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RESEARCH ARTICLE

Logistics Optimization Through Composite Payday Installment in Favor of Requisite Ultimatum Vacillating Carrying Cost and Gradual Degeneration Under Non-stocked and Continuous Circumstances Using Hexagonal Fuzzy Number

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Abstract

Throughout the course of the epidemic era, the amount of e commerce treaty to procure legacy amidst a combination of deposit and liquidity have intensified remarkably. Herein, the customer contributes a pre-determined chunk of procured price beforehand, and remaining payoff is skilled through the process of cash-on-delivery. Assuming a practicable schema in which the customer owns a warehouse with restricted room, two sustainable inventory strategies for biodegradable products are advanced. To be specific,

(i) Inventory strategy with permitted shortfall and partial backlog

(ii) Inventory strategy is perpetual

In the above strategies, the demand function and biodegradable rate exhibit an upward trajectory opus to storage time. However, the decay from the product's warehouse point, the price of storing products is assumed as a linear ramification of rising mode verses retention time. A prominent specification is advanced to scrutinize just as the everlasting circumstances beside reclamation value spares reasonable juxtaposed with the industry manager who claims infinite condition alongside structured reckoning assay.

Keywords: Sustainable EOQ model, Vacillating rot, Reimbursement, Gradual degeneration, Protracted shrivel, Fuzzy set, Hexagonal fuzzy number.

Introduction

The most perturbing fact about natural environment is the biodegradable items like food, vegetables, meat and diary products etc. which are the primary source of coherent waives into the atmosphere. As a result the evolution of ecologically conscious inventory control for biodegradable goods has evolved into audacious research domain. Managing the anticipated deadline for upcoming raw materials or furnished goods is substantial. To bring down the blockage of investment on the goods, man power and machines can be used.

Various food products like fruits, vegetables may perish naturally or due to precarious warehouse maintenance from vermin's. Some products like gasohol's and camphor putrefy on devalued in due course evolution of mechanization advances also vogue supersedes the old ones, thereby reducing the ethics of inventory. As a consequence of stock decay, fancier's augmentation may get affected considerably. So, to explore the above impact, researchers will endorse various forms of putrescence based on the nature of goods.

As the deterioration rate rises during the stockpile term, expense of ensuring goods may change than accelerating

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over the stockpile term. The arbitrator allocates extra money to stop the number of perishable goods so that they can check the quantity of loss on account of perishable goods. Rahman et.al modified the inconsequential rumination the holding cost per entity time is persistent when all products in an intermediate environment are assumed to have a marginal cost to hoard per unit time.

At the same time Yang established another inventory stratagem presuming merchandise carrying cost of goods in warehouse carrying cost in contrary to fixed cost. Lately, Dhandapani and Uthayakumar, read upon additional EOQ model for fruit at the time of retention with an outlay to transfer the goods during the storage term. For instance, fruits, vegetables etc will endure garden - fresh for some time from the hour of storage after which decay begins. This aspect is entitled as delayed deterioration or non-instantaneous deterioration in stock assessment. The entireties of the aforementioned investigations on biodegradable materials show either a steady non-instantaneous decay rate over the course of storage or a unceasing/mutating decay rate. No matter, whatever may be the decay rate of biodegradable goods it is a rising function that is not instantaneous in relation to the storage term that creates a space in governing the degradable goods. This study satisfies the above concept. From the above observation made, Mishra et.al revamps and reviews as,

- Instantaneous decay (putre scene) replaces non-instantaneous decay.
- The stockpile percentage relies on the advent time span embodied in place of constant percentage.
- Alleviating the illimitable capacity presumption the stowage is appraised as limited for a fancier.
- Forever, the circumstances in the warehouse are included.
- A composite cash and down payment accord is espoused to bring down the probability of undetermined profit.
- The coalescence of aforesaid featured fabricate the present project unique in the inventory literature.

Literature Review

Due to the fact that the effects of decay on stock can have a significant impact on a practitioner's profit, several researchers look at these effects by using various decay rate genres depending on the type of object. An inventory problem was analysed by Ghare and Schrader (1963) at a constant degradation rate. The impacts of deterioration on food items in India were examined by Jani et al.(2021), however for perishable goods, Shaikh et al.(2021) developed an EOQ approach in which deterioration begins after a predetermined period of time from the product's storage moment. A detailed analysis of an inventory system that incorporates a static patronage and percentage decay that fluctuates in relation to price was also carried out by Chowdhury, Ghosh, Chaudhuri (2015). With an legacy that is on-going deterioration, Shaikh et al(2019). Developed an additional inventory technique that takes advantage of a discount schedule in line with the buy items. Khan et al (2019). recently examined a variable decay rate based on product lifespan and storage duration. In contrast to rising over the course of storage, the cost of storing products is not necessarily unchangeable when the rate of deterioration escalates. This is because the decision maker can lower the amount of loss from deteriorated goods by allocating more funds to hinder the quantity of assets from declining.

