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An Evaluation of Second Viscosity Coefficient of Liquid He^3 Phase-B for Balian and Wethamer State as Function of Reduced Temperature

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ABSTRACT

To create a technique to used the hydrodynamics equation and Bogoliubov quasi-particle distribution function of an isotropic superfluid contain three coefficient of second viscosity in presence of momentum conservation law and the acceleration of the superfluid .The evaluated value of second viscosity coefficient is good agreement of experiment value at $\lambda = 0.7$.

Keywords: Hydrodynamic equation for isotropic superfluid, Bogoliubov distribution function, Quasi-particle, Superfluid liquid Phase- A and Phase-B, Nuclear Magnetic Resonance,

INTRODUCTION

The liquid He ^{3} is known as Fermi liquid and lies between 3 mK to 100 mK, it behaves as normal Fermi liquid. Below 3mK it under goes phase transition [1-2] to super fluid phase obtained in nuclear magnetic resonance measurement. Basically two super fluid phase are very important naming He ^{3}- A phase and He^3 – B phase. These phases are highly spin polarized phase. Theoretically He ^{3} –A phase is known as Anderson and Merel [3] state and He^3 – B phase is known as Balian and Wethamer[4] state. In Anderson etal [3] state , it has p-wave and odd parity and spin function is symmetric and on the other hand Balian *et al*[4] state is a combined of triplet pairing and spin symmetric and the inter-particle interaction is lower than Anderson *et al* [3.] .

In this paper, using the hydrodynamics equation of an isotropic superfluid and evaluated value of second viscosity for Balian *et al*[4] state as a function of reduce temperature. Taking value of $\gamma_0 = 0.12$ and $\delta_0 = 0.29$ and pressure 20 bars. Theoretical value for Balian [4] state as function of reduced temperature varies very slow. At low temperature,it decreases very firstly and after that it is almost constant and then increases very slow. The evaluation of transport coefficient of an s-wave pairing Fermi gas has been reported by Shumeiko *et al*[5]. The shear viscosity of superfluid He ^{3} has been considered by Seiden *et al*[6] , Chechetkin *et al*[7], Fujiki *et al*[8,]Shazamanian *et al*[9], Pethick *et al*[10]. These evaluations have a more or less exploratory character. On the other hand there are some exact results available on the shear viscosity and second viscosity near the transition[11, 12] . For the case of isotropic quasi-particle scattering there is also recent calculation of the B-phase viscosity for whole temperature range by Ono *et al*[13].

Mathematical Technique used in the evaluation of second viscosity coefficient of He ^{3} Phase –B liquid: The shear viscosity coefficient η describes the response of the momentum current density $\delta\pi_{ij}$ to transverse velocity field v_j^n exposed in the normal component as

$$\delta\pi_{ij} = -\eta\left(\frac{\partial v_i^n}{\partial x_i} + \frac{\partial v_i^n}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial v_i^n}{\partial x_i}\right) \tag{1}$$

Now $\delta\pi_{ij}$ is written in term of the Bogoliubov quasi-particle distributive function

$$\delta\eta_{ij} = \sum_p \rho_i (v_p)_i \delta v_p^{'} \tag{2}$$

where $\delta v_p^{'}$ is the deviation from local equilibrium .

The quasi-particle velocity is written as

$$v_p = \nabla_p E_p = \left(\frac{\xi_p}{E_p}\right)\frac{p}{m*} \tag{3}$$

where p and m^* are the momentum and effective mass of quasi-particle.

The hydrodynamics equation of an isotropic superfluid contains three coefficients of second viscosity appearing in the momentum conservation law and acceleration equation of the superfluid. Use stress tensor in corresponding terms as

$$\pi_{ij} = \delta_{ij}\left[\varsigma_1 div\left\{\rho_s\left(v^s - v^n\right)\right\} + \varsigma_2 div.v^n\right] \tag{4}$$

When the limit $T \to T_c$, then term ς_1 vanishes, while term ς_2 is identified with the usual coefficient of bulk viscosity. ς_1 and ς_2 describe the response of the momentum current to longitudinal velocity field of normal and super fluid liquid v^n and v^s. The change in the distribution function induced by such a disturbance is isotropic. The diagonal part momentum current is given by

$$\frac{1}{3}tr\left(\pi_{ij}\right) = \frac{1}{3}\sum_p p\nabla_p E_p \delta v_p'$$

Using equation (3), we have

$$\frac{1}{3}tr\left(\pi_{ij}\right) = \frac{1}{3}p_F v_F \sum_p \left(\frac{\xi_p}{E_p}\right)\delta v_p' + 0\left(\frac{T^2}{\varepsilon_p^2}\right) \tag{5}$$

