2+n+1) - n(2m+1)^2}{(m+1)}\left\{\frac{3(m+1)}{-a(2m+1)}.\exp\left(\frac{-a(2m+1)t}{3(m+1)} + n_2\right) + n_3\right\}\right]^{-1} .\exp\left(\frac{-a(2m+1)t}{3(m+1)} + n_2\right) \tag{19}$$

Therefore, the line element (5.1) becomes:

$$ds^2 = -dt^2 + (n_1^2 dx^2 + dy^2 + dy^2)\left[A(t)\right]^{\frac{2n(m+1)}{(m^2+m+1)-n(2m+1)^2}} + \left[A(t)\right]^{\frac{2(m+1)}{(m^2+m+1)-n(2m+1)^2}} dz^2 \tag{20}$$

$$\theta = (2m+1)\left[A(t)\right]^{-1}\ E(t) \tag{21}$$

$$\zeta = n(2m+1)\left[A(t)\right]^{-1}\ E(t) \tag{22}$$

$$\rho = 3n(2m+1)^2\left[A(t)\right]^{-2}\ E(t)^2 \tag{23}$$

$$H = \frac{(2m+1)}{1}\left[A(t)\right]^{-1}\ E(t) \tag{24}$$

$$\Lambda = \frac{a(2m+1)}{3}\left[A(t)\right]^{-1}\ E(t) \tag{25}$$

$$\rho_p = -m(m+2)\left[A(t)\right]^{-2}\ E(t)^2 \tag{26}$$

$$\lambda = \left(m^2 + 2 + 3n(2m+1)^2\right)\left[A(t)\right]^{-2}\ E(t)^2 \tag{27}$$

$$\sigma = \sqrt{\frac{(m+1)^2}{3}}\left[A(t)\right]^{-2}\ E(t)^2 \tag{28}$$

where

$$A(t) = \frac{(m^2+m+1) - n(2m+m+1)^2}{(m+1)}\left\{\frac{3(m+1)}{-a(2m+1)}E(t) + n_3\right\}, \tag{29}$$

and

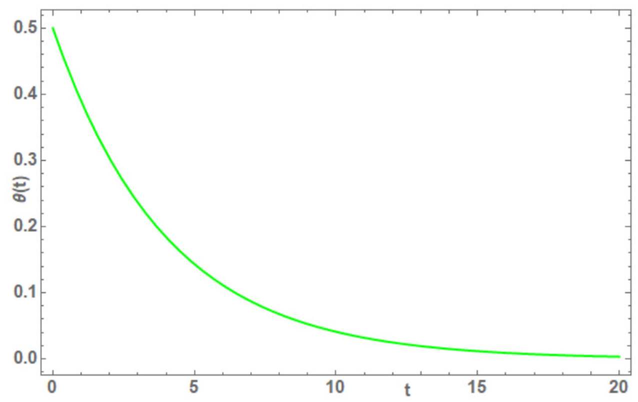

Figure 1: Expansion scalar versus cosmic time showing exponential decay from an initial high value toward zero

$$E(t) = \exp\left(\frac{-a(2m^2+1)t}{(3m+1)} + n_2\right) \tag{30}$$

$$V^3 = X_1 X_2 X_3 \tag{31}$$

Results and discussion

Numerically, the expansion scalar θ(t) begins at about 0.5 when t = 0, indicating a very high initial expansion rate. As time progresses toward t = 20, the scalar decreases smoothly and monotonically, approaching zero asymptotically. This behavior is consistent with the analytic solution $\theta(t) = \frac{2m+1}{3(m+1)}\exp\left(-\frac{a(2m+1)}{3(m+1)}t\right)$, which describes an exponential decay. The numerical trend shows that the universe expands rapidly at early times but the rate of expansion slows steadily, never reaching exactly zero but becoming negligible at large t. Physically, this plot captures the essence of a big âĂS′ bang origin followed by continual expansion. The high initial value of θ reflects the violent expansion at the birth of the universe, while the exponential decay illustrates the damping effect of bulk viscosity, which resists expansion and gradually reduces its rate. Importantly, the scalar does not vanish completely, meaning the cosmos continues to expand indefinitely, albeit at an ever âĂS′ lower pace. This supports the manuscript's conclusion that the model describes ashearing, non âĂS′ rotating, continually expanding universe that does not approach isotropy at large times.

