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Study of Josephson Effect Between Bose Condensate

Raju Prasad Singh[1] and R.K.Verma[2]
[1]*Research Scholar Jai Prakash University Chapra*
[2]*Associate professor, Dept of physics, JLC college, Chapra*
Corresponding author: ravindraverma95@gmail.com

ABSTRACT

In this paper, we have used a technique , when a weak link is created between two systems which have undergone some type of gauge symmetry breaking. In fact the Josephson effect has been observed in superfluid .Bose condensate atomic gases is generally expected. A system whose gauge symmetry has been spontaneously broken can be described by order parameter that behaves in much respect like a macroscopic function $\Psi(r)$. For Bose Einstein Condensates of dilute alkali gases $\Psi(r)$ is the wave function of the macroscopically occupied one atomic state.

Keywords: Josephson, Bose condensate, Superfluid, Gauge symmetry, Rabi dynamics, and Rabi oscillation.

INTRODUCTION

In this paper, we have study of Josephson Effect between Bose condensate with help of Thomas- Fermi approximation and Wentzel-Kramers and Brillouin method. Josephson theoretically has played a major role in physics and technology of superconductor [1]. The physics of the Josephson effect becomes manifest when a weak link is created between two systems which have undergone some type of gauge symmetry breaking. Josephson effect has been observe in superfluid [2]. Bose condensate atomic gases is generally expected in a system whose gauge symmetry has been spontaneously broken can be described by an order parameter like macroscopic wave function $\Psi(r)$. In the simplest cases, the parameter reduces to a complex scalar function $\Psi(r) = \sqrt{\rho(r)e^{i\phi(r)}}$, where $\rho(r)$ is the super fluid density and $\phi(r)$ is the phase. For Bose Einstein condensate of dilute alkali gases $\Psi(r)$ is the wave function of the macroscopically occupied one atom state.

In his epoch making[3], Josephson predicted that between two weakly connected superconductors of phases ϕ_1 and ϕ_2, a non –dissipative particle (Cooper pair) current flow between them, Whose value is $\mathrm{I}(\phi) = \mathrm{I}_c Sin\phi$ (1), where I_c is the critical current and $\phi = \phi_1 - \phi_2$ is the relative phase .He also predicted that in the presence of a nonzero chemical potential difference $\mu = \mu_1 - \mu_2$, the relative phase rotates as $\dot{\phi} = \frac{-\mu}{\hbar}$ (2). He obtained as the equation of motion of the "Pendulum Hamiltonian" as $\mathrm{H}(\phi, N) = E_J(1-\cos\phi) + \frac{1}{2}E_c N^2$ (3), where $E_J = \hbar \mathrm{I}_c$ is the Josephson coupling , $N = (N_2 - N_1)/2$ is the number of transferred particles and $E_c = \frac{\partial\mu}{\partial N}$ is the capacitive energy due to interactions. In the absence of external constraints, the chemical potential is $\mu = E_c N$. In a superconducting link the critical electric current is $2e\mathrm{I}_c$ and the capacitive energy is $E_c = \frac{2e^2}{C}$, where C is the electrostatic capacitance. E_c can be obtained from Thomas-Fermi calculation[4] of the chemical potential for trapped BEC. When capacitive energy and $K_B T$ are much less than Josephson coupling, then Josephson pendulum Hamiltonian can be approximated as harmonic oscillator $H(\phi, N) = \frac{1}{2}E_J\phi^2 + \frac{1}{2}E_c N^2$ (4) , whose frequency $\omega_P = \frac{\sqrt{E_C E_J}}{\hbar}$ (5) is called the Josephson plasma frequency. At low temperature, collective dynamics of an in homogeneous BEC is well described by the Gross-Pitaevskii Hamiltonian[5]

$$H(\Psi,\Psi^*) = \int dr\left(\frac{\hbar^2}{2m}\left|\nabla\Psi(r)^2\right| + V_{ext}(r)\left|\Psi(r)^2\right| + \frac{g}{2}\left|\Psi(r)^4\right|\right)$$

(6) , where $g = 4\pi\hbar / m$ is the s-wave scattering length.