In an interval scenario, Rahman et al (2021). Assumed a variable price to store in unit time for each item, so relaxing the trivial consideration that the cost of carrying in unit time is fixed. Recently, Dhandapani and Uthayakumar examined an additional EOQ model for fresh fruits that included transportation costs that were variable throughout the storage period and preservation equipment. For deteriorating products that met certain requirements, Chung et al (2019). Used a discount technique. Under a two-warehouse inventory approach, Khan et al(2020). Recently examined a number of decay starting possibilities brought about by improved storing conditions in a borrowed store. Every one of the previously listed research on perishable goods includes either a constant non-instantaneous degradation rate or a constant/time changing instantaneous deterioration rate over the course of storage. The degradation of numerous putrescible supplies, such as fruits, vegetables, cereals, seeds, and so forth, undergo a non-instantaneous process that lengthens with storage time.

Price of the product and supply levels have a direct impact on a product's demand. As a price rise decreases consumer demand, the inventory cycle lengthens. In a similar vein, stock that is kept in good condition will draw in more clients. The profit margin is impacted, nevertheless, because significant investment is required. The quantity of degraded items in the inventory rises as a result of an increase in stock. Sometimes, the low cost of materials even lowers demand, which has a negative impact on inventory, depending on the social standing of the local population. For a fluctuating demand pattern, Sarkar (2021). Covered pricing methods both offline and online. Two different models were created by Omair et al (2021). and Bhuniya et al(2021). In a fuzzy-typed demand environment which remains in dispute. Das et al (2021). Have concentrated on opera addressing the tie between pricing and demand for perishable goods.

The optimal pricing strategy for a practitioner that takes consumer demand as the power form of price was examined by Pando et al (2021). and Cardenas-Barron et al (2021). In order to reflect the desires of the market, Rahman et al (2021). Have developed an inventory issue that hybridizes price and current stock. The practitioners keep more goods in the warehouse in an attempt to draw in more clients, which could lead to an endless scenario in the workplace.

In the inventory model, the stock-out situation is an essential element. The store occasionally runs out of inventory when market demand is still high. Inventory backlogging, a phrase used to characterize delayed delivery of items to clients. Depending on the retailer's goodwill, customers can decide to wait for the delayed delivery or move elsewhere. Nevertheless, the store arranges the stock for the customers who are waiting, despite the adjoining store's loss. Consequently, inventory backlogs have a negative effect on inventory goodwill. A waiting time dependent partial backlog inventory model was developed by Alshanbari et al (2021). Partial backlog with reworking possibilities for damaged products was outlined by Ahmed et al (2021). Taleizadeh (2017). and Khan et al (2021). Integrated several cutting-edge features and permitted shortages to complete a number of inventory tasks.

In the corporate sector, where buyers pay the full or partial purchase price before getting the product, advance payment plays a significant role. Since buyers in this scenario generate loans from other sources, such as banks, they must be able to repay the interest on those loans. In order to solve an inventory problem, Shaikh et al (2019). Used particle swarm optimization (PSO) and integrated advance payment coordination. Duary et al (2022). Created ideal guidelines for the practitioner and provided a study on perishable commodities with trade credit and an advance payment arrangement.

Prologues

Fuzzy Set

If X is an universe of discourse and x is a particular element of X then the fuzzy set A defined on X can be written as the collection of ordered pairs

$$A = \{ X, \mu_i(x); x \in X \}$$

Fuzzy Number:

A fuzzy number is a fuzzy subset in the universe of discourse X that is both convex and normal.

Hexagonal Fuzzy Number:

A fuzzy number $(h_1, h_2, h_3, h_4, h_5, h_6, u, v)$ if its membership function is,

$$\tilde{\mu}(x) = \begin{cases} u\left(\dfrac{x-h_1}{h_2-h_1}\right), & if\ h_1 \le x \le h_2 \\ 1-(1-u)\left(\dfrac{x-h_2}{h_3-h_2}\right)^{n_2} & ,if\ h_2 \le x \le h_3 \\ 1 & ,if\ h_3 \le x \le h_4 \\ 1 & ,if\ h_3 \le x \le h_4 \\ 1-(1-v)\left(\dfrac{h_5-x}{h_5-h_4}\right)^{m_1} & ,if\ h_4 \le x \le h_5 \\ v\left(\dfrac{h_6-x}{h_6-h_5}\right)^{m_2} & ,if\ h_5 \le x \le h_6 \\ 0 & ,if\ x \succ h_6 \end{cases}$$

If $u = v, n_1 = n_2 = m_1 = m_2 = 1$, then non-linear hexagonal fuzzy number gets transformed into linear hexagonal fuzzy number and on the other hand if $u = v$, then non-linear hexagonal fuzzy number gets transformed into non-linear hexagonal symmetrical fuzzy number.

