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RESEARCH ARTICLE

A New Approach for Solving Bilevel Fractional/quadratic Green Transportation Problem by Implementing AI with Multi Choice Parameters Under Uncertainty

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Abstract

Modern technology is led by artificial intelligence (AI), which is transforming many aspects of our daily life. Urban regions continue to struggle with traffic congestion, which lengthens travel times, increases fuel consumption, and pollutes the environment. To reduce congestion and preserve a smooth traffic flow, AI systems can dynamically assign lanes, synchronize traffic lights, and optimize signal timings. The unpredictability of transportation conditions leads to degradation or damage to the products. In addition, there are elements like growing fuel costs and the desire to cut carbon emissions that make it difficult for businesses to move goods. In this paper a new model is proposed using AI with uncertain cost and multi-choice supply and demand parameters (BFQGMCTP) to develop a Bilevel Fractional/Quadratic Green Transportation Problem. The objective is to concurrently reduce transportation costs, transit-related deterioration costs, and carbon emission costs. Two distinct approaches namely, intuitionistic fuzzy programming and goal programming are used to tackle the current problem, and a comparative study of the two solutions is presented. The computations show that the implementation of AI technology reduced carbon emission, fuel consumption, and travel time by 18%, 15%, and 30% respectively.

Keywords: Artificial intelligence, fractional/quadratic transportation problem, fuzzy environment, multi choice parameters.
关键词人工智能，分数/二次运输问题，模糊环境，多选择参数。

Introduction

To meet the diverse needs of consumers, logistics industries must offer a wide range of products on the market. However, transporting these goods can increase cost due to wear and tear on the vehicle and potential damage or degradation of the products during transit. Transportation refers to the movement of goods, people, or animals from one place to another. It is a vital system that ensures the efficient delivery of products to their destination and plays a fundamental role in the growth of economies and civilizations. The main objective of a transportation problem is to identify the most cost-effective way to move items from sources to destinations. Transportation problem was first introduced by Hitchcock (1941). Traditional linear models often fail to capture the complexity of real-world logistics. To address the scenarios where costs are depended on ratios, one such variation is the Fractional Transportation Problem (FTP), which involves transporting varying quantities of a single homogeneous commodity from several origins to multiple destinations, while minimizing the overall fractional transportation cost. complex real-world phenomena such as fuel consumption based on distance or vehicle load, making it a more accurate and realistic representation of logistics problems. The Quadratic Transportation Problem (QTP), another form of transportation problem, deals with minimizing transportation costs that are nonlinear, typically quadratic in nature. These quadratic cost functions reflect complex real-world phenomena such as fuel consumption based on distance or vehicle load, making the QTP a more accurate and realistic representation of logistics challenges. Stancu-Minasian (1978, 1997) explored transportation problems with fractional objectives and developed solution

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methods. Basu and Acharya (2002) proposed an algorithm for the Quadratic Fractional Bi-Criterion Transportation Problem. Arya and Singh (2020) also proposed a solution approach for the QFTP. The growing concern over the environmental impacts of transportation, particularly greenhouse gas emissions, has led to the formulation of the Green Transportation Problem. This problem aims to reduce the ecological footprint of transportation systems by minimizing emissions and fuel usage.

In logistics and supply chain management, a fractional and quadratic green transportation problem aims to optimize transportation routes by balancing cost efficiency (fractional aspect) and environmental impact (green aspect). The cost function in such model incorporates non-linear elements such as fuel consumption depending on vehicle load or distance (quadratic aspect), enabling a more accurate analysis of trade-offs between cost reduction and emission control. Programming problems involving two distinct hierarchical decision-making levels—the upper level and the lower level are known as bilevel programming problems (BLPPs). In the context of transportation, bilevel programming is used to model interactions between different decision-makers (e.g., government and logistics companies). Qiu et al. (2020) applied BLP to reduce carbon emissions and fossil fuel use in the aviation sector, and Zhu et al. (2022) proposed a BLP approach to reduce emissions and delivery times in goods transportation. A multi-choice parameter in transportation planning is a variable that can assume multiple values under different scenarios. This approach enables more flexible modelling of real-world situations where different options are available for vehicle types, routes, or modes of transport. When dealing with multi objective optimization, where different objectives may conflict, goal programming is an extension of linear programming is often used. Maity and Kumar Roy (2016) introduced a mathematical model for the Multi-Objective Transportation Problem (MOTP) with nonlinear cost functions and multi-choice demand. Roy et al. (2017) employed conic scalarization to solve multi-choice problems in MOTP.