Now $\delta v_p' = \delta v_p - \int_p' \frac{\xi_p}{E_p}\left(\delta\varepsilon_p - \delta\mu\right)$

$$= \delta v_p - \int_p' \left(f_0 - \frac{\partial\mu}{\partial n}\right).\delta n + 0\left(\frac{T^2}{\varepsilon_F^2}\right)$$

Take into account $\frac{\partial\mu}{\partial n} = f_0 - \frac{1}{N_F}$, the expression for the density charge

$$\delta n = \sum_p\left[\frac{\xi_p}{E_p}\delta v_p' - \frac{\Delta^2}{2E_p^3}\left(\tanh\frac{E_p}{2T}\right).\left(\delta\varepsilon_p - \delta\mu\right)\right] \tag{6}$$

where $\sum_p \frac{\xi_p}{E_p}\delta V_p' = 0$

Thus equation(5) reduced to

$$\frac{1}{3}tr\left(\pi_{ij}\right) = 0\left(\frac{T^2}{\varepsilon_p^2}\right) \tag{7}$$

In the case of ς_2 there is an additional factor of $\frac{T_C^2}{\varepsilon_F^2}$ from the streaming part of the kinetic equation such that ς_2 is of order $\frac{T^4}{\varepsilon_F^4}$ small compared to the coefficient of first viscosity. ς_1 is of order $\frac{T^2}{\varepsilon_F^2}$ small. Both coefficients are negligible in liquid He^3 –Phase- B. The remaining coefficient ς_3 governs the response of the chemical potential to normal super fluid counter-flow, which introduces dissipation into the equation of motion of v_S.

$$m\ \ v^S = -\nabla\left\{\mu^1 - m\varsigma_3 div\left[\rho_S\left(v^S - v^n\right)\right] - m\varsigma_1 div.v^n\right\} \tag{8}$$

where μ^1 is the local equilibrium value of the chemical potential. $\delta\mu^1$ is determined by the requirement that the local equilibrium distribution function.

$$\delta v_p = \delta v_p' - \int_p' \frac{\xi_p}{E_p}\left(\delta\varepsilon_p - \delta\mu^1\right) \tag{9}$$

The correct charge density

$$\delta n^1 = \sum_p\left[\frac{\xi_p}{E_p}\delta v_p' - \frac{\Delta^2}{2E_p^3}\left(\tanh\frac{E_p}{2T}\right).\left(\delta\varepsilon_p' - \delta\mu'\right)\right] \equiv \delta\mu \tag{10}$$

The isotropic part of $\delta\varepsilon_p$ is given by

$\delta\varepsilon_p = \oint_0 \delta n$ (11) as local its equilibrium value.

The difference between δn and $\delta n'$ is

$$\delta n - \delta n' = \sum_p\left[\frac{\xi_p}{E_p}\left(\delta v_p - \delta v_p'\right) + \frac{\Delta^2}{2T}\left(\tanh\frac{E_p}{2T}\right).\left(\delta\mu - \delta\mu'\right)\right] = 0 \tag{12}$$

The kinetic equation is defined as

$$\delta v_p' = \delta v_p - \int_p'\left(\frac{\xi_p}{E_p}\right).\left(\delta\varepsilon_p - \delta\mu\right) = \delta v_p - \delta v_p' + \int_p'\left(\frac{\xi_p}{E_p}\right).\left(\delta\varepsilon_p - \delta\mu'\right) \tag{13}$$

Derive the relation between $\delta\mu'$ and $\delta v_p'$ from equation (12), we have

$$\sum_p\left(-\frac{\xi_p}{E_p}\int_p' + \frac{\Delta^2}{2E_p^3}\tanh\frac{E_p}{2T}\right).\delta\mu' = N_F\delta\mu' = \sum_p\frac{\xi_p}{E_p}\delta v_p' \tag{14}$$

where N_F may be interpreted as the thermodynamics derivative of the charge density with respect to the chemical potential at fixed quasi-particle distribution.