Materials and Methods

Notations

O – Cost to place a requisition
d_a, d_b – Attributes for demand function
M_p – Market price
A_p – Acquisition charge
C_c – Stabile price to clench a unit item
α_c – Racillating cost to clench a unit item
C_o – Fortuity fee
S – Rate of paucity
$\beta(t)$ – Time varying decay
ω – Coefficient for Racillating rot of items
d_{c_t} – Start off period of rot
H_{b_a} – Blustery amount for stockpile
I_{p_a} – Initial amount of product in repository
S_p – Salvage value
χ – Attribute for stockpile
P_t – Permitted time frame for filling the reimbursement
n – Number of tranches for reimbursement
γ - Contingency to snap up items in disbursement
U_w – Maximal capability of repository
T_{c_i} – Gross continual cost
t_1 – Stretch of canal stock in the repository
B – Number of products lasting in without cessation state
T – Stretch of a cycle in unit time

Assumptions

- As time passes, the rate of deterioration $\beta(t)$ increases as $\beta(t) = \omega t$ where $0 \prec \omega \prec 1$
- There is insignificance of decay on commodity measure during $[0, d_{c_t}]$. Nevertheless, decomposition of commodities with different rates of decay affects the quantity of goods in the storage period (t, t_1).
- As stated, the product's rate of demand is linearly dependent on time and unit selling price. It is found that when the parameter is positive, the ultimatum $D^* = d_a - d_b + ct, d_a, d_b \succ 0 \& c \in R$ reveals an upward shift relative to time, and when the parameter is negative, the ultimatum reveals a downward shift relative to time. Furthermore, when time is 0, the ultimatum is independent of time.
- The holding cost of a unit product for a unit period of time is proportionate to the unit's storage period, which is assumed to be $h(t) = h + \alpha_c t$.
- Shortfalls are permitted, and they are partially accumulated of the ultimatum with the proportion $[1 + \chi(T-t)]^{-1}$ where $(T-t)$ is the time span of arrival

from any duration and, once the stockless span begins where $t \in [t_1, T]$ and $\chi \succ 0$.

- For a single non-instantaneous fading product, inventory strategy is substantial.
- Renewal or renovation is not allowed for biodegradable spin off.
- The lead time and inventory planning perspective are both unlimited.
- Finally, for all perpetual circumstances, the seller will sell all of the units of stale inventory for salvage.

Friable Paradigm:

Replenishment Chain Whilst Bottlenecks Materialize:

Assume a vendor first places an order for $(I_{p_a} + H_{b_a})$ units of a product by making a payment his supplier d_b percent of the total purchase price L months prior within n identical in several chunks at identical time frames at the moment frame set aside L. The vendor then delivers the balance payment of $(1-\gamma))$ proportion at the moment of shipment. The retailer then decisively surpasses all of the reserved numerals H_{b_a}, bringing about a reduction in the momentary number of quantities to I_{p_a}. Due to client demand, the amount I_{p_a} progressively declines throughout $[0, d_{c_t}]$, and deterioration then commences. After that, a stock-out scenario arises, and shortages are accumulated depending on when the new shipment shows up.

Thereby, the inventory's rate of change at $t \in [0, T]$ must meet the requirements of the governing differential equations,

$$\frac{dI_1^*(t)}{dt} = -(d_a - d_b + ct), 0 \le t \le d_{c_t} \tag{1}$$

$$\frac{dI_2^*(t)}{dt} + \beta(t) I_2^*(t) = -(d_a - d_b + ct), d_{c_t} \prec t \le t_1 \tag{2}$$

$$\frac{dI_3^*(t)}{dt} = \frac{-(d_a - d_b + ct)}{1 + \chi(T - t)}, t_1 \prec t \le T \tag{3}$$

with the extra prerequisites,

$I_1^*(t) = S$ at $t = 0$, $I_2^*(t) = 0$ at $t = t_1$ and $I_3^*(t) = -R$ at

$$t = T \tag{4}$$

Additionally, it has to retain the continuity at d_{c_t} and t_t. When using the boundary conditions (4) to solve differential equations (1)–(3), one requires

$$I_1^*(t) = I_{p_a} - \left\{ (d_a - d_b) M_p t + \frac{c}{2} t^2 \right\}, 0 \le t \le d_{c_t} \tag{5}$$

$$I_2^*(t) = e^{-\omega t^2/2} \int_t^{t_1} (d_a - d_b M_p + cv) dv, d_{c_t} \prec t \le t_1 \tag{6}$$

$$I_3^*(t) = \frac{1}{\chi} \left[\left(d_a - d_b + c\left(\frac{1}{\chi} + T \right) \right) In \left| 1 + \chi(T - t) \right| - c(T - t) \right] - R, t_1 \le t \le T \tag{7}$$