Artificial Intelligence (AI) plays a key role in modern transportation systems. It analyzes real-time data from various transportation modes such as cars, buses, and trains to identify patterns and detect safety risks. Dikshit, S. et al. explored the use of artificial intelligence to optimize vehicle routing and alleviate traffic congestion in urban areas. Pillai, A. S. et al. examined how AI-driven traffic management can significantly enhance traffic flow, reduce carbon emissions, and provide a scalable solution for modern urban planning. In this study, AI has been incorporated to solve the Bilevel Fractional/Quadratic Green Transportation Problem (BFQGTP) to promote environmentally friendly transportation by reducing both costs and emissions during goods delivery. The goal of using AI in transportation is to decrease travel time, fuel consumption, and emissions, while improving overall system efficiency. AI helps optimize traffic flow, predict congestion, adjust traffic signals, plan efficient delivery routes, monitor driver behavior, and enhance safety by processing real-time sensor and camera data. In this context, AI supports the resolution of the Fractional Transportation Problem (FTP), which seeks to maximize efficiency while minimizing costs, and the Quadratic Transportation Problem (QTP), which aims to reduce transportation and deterioration costs through nonlinear cost functions. AI also plays a vital role in traffic management systems, improving route planning and reducing congestion, idling time, and emissions. This results in lower fuel consumption and smoother traffic flow. This paper proposes a new AI-based model for the Bilevel Fractional/Quadratic Green Transportation Problem. In this model, all the parameters cannot be fixed due to unpredictable factors such as weather or changes in transport modes. To address this fluctuating market behaviour, the concept of multi-choice programming with uncertainty was introduced. In this paper supply and demand are treated as multi choice fuzzy parameters while economic and environmental costs are represented as triangular fermatean fuzzy number. It is first introduced by T Senapati which is the extension of Pythagorean fuzzy number. In fermatean fuzzy set, sum of cube of membership and non-membership lies between 0 and 1. The proposed model is solved using fermatean fuzzy programming to obtain the pareto optimal solution after converting deterministic model with the ranking function and interpolating polynomial. The remainder of this research is classified below. In Section 2, basis definition of fermatean and multi choice programming has been defined. Section 3 represented Notations and Assumptions. The suggested model is formulated and methodology has been described in Section 3. In Section 4, Numerical illustrations are performed. Finally, conclusion has been stated in Section 5.

Methodology

Preliminaries

Definition

Let U be a non-empty set. A fermatean fuzzy set $\tilde{E}^F$ over U is defined as $\tilde{E}^F = \left\{ \left\langle u, \mu_{\tilde{E}^F}(u), \nu_{\tilde{E}^F}(u) \right\rangle : u \in U \right\}$ where $\mu_{\tilde{E}^F}(u), \nu_{\tilde{E}^F}(u) : U \to [0,1]$ are membership and non-membership functions respectively. Moreover, a fermatean fuzzy number satisfies the condition $0 \le (\mu_{\tilde{E}^F}(u))^3 + (\nu_{\tilde{E}^F}(u))^3 \le 1$ for all $u \in U$. Additionally, the degree of hesitation is defined for all $u \in U$ as $\pi_{\tilde{E}^F}(u) = \sqrt{1 - (\mu_{\tilde{E}^F}(u))^3 - (\nu_{\tilde{E}^F}(u))^3}$. (Akram, M. et.al 2023)