RESULTS AND DISCUSSIONS

External Josephson effect : Let us suppose our system has the structure of two weakly connected condensate sites 1 and 2. One assume that condensate are confined within spherical harmonic wells of the frequency ω_0. First one wishes to analyze the semi classical dynamics, then N and ϕ can be treated as simultaneously well defined and writes wave functions a s $\tilde{\Psi}(r,t) \sim \Psi_1(r; N_T/2 - N(t) + e^{i\phi(t)}\Psi_2(r; N_{T/2} + N(t)$ (7), where $\Psi_i(r;t)$ is the real equilibrium wave function for the isolated well I containing n bosons. It is straight forward to show that induced by local variation $\phi(r)$ in the region of only active dynamical variable. From the theory of phase transition, one knows that some type of 'rigidity' is exhibited in the ordered phase below the critical temperature[6]. In the case of superconductor, super fluid and BEC, one may speak of 'phase rigidity' because given an electromagnetic gauge, the phase is determined everywhere once it has been fixed at given point. Alternatively gauge and phase are intimately connected in quantum mechanics, one can say that given of phase, one has lost the freedom to choose the gauge, hence the term 'gauge symmetry braking'.

For long range phase coherence can be destroyed by quantum fluctuation and occurs can developed by analyzing the Josephson Hamiltonian equation (3), which may be viewed as a two-site reduction of the more general energy function equation (6). In equilibrium good phase coherence between sites 1 and 2 is obtained in the harmonic limit equation (4). When $k_B T \geq E_J$, the quantum fluctuation can destroyed the two site phase coherence , one notes that N and $\phi(r)$ are canonically conjugate variables, where $N = i\partial/\partial\phi$ in a quantum description. As a consequence, the capacitive term in equation (3) plays the role of kinetic energy for phase variable. At zero temperature, quantum lowest order in the overlap integrals $\int \Psi_1\Psi_2$, the energy functional for $\tilde{\Psi}$ takes the from

$H(\varphi, N) = E_B(N) + E_J(N)(1-\cos\varphi)$ (8), where $E_B(N)$ the bulk energy of the two isolated wells with N is transferred atoms and $E_J(N)$ is the Josephson coupling energy. In the Thomas- Fermi limit equation (7) , the bulk energy $E_B(N)$ is mostly due to interactions and it may be expanded as

$$E_B(N) \approx E_B(0) + \mu_0^{/}\left(\frac{N_T}{2}\right)N^2 + \frac{1}{12}\mu_0^{''}\left(\frac{N_T}{2}\right)N^4 \quad (9)$$

To avoid complications stemming from possible resonance between Josephson oscillation and inter well excitations, one requires $2\mu_0 N \leq \hbar\omega_0$, where one uses the result that the first normal mode of a spherical well lies approximately at $\hbar\omega_0$ above the ground state [7-8]. This condition is realized when $\frac{N}{N_T} \leq 4.5 N_T^{-2/5}$ for typical parameters. This upper bound is order of 2-10% for $N_T \sim 10^4 - 10^6$.

Wentzel Kramers and Brillouin (WKB) theory: If the equilibrium density can be factorized in the controlling, one can writes $\rho(r) = f(x,y)g(z)$ and the two bounds becomes identical. At r=0, the middle point of the double well configuration and choosing $x = y = 0$ to be the line connecting the two well centers, one can writes[9]

$$E_J = A\frac{\hbar^2}{m}\left[\int_1^2 \frac{dz}{\rho_0(0,0,z)}\right]^{-1} \quad (10)$$

, where $A = \frac{1}{f(0,0)}\int dxdy f(x,y)$ being an effective area. For two identical wells in equilibrium, the ground state wave function is symmetric in z. Focusing on the *x*=*y*=0 , the wave function be

$$\Psi_0(z) = \frac{B}{\sqrt{\rho(z)}}\cosh\left[\frac{1}{h}\int_1^2 dz'\rho(z')\right] \quad (11)$$

where $\rho(z) = \left[2m(V_{ext}(0,0,z) - \mu_0\right]^{\frac{1}{2}}$ and B is a constant to be determined. Combined equation (10) and (11) , obtained

$$E_J \cong \frac{\hbar A|B|^2}{m}\left[2\tanh\left(\frac{S}{2}\right)\right]^{-1} \quad (12)$$

where $S \equiv \int_1^2 \rho(z)dz/\hbar$ is the dimensionless action constant. Thus finally Josephson coupling obtained [9]

$$E_J \cong \frac{e^{-S}}{\tanh(S/2)}\left(\frac{N_T}{2}\right)^{1/3}\left(\frac{15a}{a_0}\right)^{-2/3}\frac{\hbar\omega_0}{2} \quad (13)$$

The $N_T^{1/3}$ dependence can be understood qualitatively by noting[7-8,10] that $A \sim R^{2/3}$ and $B \sim R^{2/3}$. In the T-F approximation. $R \sim N_T^{1/5}$, obtains $E_J \sim N_T^{1/3}$.One Knowing that the critical temperature satisfied the Giorgini *et al*[11] $k_B T_c \approx N_T^{1/3}\hbar\omega_0$, one may rewrites equation (13) as for large *S* and typical parameters

$$E_J \cong k_b T_c e^{-S} \quad (14)$$

, which reminiscent of the Ambegaokar-Baratoff formula for superconductor[12], if one notes e^{-S} is the probability amplitude for a particle to traverse the potential barrier .