The most substantial extent of merchandise I_{p_a} and maximum shortages H_{b_a}, can be easily obtained by "profiting of the continuity criteria, which are points $t = d_{c_t}, t_1$ and the expansion of the exponential function. This is because the inventory level $I'(t)$ holds the continuity at $t = d_{c_t}$ and $t = t_1$."

$$I_{p_a} = (d_a - d_b) \left[\left(t_1 - d_{c_t} \right) + \frac{\omega}{6} \left(t_1^3 - d_{c_t}^{\ 3} \right) + \frac{\omega^2}{40} \left(t_1^5 - d_{c_t}^{\ 5} \right) - \frac{\omega}{2} \left(t_1 t_d^{\ 2} - d_{c_t}^{\ 3} \right) - \frac{\omega^2}{12} \left(t_1^3 d_{c_t}^{\ 2} - d_{c_t}^{\ 5} \right) + \frac{\omega^2}{8} \left(t_1 d_{c_t}^{\ 4} - d_{c_t}^{\ 5} \right) \right] +$$
$$c \left[\frac{1}{2} \left(t_1^2 - d_{c_t}^{\ 2} \right) + \frac{\omega}{8} \left(t_1^4 - d_{c_t}^{\ 4} \right) + \frac{\omega^2}{48} \left(t_1^6 - d_{c_t}^{\ 6} \right) - \frac{\omega}{4} \left(t_1^2 d_{c_t}^{\ 2} - d_{c_t}^{\ 4} \right) - \frac{\omega^2}{16} \left(t_1^4 d_{c_t}^{\ 2} - d_{c_t}^{\ 6} \right) + \frac{\omega^2}{16} \left(t_1^2 d_{c_t}^{\ 4} - d_{c_t}^{\ 6} \right) \right] +$$
$$(d_a - d_b) d_{c_t} + \frac{c}{2} d_{c_t}^{\ 2} \tag{8}$$

and

$$H_{b_a} = \frac{1}{\chi} \left[\left(d_a - d_b + c\left(\frac{1}{\chi} + T \right) \right) In \left| 1 + \chi(T - t_1) \right| - c(T - t_1) \right] \tag{9}$$

For every single cycle, the net leveraging cost, including degradation costs, is $C_P = A_p(I_{p_a} + H_{b_a})$. Since this inventory system's inventory level $I'(t)$ is invariably non-negative across $[0, t_1]$ the holding cost is divided into two parts: the cost of storing goods during $[0, d_{c_t}]$ and the cost of keeping items during $[d_{c_t}, t_1]$.

Levy of equities during $[0, d_{c_t}]$ is,

$$\int_0^{d_{c_t}} C_c(t) I_1'(t) dt = \int_0^{d_{c_t}} (C_c + \alpha_c t) \left[I_{p_a} - \left\{ (d_a - d_b)t + \frac{c}{2} t^2 \right\} \right] dt$$

Afresh the cost of equities in $[d_{c_t}, t_1]$,

$$\int_{d_{c_t}}^{t_1} C_c(t) I_2'(t) dt = \int_{d_{c_t}}^{t_1} (C_c + \alpha_c t) e^{-\omega t^2/2} \int_t^{t_1} e^{-\omega t^2/2} \left(d_a - d_b M_p + cv \right) dv\, dt$$

As a result, the total expense of equities every item for a single refill is, $E_C = \int_0^{d_{c_t}} C_c(t) I'(t) dt + \int_{d_{c_t}}^{t_1} C_c(t) I_2'(t) dt$

In this case, shortages are categorised based on when the new shipment arrives and the market capitalisation scenario that takes place immediately following the stock level dropping to zero at $[t_1, T]$. Therefore, the price for a single resupply under stock out conditions is, $C_S = -S \int_{t_1}^{T} I_3'(t) dt$

Due to incomplete inventory, the practitioner experiences some sales catastrophes, and the ensuing declaim in the retail price is,

$$L_{C_s} = c_0 \int_{t_1}^{T} D \left(1 - \frac{1}{1 + \chi(T - t)} \right) dt$$