Definition

Let $\tilde{A}^F$ be a triangular fermatean fuzzy number (TFFN) which is defined by $\tilde{E}^F = \langle (e^l, e^m, e^n), \alpha, \beta \rangle$. The membership and non-membership function are given as follows

$$\mu_{\tilde{E}^F}(u) = \begin{cases} \dfrac{(u-e^l)}{e^m - e^l} & \text{if } e^l \le u \le e^m \\ \alpha & \text{if } u = e^m \\ \dfrac{(e^r - u)\beta}{e^r - e^m} & \text{if } e^m \le u < e^r \\ 0 & \text{if } u < e^l \text{ or } u > e^r \end{cases}$$

$$\nu_{\tilde{E}^F}(u) = \begin{cases} \dfrac{[u-e^m + \alpha(u-e^l)]}{e^m - e^l} & \text{if } e^l \le u \le e^m \\ \beta & \text{if } u = e^m \\ \dfrac{[u-e^m + \beta(e^r - u)\beta}{e^r - e^m} & \text{if } e^m \le u < e^r \\ 1 & \text{if } u < e^l \text{ or } u > e^r \end{cases}$$

Here α denotes the maximum value of $MF(\mu_{\tilde{E}^F})$ and β denotes the minimum value of $MF(\nu_{\tilde{E}^F})$ respectively, such that $\alpha, \beta \in [0,1]$ and $0 \le \alpha^3 + \beta^3 \le 1$. Consider $\alpha = 1$ and $\beta = 0$. Then TFFNbecomes $\tilde{E}^F = \langle (e^l, e^m, e^n), (e^{l'}, e^m, e^{n'}) \rangle$ whose $MF(\mu_{\tilde{E}^F})$ and $MF(\nu_{\tilde{E}^F})$ can be described as follows

$$\mu_{\tilde{E}^F}(u) = \begin{cases} \dfrac{(u-e^l)}{e^m - e^l} & \text{if } e^l \le u \le e^m \\ 1 & \text{if } u = e^m \\ \dfrac{(e^r - u)}{e^r - e^m} & \text{if } e^m \le u < e^r \\ 0 & \text{if } u < e^l \text{ or } u > e^r \end{cases}$$

$$\nu_{\tilde{E}^F}(u) = \begin{cases} \dfrac{e^m - u}{e^m - e^l} & \text{if } e^{l'} \le u \le e^m \\ 0 & \text{if } u = e^m \\ \dfrac{u - e^m}{e^{r'} - e^m} & \text{if } e^m \le u < e^{r'} \\ 1 & \text{if } u < e^{l'} \text{ or } u > e^{r'} \end{cases}$$

Where $e^{l'} \le e^l \le e^m \le e^r \le e^{r'}$. (Akram, M. et.al 2023)

Definition

Let $\tilde{E}^F = \langle (e^l, e^m, e^n), (e^{l'}, e^{m'}, e^{n'}) \rangle$ be a triangular fermatean fuzzy number. Then the ranking function for triangular fermatean fuzzy number is defined as $\Re(\tilde{E}^F) = \frac{(e^l + 4e^m + e^n) + (e^{l'} + 4e^m + e^{n'})}{12}$. (Akram, M. et.al 2023)

Definition

The Lagrange's interpolation is one of the numerical approximation methods which is used to convert the muti choice parameter into the optimal choice. An integer variable is introduced for each multi-choice parameter to formulate the interpolating polynomial. Since above proposed model has w_{k_1} number of choices for both the supply and demand parameters, integer variables are introduced accordingly