Numerically, the energy density ρ(t) begins at a high value of about 5 when t = 0, reflecting the dense, hot initial state of the universe. As time increases toward t = 20, the curve decreases smoothly and exponentially, approaching zero asymptotically. This behavior is consistent with the analytic solution $\rho(t) = 3n(2m+1)^2\exp\left(-\frac{a(2m+1)}{3(m+1)}t\right)$, which predicts a monotonic decay. Thenumerical trend shows that matter density dilutes rapidly at early times and becomes negligible at large t. Physically, this plot illustrates the transition from a primordial universe dominated by high energy density to a late âĂŞ time universe that is nearly empty. The exponential decay reflects the combined effects of cosmic expansion and bulk viscosity, which dissipates energy and accelerates dilution. This supports the manuscript's conclusion that the model describes a continually expanding cosmos with a big âĂŞ bang origin, where matter content fades with time and the universe asymptotically approaches an empty state.

Numerically, the cosmological constant $\Lambda(t)$ begins at about 0.25 when t = 0, reflecting a finite initial value tied to the Hubble parameter. As time increases toward t = 20, the curve decreases smoothly and monotonically, approaching zero asymptotically. This behavior is consistent with the analytic solution $\Lambda(t) = \frac{a(2m+1)}{3(m+1)} \exp\left(-\frac{a(2m+1)}{3(m+1)}t\right)$, which predicts exponential decay. The numerical trend shows that the cosmological constant diminishes steadily with time, mirroring the decay of the expansion scalar and Hubble parameter. Physically, this plot illustrates the concept of a dynamical cosmological term rather than a fixed constant. The high initial value of Λ corresponds to strong vacuum energy driving rapid expansion in the early universe. Its exponential decay reflects the assumption $\Lambda \propto H$, meaning that as the expansion slows, the effective cosmological constant also weakens. This supports the manuscript's conclusion that the model describes a continually expanding universe with bulk viscosity damping expansion, where vacuum energy fades with time and the cosmos asymptotically approaches an empty state.

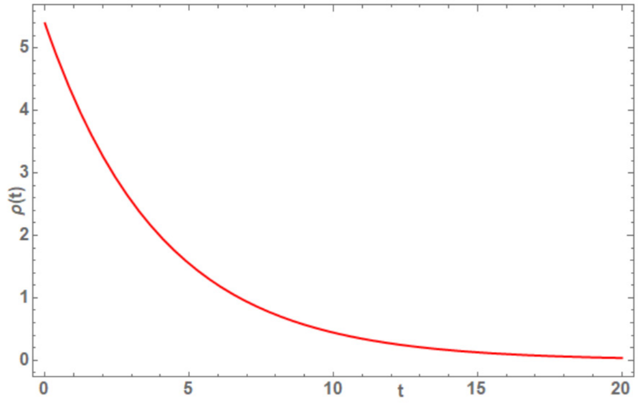

Figure 2: Energy density versus cosmic time showing exponential decay from a dense initial state toward near âĂŞ zero values

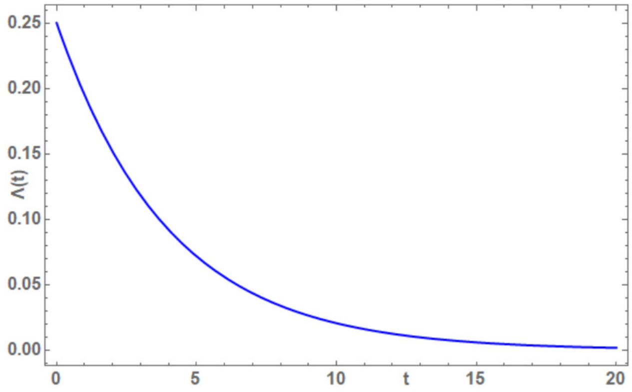

Figure 3: Cosmological constant versus cosmic time showing exponential decay from an initial finite value toward zero