The amount paid in advance for a single refill is obtained by,

$$I_c' \left[\frac{\gamma A_p (I_{p_a} + H_{b_a})}{n} \cdot \frac{P_t}{n} (1 + 2 + 3 + ...+ n) \right] = \frac{n+1}{n} I_c' P_t \gamma A_p (I_{p_a} + H_{b_a})$$

The overall cost incurred by the practitioner throughout the course of the full cycle is now made up of the sum of the following costs: Requisition costs (A_p), leveraging cost (C_P), Equity costs (E_C), Paucity costs (C_S), catastrophe costs (L_{C_S}), and capex cost. Thus, the bottom line of complete succession is,

$$O + \left(1 + \frac{n+1}{2n} I_c' P_t \gamma \right) A_p (I_{p_a} + H_{b_a}) + E_c + C_S + L_{C_S}$$

In summary, the practitioner's fee per unit of time is briefed by the following expression,

$$TC(t_1,T)=\frac{1}{T}\left[O+\left(1+\frac{n+1}{2n}I_c{}'P_t\gamma\right)A_P(I_{p_a}+H_{b_a})+E_c+C_S+L_{C_S}\right] \tag{10}$$

The ultimate goal is to maximize the practitioner's cost $TC_1(t_1,T)$ by obtaining the ideal value of t_1 and T.

Unremitting Repetoire Strategem:

In this instance, after paying the entire purchase price, the order is given to the merchant via new I_{p_a} units when $t=0$, proceeding similarly to the aforementioned paradigm Because of the customer's need for $D=(d_a-d_bM_P-ct)$ alone, this inventory level drops during $[0,d_{c_t}]$ and deterioration begins precisely at $t=d_{c_t}$. The stock level then declines due to both client demand and deterioration over time, eventually depleting to battery temperature. Before receiving the subsequent order of new units, the vendor sells these units for salvage values near the conclusion of the cycle.

As a result, the following is a description of the amount of commodities in stock at $t\in[0,T]$:

$$\frac{dI_1^*(t)}{dt}=-(d_a-d_bM_P+ct),0\le t\le d_{c_t} \tag{11}$$

$$\frac{dI_2^*(t)}{dt}+\beta(t)I_2^*(t)=-(d_a-d_bM_P+ct),d_{c_t}\prec t\le t_1 \tag{12}$$

under the additional criteria,

$$I_1^*(t)=S \text{ at } t=0,\ I_2^*(t)=B \text{ at } t=T \tag{13}$$

Furthermore, it must maintain continuity at $t=d_{c_t}$. By applying the exponential function expansion and the boundary conditions (13) to differential equations (11) and (12), one can obtain

$$I_1^*(t)=I_{p_a}-\left\{(d_a-d_bM_p)t+\frac{c}{2}t^2\right\},0\le t\le d_{c_t} \tag{14}$$

$$I_2^*(t)=\Phi\left(1-\frac{\omega}{2}t^2+\frac{\omega^2}{8}t^4-\frac{\omega^3}{48}t^6\right)-(d_a-d_bd_bM_p)\left(t-\frac{\omega}{3}t^3+\frac{\omega^2}{15}t^5\right)-c\left(\frac{1}{2}t^2-\frac{\omega}{8}t^4+\frac{\omega^2}{48}t^6\right) \tag{15}$$

where,

$$\Delta=Be^{\omega T^2/2}+(d_a-d_bM_P)\left(T+\frac{\omega}{6}T^3+\frac{\omega^2}{40}T^6+\frac{\omega^3}{336}T^7\right)+c\left(\frac{1}{2}T^2+\frac{\omega}{8}T^4+\frac{\omega^2}{48}T^6+\frac{\omega^3}{384}T^8\right)$$

At the point $t=d_{c_t}$, applying the continuity requirement. The maximum inventory level I_{p_a} can be explained as follows using the continuity criterion at the moment $t=d_{c_t}$, since the inventory level preserves continuity.

$$I_{p_a}=\Phi\left(1-\frac{\omega}{2}t^2+\frac{\omega^2}{8}t^4-\frac{\omega^3}{48}t^6\right)-(d_a-d_bd_bM_p)\left(t-\frac{\omega}{3}t^3+\frac{\omega^2}{15}t^5\right)-c\left(\frac{1}{2}t^2-\frac{\omega}{8}t^4+\frac{\omega^2}{48}t^6\right) \tag{16}$$