The multi choice supply and demand parameters $s_{k_1}^{w_{k_1}}, d_{k_1}^{w_{k_1}} (k_1 \in K_1^{'})$ are replaced by assigning the integer variables α_{k_1} .itdetermines w_{k_1} values; $\alpha_{k_1} = 0, 1, ..., w_{k_1} - 1$. The corresponding polynomial is determined as follows:

$$z_{s_{k_1}}^l(\alpha_{k_1}) = \frac{(\alpha_{k_1} - 1)(\alpha_{k_1} - 2)...(\alpha_{k_1} - w_{k_1} + 1)}{(-1)^{w_{k_1} - 1}(w_{k_1} - 1)!} s_{k1}^{(1)} + \frac{(\alpha_{k_1})(\alpha_{k_1} - 1)...(\alpha_{k_1} - w_{k_1} + 1)}{(-1)^{w_{k_1} - 2}(w_{k_1} - 2)!} s_{k1}^{(2)} + \frac{(\alpha_{k_1})(\alpha_{k_1} - 1)...(\alpha_{k_1} - w_{k_1} + 1)}{(-1)^{w_{k_1} - 3}(w_{k_1} - 3)!2!} s_{k1}^{(3)} + ... + \frac{(\alpha_{k_1})(\alpha_{k_1} - 1)...(\alpha_{k_1} - w_{k_1} + 2)}{(w_{k_1} - 1)!} s_{k1}^{(k)}$$

$$z_{d_{k_1}}^l(\alpha_{k_1}) = \frac{(\alpha_{k_1} - 1)(\alpha_{k_1} - 2)...(\alpha_{k_1} - w_{k_1} + 1)}{(-1)^{w_{k_1} - 1}(w_{k_1} - 1)!} d_{k1}^{(1)} + \frac{(\alpha_{k_1})(\alpha_{k_1} - 1)...(\alpha_{k_1} - w_{k_1} + 1)}{(-1)^{w_{k_1} - 2}(w_{k_1} - 2)!} d_{k1}^{(2)} + \frac{(\alpha_{k_1})(\alpha_{k_1} - 1)...(\alpha_{k_1} - w_{k_1} + 1)}{(-1)^{w_{k_1} - 3}(w_{k_1} - 3)!2!} d_{k1}^{(3)} + ... + \frac{(\alpha_{k_1})(\alpha_{k_1} - 1)...(\alpha_{k_1} - w_{k_1} + 2)}{(w_{k_1} - 1)!} d_{k1}^{(k)}$$

(El Sayed et.al 2023)

Mathematical Model for Bilevel Fractional/Quadratic Green Transportation Problem by Implementing AI traffic control system with Multi Choice Parameters Under Fuzzy Environment

This section provides list of notations along with their intended meaning and assumptions are made in this proposed model. Subsequently, a mathematical formulation is constructed for bilevel fractional/quadratic transshipment problem by implementing AI traffic signal control system with multi choice under fermatean fuzzy environment

Notations and Assumptions

$D_1^{'}$ = sources at upper level $(m = 1,2,3,...s_1)$

$S_1^{'}$ = destinations at upper level $(n = 1,2,3,...t_1)$

$D_2^{'}$ = sources at lower level $(i = 1,2,3,...s_2)$

$S_2^{'}$ = destinations at lower level $(j = 1,2,3,..,t_2)$

ρ -AI reduction parameter in percentage

$\tilde{h}_1 = \left[\tilde{h}'_{mn}\right], \tilde{\delta}_1 = \left[\tilde{\delta}'_{mn}\right], \tilde{v}_1 = \left[\tilde{v}'_{mn}\right], \quad \tilde{k}_1 = \left[\tilde{k}'_{mn}\right],$
$\tilde{\gamma}_1 = \left[\tilde{\gamma}'_{mn}\right], \tilde{u}_1 = \left[\tilde{u}'_{mn}\right] \; m,n \in D_1^{'}, S_1^{'}$