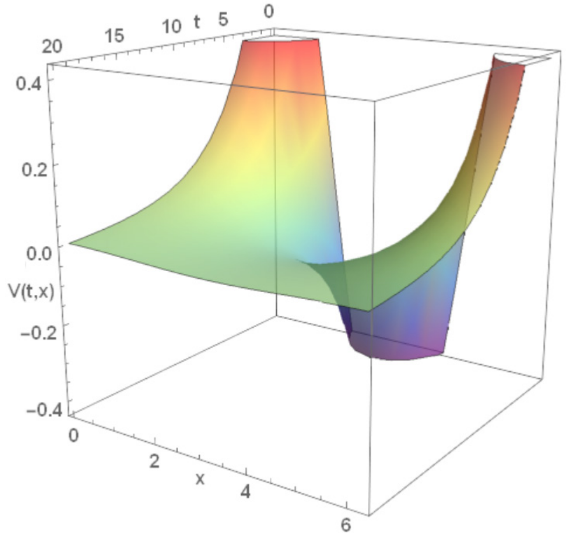

Figure 4: Three âĂŞ dimensional spatial volume evolution showing exponential decay in amplitude with persistent anisotropic oscillations over time

Numerically, the spatial volume function V(t, x) begins with oscillatory variations in x at t = 0,with amplitudes around ± 0.4. As time increases toward t = 20, the amplitude of these oscillations decays steadily, reflecting the exponential damping factor in the analytic solution. The surface plot shows ripples along the spatial direction x, but their magnitude diminishes with time, eventually flattening toward zero. This confirms that the volume contribution decreases monotonically with cosmic time, consistent with the model's prediction that expansion slows and the effective volume tends toward emptiness at late epochs. Physically, this figure illustrates the combined effects of expansion and anisotropy in the Bianchi type âĂŞ I viscous string cosmology. The oscillatory ripples represent directional dependence, showing that anisotropy persists even as the universe evolves. The exponential decay in amplitude reflects the damping influence of bulk viscosity, which reduces expansion and energy density over time. Together, the plot demonstrates that the universe begins with significant anisotropic volume fluctuations, but these fade as time progresses, leaving a continually expanding yet anisotropic cosmos that does not approach isotropy at large t.

- As a final result shows, the scalar of expansion θ increases as time t drops, and the scalar of expansion θ decreases as time t increases. As a result, the rate of expansion decreases as time increases.
- Energy density ρ falls with increasing time t and rises with decreasing time t. Consequently, a shearing, non-

rotating, continually expanding universe with a big-bang beginning is described by the model.

- Additionally, as time t increases, λ, ρ_p, ζ, Λ and σ^2 decreases. In contrast, the volume remains constant as $t \rightarrow \infty$. The following tend to zero: λ, ρ_p, ζ, Λ and σ^2. As a result, for a large value of t, the model would basically give an empty universe.
- For a large value of t, the model does not approach isotropy since $\lim_{t\rightarrow\infty} \frac{\sigma}{\theta} \neq 0.$
- The spatial volume V diminishes as time t grows. As time increases, the rate of expansion decreases.

Conclusion

We have investigated the string cosmological model of Bianchi type-I with bulk viscosity. The field equations are solved assuming the following: the expansion scalar is proportional to the shear scalar, i.e., $\theta \propto \sigma$, the bulk viscosity is proportional to the expansion scalar, i.e., $\zeta \propto \sigma$ and Λ is proportional to the Hubble parameterb Xing (2006); Tiwari and Dwivedi (2009); Schutzhold (2002). Next, a string cosmology with bulk viscosity is modeled cosmologically. There is additional discussion of the model's geometric and physical features. A shearing, non-rotating, continually expanding cosmos with a big-bang beginning is described by the mode.

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