Consequently, the entire cost of acquisition, including deterioration costs, for a single cycle is outlined by,

$$C_p=A_PI_{p_a} \tag{17}$$

"The cost to store goods during $[0,d_{c_t}]$ and the cost to hold items during $[d_{c_t},T]$ make up the holding cost since the inventory volume $I'(t)$ under this inventory procedure is always non- adverse over" $[0,T]$. At $[0,d_{c_t}]$ the cost of ensuring the model is drawn as,

$$\int_0^{d_{c_t}}C_c(t)I_1'(t)dt=\int_0^{d_{c_t}}(C_c+\alpha_ct)\left[I_{p_a}-\left\{(d_a-d_bM_P)t+\frac{c}{2}t^2\right\}\right]dt$$

Furthermore, using the expansion of the exponential function, the holding cost in $[d_{c_t},T]$ is,

$$\int_{d_{c_t}}^{t_1}C_c(t)I_2'(t)dt=\int_{d_{c_t}}^{t_1}(C_c+\alpha_ct)I_2'(t)dv\,dt$$

Consequently, for a unique refill, the aggregate retention cost is outlined by,

$$E_C=\int_0^{d_{c_t}}C_c(t)I'(t)dt+\int_{d_{c_t}}^{t_1}C_c(t)I_2'(t)dt \tag{18}$$

At the end of the cycle, the seller proceeds to sell all remaining inventory at the salvage price per unit at $t=T$.

Consequently, the importance of salvation is, $S_V=S_PB$. The amount that must be paid in advance for a single refill is calculated solely. Therefore, the capital expense is,

$$I_c{}'\left[\frac{\gamma A_p(I_{p_a}+H_{b_a})}{n}.\frac{P_t}{n}(1+2+3+......+n)\right]=\frac{n+1}{n}I_c{}'P_t\gamma A_pI_{p_a} \tag{19}$$

Consequently, the practitioner's overall expense at the end of the entire cycle is,

$$O+\left(1+\frac{n+1}{2n}I_c{}'P_t\gamma\right)A_pI_{p_a}+E_c-S_V$$

Lastly, the practitioner's recurring fee for a specific duration is given by the utterance,

$$TC(B,T)=\frac{1}{T}\left[O+\left(1+\frac{n+1}{2n}I_c{}'P_t\gamma\right)A_pI_{p_a}+E_c-S_V\right] \tag{20}$$

Subpoena

The following prerequisites must be met concurrently in order to determine the optimal value of "t_1 and T to minimize the total cyclic cost $TC(t_1,T)$ for a unit time, $\frac{\partial TC}{\partial t_1}=0$ and $\frac{\partial TC}{\partial T}=0$ given that the following necessary requirements must be met".

$$\left.\frac{\partial^2TC}{\partial t_1^2}\right|_{(t_1,T)}\prec 0,\left.\frac{\partial^2TC_1}{\partial T^2}\right|_{(t_1,T)}\prec 0\ and\left.\left\{\frac{\partial^2TC_1}{\partial t_1^2}\frac{\partial^2TC_1}{\partial T^2}-\left(\frac{\partial^2TC_1}{\partial t_1\partial T}\right)\right\}\right|_{(t_1,T)}\prec 0$$

Fuzzy Paradigm Using Ranking of Symmetrical Hexagonal Fuzzy Number

This approach divides the hexagon into two triangles and two trapeziums, and it uses the centroid formula for triangles and trapeziums to determine the ranking.

Here T_1 indicates the centroid of the triangle having vertices $(d_1,0),(d_2,0),(d_3,u/2)$ and T_2 indicated the centroid of the triangle having the vertices $(d_4,0),(d_6,0),(d_5,u/2)$ and T_3 indicates the centroid of the trapezium having vertices $(d_3,0),(d_4,0),(d_2,u/2),(d_5,u/2)$ and T_4 indicates the centroid of the trapezium having vertices $(d_3,v),(d_4,v),(d_2,u/2),(d_5,u/2)$ where

$$N_1T_1=\left(\frac{d_1+d_2+d_3}{3},\frac{v}{6}\right);N_2T_2=\left(\frac{d_4+d_5+d_6}{3},\frac{v}{6}\right);N_3T_3=\left(\frac{d_1+2d_3+2d_4+d_5}{4},\frac{v}{4}\right)$$

and $N_4T_4=\left(\frac{d_2+d_3+2d_4+d_5}{4},\frac{v}{4}\right)$.