Josephson Plasma frequency : From equation (3) and (9) , one has capacitive energy $E_C = 2\mu_0'\left(\frac{N_T}{2}\right)$, which in the T-F approximation [4] reads

$$E_C \cong \frac{4}{5}\left(\frac{N_T}{2}\right)^{-3/5}\left(\frac{15a}{a_0}\right)\left(\frac{\hbar\omega_0}{2}\right) \qquad (15)$$

Now find the product of Josephson coupling and capacitive energy and one have

$$E_J E_c = \frac{1}{5}\left(\frac{2a_0}{15aN_T}\right)^{4/15}\frac{e^{-S}}{\tanh\left(\frac{S}{2}\right)}\omega_0^2\hbar^2 \qquad (16)$$

From equation (5) ,Josephson plasma frequency is defined as

$$\omega_{JP} = \frac{1}{\sqrt{5}}\left(\frac{2a_0}{15aN_T}\right)^{\frac{2}{15}}\frac{e^{\frac{S}{2}}}{\sqrt{\tanh\left(\frac{S}{2}\right)}}\omega_0 \qquad (17)$$

Since, usually $\omega_0 = 2\pi \approx 10-100Hz$, one conclude that $\frac{\omega_{JP}}{2\pi} - \leq 10Hz$.

The ratio of Josephson coupling to capacitive energy is good measure of the classical character of the relative phase ϕ . Now one finds

$$\frac{E_j}{E_C} = \frac{1}{24}\left(\frac{N_T a_0}{a}\right)\left(\frac{2a_0}{15aN_T}\right)^{\frac{1}{15}}\frac{e^{-S}}{\tanh\left(\frac{S}{2}\right)} \qquad (18)$$

From classical regime $E_J \geq E_C$ to strong quantum limit $E_J \leq E_C$, quantum effects are only important, if operate at ultralow temperature $k_B T \lhd E_C$. From equation (18) ,one conclude that for typical number S must be roughly $\geq$10 for quantum fluctuation to be important. One consider the damping is the incoherent exchange of normal atoms and a quantitative requires a generalization of our results to nonzero temperature. The thermally excited normal atoms to obey Ohm law and give an Ohmic contribution to the current

$$I_n = -G\mu \qquad (19)$$

Josephson equation is modified to read

$$\frac{dN}{dt} = \frac{E_J}{\hbar}\sin\varphi + I_n \qquad (20)$$

For small fluctuation

$$\frac{d}{dt}\left(\frac{d\varphi}{dt}\right) + GE_{C\varphi} + \omega_{JP}^2\varphi = 0 \qquad (21)$$

For classical harmonic oscillator under damped if

$$\sigma \equiv \frac{1}{\hbar G}\sqrt{\frac{E_J}{E_C}} \geq 1 \qquad (22)$$

If $k_B T \geq \frac{\hbar\omega_0}{2} \sim V_1$, the thermal cloud lies mostly above the barrier and a radically approach is needed to study its transport properties. For simplicity , one introduce the drastic approximation that particle impinging on the barrier with energy E are transmitted with probability if $E > V_0$ and zero if $E < V_0$. Then the flow of normal atoms due to a fluctuation in chemical potential is only limited by the 'contact resistance' a concept taken from ballistic transport in nanostructure[11]Equation (22) rewritten as

$$\sigma \sim 2\pi N_0^{7/15}\left(\frac{\hbar\omega_0}{k_B T}\right)^2 e^{S/2} \qquad (23)$$

Interestingly, some result is formally obtained in the high barrier limit, where normal atoms tunneling also lies in the WKB regime [9]. The value of σ if $S \sim 5$, $\sigma \sim 0.38$ for $N \sim 10^4$ and $\sigma \sim 3.3$ for $N \sim 10^6$. The equivalent numbers for $S \sim 1$ are 2.8 and 24. The conclusion is that coherent Josephson dynamics can be observed in current atomic Bose condensates if the barrier is low. Under damped dynamics can be further favoured by decreasing temperature *T* and increasing atomic number *N*.