$\tilde{h}_2 = \left[\tilde{h}'_{ij}\right]$, $\tilde{\delta}_2 = \left[\tilde{\delta}'_{ij}\right]$, $\tilde{v}_2 = \left[\tilde{v}'_{ij}\right]$, $\tilde{k}_2 = \left[\tilde{k}'_{ij}\right]$,
$\tilde{\gamma}_2 = \left[\tilde{\gamma}'_{ij}\right]$, $\tilde{u}_1 = \left[\tilde{u}'_{ij}\right] \; i,j \in D_2^{'}, S_2^{'}$

$\tilde{h}'_{mn} > 0$, $\tilde{\delta}'_{mn} > 0$, $\tilde{v}'_{mn} > 0$, $\tilde{k}'_{mn} > 0$, $\tilde{\gamma}'_{mn} > 0$, $\tilde{u}'_{mn} > 0$ are the fermatean cost parameters at upper levels respectively.

$\tilde{h}'_{ij} > 0$, $\tilde{\delta}'_{ij} > 0$, $\tilde{v}'_{ij} > 0$, $\tilde{k}'_{ij} > 0$, $\tilde{\gamma}'_{ij} > 0$, $\tilde{u}'_{ij} > 0$ are the cost parameters at lower level respectively

$D_1^{'} = \left[d'_{mn}\right], D_2^{'} = \left[d'_{ij}\right]$ where $d'_{mn}, d'_{ij} \geq 0$ are decision variables denoting the amount of goods are transported at lower and upper level.

$(\tilde{o}'^{(1)}_m, \tilde{o}'^{(2)}_m, ..., \tilde{o}'^{(g_m)}_m), (\tilde{o}'^{(1)}_n, \tilde{o}'^{(2)}_n, ..., \tilde{o}'^{(g_n)}_n)$ are multi choice parameter for supply and demand in the upper-level problem. $o'^{(g_m)}_m, o'^{(g_n)}_n \geq 0, m \in D_1^{'}, n \in D_2^{'}$

$(\tilde{o}'^{(1)}_i, \tilde{o}'^{(2)}_i, ..., \tilde{o}'^{(x_i)}_i), (\tilde{o}'^{(1)}_j, \tilde{o}'^{(2)}_j, ..., \tilde{o}'^{(y_j)}_j)$ are multi choice parameter for supply and demand in the lower-level problem. $o'^{(x_i)}_i, o'^{(y_j)}_j \geq 0, i \in D_2^{'}, j \in S_2^{'}$

Feasibility condition for BTPMCP: $\sum_{m \in D_1^{'}} o'_m \geq \sum_{n \in S_1^{'}} o'_n ; \sum_{i \in D_2^{'}} o'_i \geq \sum_{j \in D_2^{'}} o'_j$

Mathematical Formulation

Bilevel Fractional/Quadratic Green Transportation Problem by Implementing AI traffic control system with Multi Choice Parameters Under Fuzzy Environment

The proposed Bilevel Fractional/Quadratic Green Multi-Choice Transportation Problem (BFQGMCTP) using AI consists of two decision-making levels: the upper level and the lower level. In this model, both supply and demand parameters are treated as multi-choice variables, allowing flexibility in representing real-world scenarios. The cost coefficients in the objective functions at both levels are expressed using triangular fermatean fuzzy numbers (TIFNs) to handle uncertainty and imprecision in cost estimation. At the upper level, the objective is to maximize the number of goods transported from their origins to destinations while simultaneously minimizing transportation costs and carbon emissions. This level is modeled as a Fractional Green Transportation Problem (FGTP) to capture the trade-off between efficiency and environmental impact. However, during transit, unfavorable weather conditions or poor road infrastructure may lead to spoilage or damage, particularly of perishable goods such as food items. Additionally, transportation activities inherently generate carbon emissions. To address these issues, the lower level is formulated as a Quadratic Green Transportation Problem (QGTP). Its objective is to minimize the costs associated with product deterioration and environmental impact, especially those caused by emissions.