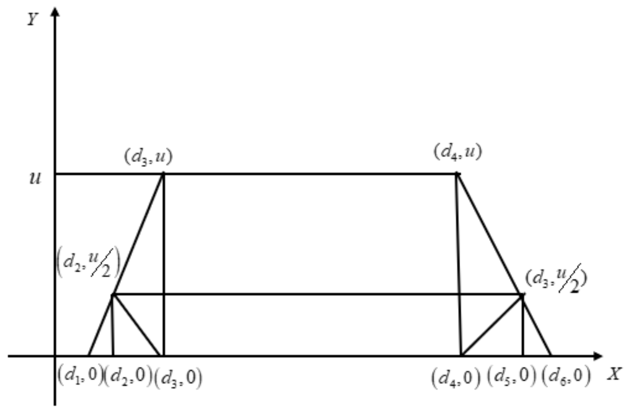

Figure 1: Ranking of Hexagonal fuzzy number

Summing up all these we get,

$$R\left(\tilde{H}_F\right)=\left(\frac{4d_1+10d_2+16d_3+10d_5+4d_6}{12},\frac{5v}{6}\right)$$

For symmetrical hexagonal fuzzy number we obtain the ranking formula using centroid technique as,

$$\tilde{R}_{H_F}=\frac{\vartheta\left(d_6^2+d_5^2+d_5.d_6-d_1.d_2-d_2^2-d_1^2\right)}{3\left(d_6+d_5-d_2-d_1\right)}$$

By taking into consideration the cost parameters $O, M_P, A_P, \alpha_c, C_0, S, I_{p_a}, H_{b_a}, S_P$ as hexagonal fuzzy number and enacting the ranking technique of hexagonal fuzzy number for this propounded model the net cost of the replenishment chain whilst bottlenecks materialize system in fuzzy sense is drawn by the expression as delineated as,

$$TC^*(t_1^*,T^*)=\frac{1}{T^*}\left[\tilde{O}+\left(1+\frac{n+1}{2n}\tilde{I}_c{}'P_t\gamma\right)\tilde{A}_p(\tilde{I}_{p_a}+\tilde{H}_{b_a})+\tilde{E}_c+\tilde{C}_S+\tilde{L}_{C_S}\right]$$

and the net cost for the unremitting repertoire stratagem is underlined as,

$$TC^*(B,T^*)=\frac{1}{T^*}\left[\tilde{O}+\left(1+\frac{n+1}{2n}\tilde{I}_c{}'P_t\gamma\right)\tilde{A}_p\tilde{I}_{p_a}+\tilde{E}_c-\tilde{S}_V\right] \quad (21)$$

Numerical Example

Friable Paradigm

Precedent 1

In order to see the prototype of their improvement, Mishra et al.'s numerical problem is taken into consideration and resolved when a few more data points are added to conform to the developed model and the values are outlined as

$O=\$2500/order; d_a=250units/month; d_b=0.25; c=50; M_p=\$30/unit; A_p=\$20/unit; d_{c_t}=0.1month;$
$c_0=\$15/unit/month; C_c=\$0.25/unit/month; \alpha_c=20/unit/month; \omega=0.03; P_t=0.5month; \chi=0.04;$
$I_c'=\$0.01/month; n=10; \gamma=0.6; S=\$10/unit/month$

The ideal inventory oversight strategy turns out as
$t_1=0.6987month; T=1.2566month$
and

$I_{p_a}=165.565units; H_{b_a}=131.818units; TC(t_1,T)=Rs.6093.712$

Precedent 2

Replenishment chain whilst bottlenecks materialize

To first examine a preceding inventory technique in which the values of $O, d_a, d_b, c, \omega, A_p, S, c_0, C_c$ are modified in order to determine the suitability of the suggested model.
$O=\$1000/order; d_a=250units/month; d_b=0.5; c=4; M_p=\$30/unit; A_p=\$20/unit; d_{c_t}=0.8month;$
$c_0=\$15/unit/month; C_c=\$0.25/unit/month; \alpha_c=20/unit/month; \omega=0.03; P_t=0.5month; \chi=0.04;$
$I_c'=\$0.01/month; n=10; \gamma=0.6; S=\$10/unit/month$

Here, the ideal inventory oversight strategy turns out as
$t_1=0.6987month; T=1.2566month$
and $I_{p_a}=165.565units; H_{b_a}=131.818units; TC(t_1,T)=Rs.6093.712$

Precedent 3

Enduring merchandise strategy

To investigate the viability of the proposed enduring merchandise, let's explore another inventory approach that solely modifies the deterioration-free period $d_{c_t}=0.8month$ and $U_W=300; S_P=\$20/unit$

So, the ideal inventory oversight strategy for this precedent turns out as $B=110.72units; I_{p_a}=300units; T=0.8months; TC(t_1,T)=Rs.6469.181$

