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On a Special (γ, β) - Metric and its Hypersurfaces

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ABSTRACT

In this paper we have considered a special (γ, β) – metric (1.2) and find some important tensors. We have also considered the hypersurface given by the equation *b(x)* = constant with the special (γ, β) – metric (1.2).

Keywords : Finsler Space with (γ, β) – metric, cubic metric, one form metric, angular metric tensor, fundamental tensor and reciprocal tensor.

INTRODUCTION

In 1979 Matsumoto.M., [1] introduced the concept of cubic metric given by

$$L(x, y) = \left\{a_{ijk}(x) y^i y^j y^k\right\}^{1/3} \tag{1.1}$$

where $a_{ijk}(x)$ are the components of symmetric tensor field of (0, 3) type which depends on the position *x* alone. In 2011 Pandey. T. N. and Chaubey. V.K.[9], introduced the concept of (γ, β) – metric

$$L = L(\gamma, \beta)$$

where γ is a cubic metric and $\beta = b_i y^i$ is a one form metric. We have also some research papers on (γ, β)– metric [2,7,9,10,11,12], whose studies have definitely contributed to the growth of Finsler geometry. In this paper we have introduced a special (γ, β)– metric given by

$$L^3 = C_1\gamma^3 + 3C_2\gamma^2\beta + 3C_3\gamma\beta^2 + C_4\beta^3 \tag{1.2}$$

By taking this metric, we have calculated basic tensors such as $l_i, h_{ij}, g_{ij}, g^{ij}$ and some important theorems.

In 1995, Matsumoto. M., [6], had discussed the properties of a special hypersurface of Rander's spaces with $b_i(x) = (\partial_i b)$ being the gradient of a scalar function *b(x)*. He had considered a hypersurface given by the equation *b(x)* = constant.

In this paper we have also used the hypersurface given by the equation *b(x)* = constant, of the Finsler space with a special (γ, β) – metric given by the equation (1.2).

2. The Finsler space with metric (1.2)

Let $F^n = (M^n, L)$ be an n –dimensional Finsler space with (γ, β) – metric given by the equation (1.2), where $\gamma = \left\{a_{ijk}(x) y^i y^j y^k\right\}^{1/3}$ is a Cubic metric in M^n and $\beta = b_i(x) y^i$ is a 1 – form in M^n. The derivatives of L (γ,β) with respect to γ and β are given by

$$L_\gamma = L^{-2}\left(C_1\gamma^2 + 2C_2\gamma\beta + C_3\beta^2\right) \tag{2.1}$$

$$L_\beta = L^{-2}\left(C_2\gamma^2 + 2C_3\gamma\beta + C_4\beta^2\right) \tag{2.2}$$

$$L_{\gamma\gamma} = 2L^{-2}\left(C_1\gamma + C_2\beta - LL^2\gamma\right) \tag{2.3}$$

$$L_{\beta\beta} = 2L^{-2}\left(C_3\gamma + C_4\beta - LL^2\beta\right) \tag{2.4}$$

$$L_{\gamma\beta} = 2L^{-2}\left(C_2\gamma + C_3\beta - LL_\gamma L_\beta\right) \tag{2.5}$$

Where

$$L_\gamma = \frac{\partial L}{\partial \gamma} \qquad L_\beta = \frac{\partial L}{\partial \beta}$$

$$L_{\gamma\gamma} = \frac{\partial L_\gamma}{\partial \gamma} \qquad L_{\beta\beta} = \frac{\partial L_\beta}{\partial \beta}$$

The normalized element of support $l_i = \dot{\partial}_i L$ is given by

$$l_i = \frac{L_\gamma}{\gamma^2} a_i + L_\beta b_i \tag{2.6}$$

The angular metric tensor $h_{ij} = L\dot{\partial}_i\dot{\partial}_j L,$ is obtained as

$$h_{ij} = P_{-1}a_{ij} + P_0 b_i b_j + P_{-2}\left(a_i b_j + a_j b_i\right) + P_{-4}a_i a_j \tag{2.7}$$

Where $P_0 = LL_{\beta\beta} = 2L^{-1}\left(C_3\gamma + C_4\beta - LL^2\beta\right)$

$$P_{-1} = \frac{2LL_\gamma}{\gamma^2} = 2\gamma^{-2}L^{-1}\left(C_1\gamma^2 + 2C_2\gamma\beta + C_3\beta^2\right)$$

$$P_{-2} = \frac{LL_{\gamma\beta}}{\gamma^2} = 2\gamma^{-2}L^{-1}\left(C_2\gamma + C_3\beta - LL_\gamma L_\beta\right)$$

$$P_{-4} = \frac{L}{\gamma^4}\left(L_{\gamma\gamma} - 2\frac{L_\gamma}{\gamma}\right) = -2\gamma^{-5}L^{-1}\left(C_2\gamma\beta + C_3\beta^2 + \gamma LL_\gamma^2\right)$$

In equation (2.7) the subscripts of coefficients P_{-1}, P_0, P_{-2} and P_{-4} are used to indicate respective degrees of homogeneity.