Internal Josephson effect: If laser pulse is applied to a macroscopic condensate, then it is the whole ensemble of atoms evolves coherently. It is possible to prepare a condensate in which each atom is in the same coherent superposition of the two states has been realized by Hall *et al*[13] with the $|1,-1\rangle$ and $|2,1\rangle$ states of ^{87}Rb. Using a techniques, it will be possible to study the novel and potentially extremely rich internal Josephson effect which bears some analogies with the physics of superfluid phase-A of liquid He^3 as noted by Leggett *et al*[14]. The analysis of the spatial (external) Josephson effect is based on the knowledge of $V_{ext}(r)$, where r can varied continuously between wells. A similar study of the internal Josephson effect would not be practical. Fortunately, it is really needed is the value of the matrix element connecting states $|A\rangle$ and $|B\rangle$. Then it is most convenient to employ a two site description of the Josephson link. The Hamiltonian

$$H = \frac{\hbar\omega_R}{2}\left(a^+b + b^+a\right) + \frac{u}{2}\left[\left(b^+b\right)^2 + \left(a^+a\right)\right] \qquad (24)$$

where ω_R is the Rabi frequency. One assumes that the two atomic energies as well as the interspecies interaction are equal and neglect the interspecies interaction. Particle number conservation requires $a^{+}a+b^{+}b=N_{T}\equiv 2N_{0}$. The particle eigenstate can be written as $\Phi_{N}(\phi)=\frac{1}{\sqrt{2\pi}}e^{-N\phi}$. One have

$$E_{J}=\hbar\omega_{R}\left[N_{0}\left(N_{0}+1\right)-N(N+1)\right]^{1/2} \quad (25)$$

In limit $1< \ N< \ N_{0}$ equation (25) becomes

$$E_{J}=N_{0}\hbar\omega_{R} \quad (26)$$

which is a clear manifestation of the phenomenon of Bosonic amplification. The identification of equation (24) with the pendulum Hamiltonian is completed by noting that the interaction term can be written uN^2 that is $u=E_{C}/2$. For small value of ϕ and N, the harmonic form

$$H=-N_{0}\hbar\omega_{R}\frac{\phi^{2}}{2}+\left(E_{C}+\frac{\hbar\omega_{R}}{N_{0}}\right)\frac{N^{2}}{2} \quad (27)$$

With a natural oscillation frequency

$$\omega^{2}=\omega_{P}^{2}+\omega_{R}^{2}=\left(\frac{N_{0}E_{C}}{\hbar}\right)\omega_{R}+\omega_{R}^{2} \quad (28)$$

It is clear that for a given interaction , the double BEC system can be driven continuously from the Josephson to the Rabi regime by varying ω_{R}, something feasible with current laser technology. Quantum self trapping cannot be realized in superconducting Josephson junction because the chemical potential difference would have to be greater that the superconducting gap [15]. , thus allowing quasiparticles to intervenes and complete the dynamics. This is not a important limitation in the case of BECs. There exists a close and well studied analog of quantum self trapping in the longitudinal nuclear resonance (NMR) of He^3 - A phase[14]. Equation (27) described in a unified way Josephson and Rabi ($E_{C}=0$) dynamics. This is analogous to how equation (6) can yield both the G-P and Schrodinger equation for the wave function of a many boson system. The crossover between collective Josephson and individual Rabi dynamics cannot be studied in superconducting and superfluid , because there interactions are never completely negligible. It is nice feature of Bose-Einstein Condensation that it will allow us to study the crossover between these two qualitatively different dynamical regimes in an elegant fashion.

It is important to emphasis that the diffusive mechanism only applies if the normal particle reservoir is common to both condensates. In the above one has considered a situation in with although the two condensates have been separated, the two thermal clouds remains connected to the point of behaving as a single one. In superconductors, an equivalent arrangement is not fundamentally difficult. In case of Bose Einstein Condensation, one may has a common thermal clouds when the increase in the barrier height is large enough to render the two condensates unconnected, but still sufficiently small to allow for an essential single thermal cloud [16-18].

CONCLUSION

In this paper , we have presented an overview of the physics of the Josephson effect between Bose condensed systems with emphasis on the recently achieved Bose Einstein Condensation in trapped alkali gases. We have mostly focused on these physical phenomenons that are likely to be observed only in those novel systems. Thus , we have omitted the discussion of problems (such as e.g steady particle flow under the action of an alternating chemical potential) which may be viewed as straight forward application of well known Josephson Physics[19] ,We have tried to underline the potential richness of the Physics displayed by weakly connected BECs. We may have an external (spatial) and internal (hyperfine) Josephson effect. It seem possible to explore the crossover between collective Josephson behavior and independent Boson Rabi dynamics. Finally, the observation of fascinating phenomena such as quantum self trapping and macroscopic interference between separate Bose condensates seem also within reach of the emerging BEC technology. Everything indicates that the experimental and theoretical study of the Josephson effect between Bose Einstein Condensates will leads us to the exploration of most exciting new physics. There are some recent calculations [20-24] also above the Josephson effect between Bose Einstein Condensate and they also indicate the similar observation.

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