Friable Paradigm

Precedent 1

$O=\$850/order; d_a=250units/month; d_b=0.5; c=4; M_p=\$27/unit; A_p=\$18/unit; S=\$9/unit/month;$
$c_0=\$13.5/unit/month; C_c=\$0.25/unit/month; \alpha_c=18/unit/month; d_{c_t}=0.1month; \omega=0.03; P_t=0.5month;$
$I_c'=\$0.01/month, n=10; \gamma=0.6\ \chi=0.04$

The ideal inventory oversight strategy turns out as
$t_1^*=0.6873month; T^*=1.1632month$
and
$I_{p_a}{}^*=165.4470units; H_{b_a}{}^*=113.2623units; TC^*(t_1^*,T^*)=Rs.5491.8741$

Precedent 2

Replenishment chain whilst bottlenecks materialize

In order to examine an antecedent inventory technique in a friable paradigm where the values of $O, d_a, d_b, c, \omega, A_p, S, c_0, C_c$ are modified in a fuzzy sense in order to determine whether the suggested model is adequate.
$O=\$850/order; d_a=250units/month; d_b=0.5; c=4; M_p=\$27/unit; A_p=\$18/unit; d_{c_t}=0.64315month;$
$c_0=\$13.5/unit/month; C_c=\$0.25/unit/month; \alpha_c=18/unit/month; \omega=0.03; P_t=0.5month; \chi=0.04;$
$I_c'=\$0.01/month; n=10; \gamma=0.6; S=\$9/unit/month$

Here, the ideal inventory oversight strategy for this precedent turned to be as $t_1^*=0.2879month;$ $T^*=0.64315\,month; I_{p_a}{}^*=261.8113\,units; H_{b_a}{}^*=187.5463\,units$ and
$TC^*(t_1^*,T^*)=Rs.5567.3253$

Precedent 3

Enduring merchandise strategy
$O=\$540/order; d_a=200units/month; d_b=0.5; c=4; M_p=\$22.5/unit; A_p=\$13.5/unit; d_{c_t}=0.5month;$
$c_0=\$15/unit/month; C_c=\$0.20/unit/month; \alpha_c=22.5/unit/month; \omega=0.05; P_t=0.6month; \chi=0.04;$
$I_c'=\$0.01/month; n=15; \gamma=0.6; S_P=\$18/unit/month; U_W=300$

So, the ideal inventory oversight
strategy for this precedent turns out as

$B^*=109.56units; I_{p_a}{}^*=238.0698units; T^*=1.25months; TC^*(t_1^*,T^*)=Rs.4116.0048$

Discussion

The ultimatum in the present study is finite to examine all upward and downward linearly with regard to period of storage. Therefore, it is more realistic to adopt non-linear time-sensitive ultimatum rather than linear time-sensitive ultimatum. Nevertheless, due to lack of raw materials and furnished goods in their warehouses, the enterprise and the market system may crumple. The financial gain process may get affected due to the outlay, defective product during storing and depreciation of the commodities. Subsequently, a genuine inventory management is the need of the enterprise to remain unaffected. Therefore, traditional methods are employed to solve the aforementioned issues. A fundamental principle is established to ascertain the best inventory strategy for either a continuous inventory problem or a non-ending inventory problem that necessitates the excess products in stock or stock-out. The advanced notion is used to provide an algorithm approach for determining the optimal inventory strategy for a perpetual inventory problem. By figuring up different fictitious inventory procedures, this method is evaluated.

Conclusion

All organisations are in a quest of a perfect inventory strategy for biodegradable items to attain productive profit and to make certain that the natural resources are properly used thereby the environment is protected. In this paper, a variety of prepayment mechanisms are obtained and two sustainable inventory strategies for both time-sensitive and delayed biodegradable commodities are examined. To enmesh an issue more typical, both time associated demand and includes incompetence in the fancier's stockroom and the time associated with each unit holding cost. Two models are worked out in which far-fetched condition is overcome by espousing period of waiting linked rate of backlogging in the first model and a continuous occurrence wherein the latter model, salvage value is determined. The primary goal is to lower the fancier's value for the two inventory procedures mentioned above. But on account of exceptional non-linearity, closed form solutions cannot be applied to accomplish the above two cases. So, classic techniques are used to accomplish solutions for above cases. To regardless, the paradigm demands for the optimal inventory strategy in a continual inventory problem that never ends in a non-ending inventory issue in a continuous inventory issue demands the extra goods in the stock or stock-out a prime principle is established. By using the advanced principle, an algorithm methodology is furnished to come up with the optimal inventory strategy for a perpetual inventory replica. This epitome is evaluated by the solution of several fictitious inventory tactics.

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