The left hand side of the kinetic equation in this case is approximated by substituting the local equilibrium distribution function

$$\delta v_p^{loc} = \int_p'\left(\delta E_p' p.v^n\right) \tag{15}$$

The isotropic part of δE_p^1 is given by

$$\delta E_p^1 = \frac{\xi_p}{E_p}\left(\int_0 \delta n - \delta\mu^1\right) = \frac{\xi_p}{E_p}.\frac{1}{N_F}.\delta n \quad (16)$$

Use the continuity equation to eliminate $\omega\delta n$ in favour of $q.j$, obtain the isotropic part of the kinetic equation

$$\int_p{}' \frac{\xi_p}{E_p}\frac{1}{N_F} q\left(j - \rho v^n\right) = -\frac{i}{\tau_p}\delta v_p' + I_p^{in} \quad (17)$$

The scattering term of isotropic and odd parity in ξ_p, is approximated by

$$I_p^{in} = \frac{i}{\tau_p}\frac{\xi_p}{E_p}\int_p{}' \frac{\sum_{p'}\left(\xi_{p'}/E_{p'}\right).\left(\delta v_{p'}'/\tau_{p'}\right)}{\sum_{p'}\int_{p'}{}'/\tau_{p'}} \quad (18)$$

The scattering term of equation (18) satisfies approximately the relation of the exact inscattering integral.A approximation involving two relaxation time. Introducing a dimensionless function $\phi(\xi)$ by

$$\delta v_{p'}' = -i\frac{\xi_p}{E_p}\int_p{}' \frac{1}{N_F} q\left(j - \rho v''\right).\varphi\left(\xi_p\right) \quad (19)$$

Again rewrite the kinetic equation

$$\tau\left(\xi\right) = \varphi\left(\xi_p\right) - \frac{\left\langle\left(\frac{\xi^2}{E^2}\right)..\frac{\varphi\left(\xi\right)}{\tau}\right\rangle_f}{\left\langle\left(\frac{1}{\tau}\right)\right\rangle_f} \quad (20)$$

This separable integral can be easily solved. Substituting the result of equation (20) in equation (17) and find the second viscosity coefficient as

$$\xi_3 = \frac{1}{N_F}\left\langle\frac{\xi^2}{E^2}\tau\right\rangle_f + \frac{\left\langle\frac{\xi^2}{E^2}\right\rangle_f}{\left\langle\left(\frac{\Delta^2}{E^2\tau}\right)\right\rangle_f} \quad (21)$$

At the transition, second term in the square brackets diverges as $\frac{1}{\Delta}$, and also

$$\xi_3 = \frac{4}{m^* p_F}\tau_N(0).\left(\frac{3}{2}\frac{C_N}{\Delta C}\right)^{\frac{1}{2}}\left(1 - \frac{T}{T_C}\right)^{-\frac{1}{2}} \quad (22)$$

Under the limit of low temperature, the second viscosity of coefficient ξ_3 becomes independent of temperature, thus

$$\lim_{\tau\to 0}\xi_3 = \frac{2\pi}{3}\frac{1}{N_F}\left(\frac{T_c}{\Delta_0}\right)^2\frac{1}{\omega_0}\left(0, T_c\right) \quad (23)$$

Numerically evaluated value from equation (21) by using the value $\gamma_0 = 0.12$, $\delta_0 = 0.29$ and appropriate pressure of 20 bar. Evaluated value shown in table (T_1).

Evaluated result of second viscosity of coefficient of He^3 liquid of phase-B for Balian and Wethamer state[4] as function of reduced temperature for $\lambda = 0.7$

Table-T_1

$\left(1 - \frac{T}{T_c}\right)$	$\left(\frac{N_F\xi_3}{\tau_N(0)T_c}\right)\frac{\Delta}{\Delta_0}$
0.100	0.762
0.200	0.723
0.300	0.656
0.400	0.629
0.500	0.607
0.600	0.584
0.700	0.558
0.800	0.535
0.900	0.598
1.000	0.612

DISCUSSIONS

In this paper, I have evaluated the second viscosity of coefficient of normalized liquid He ^{3} of phase-B at low temperature. The hydrodynamics equation of an isotropic super fluid contains three coefficient of viscosity appearing in the momentum conservation law and the acceleration equation of super fluid. By using equation (22), I have evaluated second viscosity coefficient ξ_3 for the Balian and Wethamer [4] state as a function of reduced temperature $\left(1 - \frac{T}{T_c}\right)$. In this calculation, I have taken the parameters value $\gamma_0 = 0.12$ and $\delta_0 = 0.29$ and pressure 20 bars. Our theoretical evaluated value shows that the normalized second viscosity coefficient ξ_3 for the Balian and Wethamer state as a function of reduced temperature varies very slowly. At lower value of temperature,it decreases very fastly and after that it is almost constant and then increases very slowly . The evaluated results are shown in table (T_1). The theoretical results were evaluated by using kinetic equation for Bogoliubov quasi-particle in a well control approximation [14-27.]

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