The fundamental tensor $g_{ij} = h_{ij} + l_i l_j$ is given by

$$g_{ij} = P_{-1}a_{ij} + q_0 b_i b_j + q_{-2}\left(a_i b_j + a_j b_i\right) + q_{-4}a_i a_j \quad (2.8)$$

If

$$q_0 = P_0 + L^2\beta = 2L^{-1}\left(C_3\gamma + C_4\beta - \frac{1}{2}LL^2\beta\right)$$

$$q_{-2} = P_{-2} + \frac{L_\gamma L_\beta}{\gamma^2} = 2\gamma^{-2}L^{-1}\left(C_2\gamma + C_3\beta - \frac{1}{2}LL_\gamma L_\beta\right)$$

$$q_{-4} = P_{-4} + \frac{L^2\gamma}{\gamma^4} = -\gamma^{-5}L^{-1}\left(2C_2\gamma\beta + 2C_3\beta^2 + \gamma LL_\gamma^2\right)$$

(2.9)

We know that $g^{hj}g_{ij} = \delta_i^h$

The reciprocal tensor g^{ij} of g_{ij} is given by

$$g^{ij} = \frac{a^{ij}}{P_{-1}} - \frac{d\left(q_{-2}\pi_{-1} - q_{-4}\tau_{-1}\right)}{P_{-1}}a^i a^j - \frac{d\left(q_{-2}\tau - q_0\pi\right)}{P_{-1}}b^i b^j - \frac{d\left(q_0\pi_{-1} - q_{-2}\tau_{-1}\right)}{P_{-1}}a^i b^j - \frac{d\left(q_{-4}\tau - q_{-2}\pi\right)}{P_{-1}}a^j b^i$$

Where $P_{-1} + q_{-4}\gamma^3 + q_{-2}\beta = \pi, \qquad q_{-2}\gamma^3 + q_0\beta = \tau$

$$q_{-4}\beta + q_{-2}b^2 = \pi_{-1}, \qquad P_{-1} + q_{-2}\beta + q_0 b^2 = \tau_{-1}$$

$$d = \frac{1}{\tau\pi_{-1} - \pi\tau_{-1}}$$

$$g^{ij} = \frac{a^{ij}}{P_{-1}} - S_0' b^i b^j - S_1'\left(a^i b^j + a^j b^i\right) - S_2' a^i a^j \quad (2.10)$$

Where

$$d = \frac{1}{\left\{q_0 q_{-4} - \left(q_{-2}\right)^2\right\}\left(\beta^2 - b^2\gamma^3\right) - P_{-1}\left(P_{-1} + 2q_{-2}\beta + q_0 b^2 + q_{-4}\gamma^3\right)}$$

$$S_0' = \frac{d\left(q_{-2}\tau - q_0\pi\right)}{P_{-1}} = \frac{d\left[\left\{\left(q_{-2}\right)^2 - q_0 q_{-4}\right\}\gamma^3 - q_0 P_{-1}\right]}{P_{-1}}$$

$$S_1' = \frac{d\left(q_0\pi_{-1} - q_{-2}\tau_{-1}\right)}{P_{-1}} = \frac{d\left(q_{-4}\tau - q_{-2}\pi\right)}{P_{-1}} = \frac{d\left[\left\{q_0 q_{-4} - \left(q_{-2}\right)^2\right\}\beta - P_{-1}q_{-2}\right]}{P_{-1}}$$

$$S_2' = \frac{d\left(q_{-2}\pi_{-1} - q_{-4}\tau_{-1}\right)}{P_{-1}} = \frac{d\left[\left\{\left(q_{-2}\right)^2 - q_0 q_{-4}\right\}b^2 - q_{-4}P_{-1}\right]}{P_{-1}}$$

Theorem (2.1) The angular metric tensor h_{ij}, the fundamental tensor g_{ij} and its reciprocal tensor g^{ij} of (*γ, β*)−metric are given by equations (2.7), (2.8) and (2.10) respectively.

3. The Hypersurfaces $F^{(n-1)}(c)$

In this section, we have taken the special (*γ,β*) – metric with a gradient $b_i(x)=\partial_i b$ for a scalar function *b(x)* and a hypersurface $F^{n-1}(c)$ which is given by the equation *b(x) = c* (constant).

Since the parametric equation of $F^{n-1}(c)$ is $x^i = x^i(u^\alpha)$, hence

$(\partial/\partial u^\alpha) b(x(u)) = 0 = b_i(x) X^i_\alpha$, where $b_i(x)$ are covariant components of a normal vector field of $F^{n-1}(c)$

Therefore, along the $F^{n-1}(c)$, we have

$$b_i X^i_\alpha = 0 \qquad and \qquad b_i y^i = 0 \tag{3.1}$$

In general, the induced metric

$$\underline{L}(u,v) = C_1\left\{a_{\alpha\beta\gamma}(u)v^\alpha v^\beta v^\gamma\right\} \tag{3.2}$$

where

$$a_{\alpha\beta\gamma}(u) = a_{ijk}(x(u)) X^i_\alpha X^j_\beta X^k_\gamma$$

$$X^i_\alpha = \frac{\partial x^i}{\partial u^\alpha}$$

By using equation (3.1) and (2.10), we have

$$g^{ij}b_i b_j = b^2\left(\frac{1}{P_{-1}} - S'_0 b^2\right)$$

where

$$b^2 = a^{ij}b_i b_j$$

Hence, we get

$$b_i = b\sqrt{\frac{1}{P_{-1}} - b^2 S'_0}\, N_i \tag{3.3}$$

Hence from (2.10) and (3.3) we have

$$b^i = a^{ij}b_j = \frac{b}{\sqrt{\frac{1}{P_{-1}} - b^2 S'_0}} N^i + \left(\frac{b^2 S'_1}{\frac{1}{P_{-1}} - b^2 S'_0}\right) a^i \tag{3.4}$$

Theorem (3.1). Let F^n be a Finsler space with (*γ,β*) – metric (1.2), $b_i(x) = \partial_i b(x)$ and $F^{n-1}(c)$ be a hypersurface of F^n given by *b(x) = c* (constant). If b_i is a non- zero field, then the induced metric of $F^{n-1}(c)$ is given by (3.2) and relations (3.3) and (3.4) hold.

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