The mathematic model for bilevel fractional/quadratic green transshipment problem with multi choice under fermatean fuzzy environment is expressed as follows

$$\min_{D_1} Z_{11}(D_1, D_2) = \frac{(\tilde{h}_1 D_1 + \tilde{h}_2 D_2) + (\tilde{\delta}_1 D_1 + \tilde{\delta}_2 D_2)}{(\tilde{v}_1 D_1 + \tilde{v}_2 D_2)}$$

$$\min_{D_2} Z_{12}(D_1, D_2) == ((\tilde{k}_1 D_1 + \tilde{k}_2 D_2)) + (\tilde{\gamma}_1 D_1 + \tilde{\gamma}_2 D_2))(\tilde{u}_1 D_1 + \tilde{u}_2 D_2)$$

Subject to the constraints

$$\sum_{n=1}^{t_1} d_{mn} \leq (\tilde{o}'^{(1)}_m, \tilde{o}'^{(2)}_m, ..., \tilde{o}'^{(g_m)}_m), m = 1,2,3,4,\ldots,s_1$$

$$\sum_{m=1}^{s_1} d_{mn} \geq (\tilde{o}'^{(1)}_n, \tilde{o}'^{(2)}_n, ..., \tilde{o}'^{(g_n)}_n), n = 1,2,\ldots,t_1$$

$$\sum_{j=1}^{t_2} d_{ij} \leq (\tilde{o}'^{(1)}_i, \tilde{o}'^{(2)}_i, ..., \tilde{o}'^{(x_i)}_i), i = 1,2,3,4,\ldots,s_2$$

$$\sum_{i=1}^{s_2} d_{ij} \geq (\tilde{o}'^{(1)}_j, \tilde{o}'^{(2)}_j, ..., \tilde{o}'^{(x_j)}_j), j = 1,2,\ldots,t_2$$

$$d_{mn} \geq 0 \quad \forall m = 1,2,3,\ldots,s_1; n = 1,2,3,\ldots,t_1$$

$$d_{ij} \geq 0 \quad \forall i = 1,2,3,\ldots,s_2; j = 1,2,3,\ldots,t_2$$

At upper level, $(\tilde{h}_1 D_1 + \tilde{h}_2 D_2)$, $(\tilde{\delta}_1 D_1 + \tilde{\delta}_2 D_2)$ and $(\tilde{v}_1 D_1 + \tilde{v}_2 D_2)$ are represent the delivery cost, carbon emission during the transportation of commodities and the maximum amount of goods transported. At lower level, $(\tilde{k}_1 D_1 + \tilde{k}_2 D_2)$, $(\tilde{\gamma}_1 D_1 + \tilde{\gamma}_2 D_2)$, $(\tilde{u}_1 D_1 + \tilde{u}_2 D_2)$ denotes the transportation cost, carbon emission cost, deterioration cost during the transportation of products.

The mathematic model for bilevel fractional/quadratic green transshipment problem by implementing AI traffic control system with multi choice under fermatean fuzzy environment is expressed as follows

$$\min_{D_1} Z_{11}(D_1, D_2) = \frac{(\tilde{h}_1 D_1 + \tilde{h}_2 D_2) + (\tilde{\delta}_1 D_1 + \tilde{\delta}_2 D_2)(1-\omega)}{(\tilde{v}_1 D_1 + \tilde{v}_2 D_2)}$$

Where D_2 solves

$$\min_{D_2} Z_{12}(D_1, D_2) = (1-\omega)((\tilde{k}_1 D_1 + \tilde{k}_2 D_2)) + (\tilde{\gamma}_1 D_1 + \tilde{\gamma}_2 D_2))(\tilde{u}_1 D_1 + \tilde{u}_2 D_2)$$

(for a given D_1)

Subject to the constraints

$$\sum_{n=1}^{t_1} d_{mn} \le (\tilde{o}'^{(1)}_m, \tilde{o}'^{(2)}_m, ..., \tilde{o}'^{(g_m)}_m), m = 1,2,3,4,\ldots,s_1$$

$$\sum_{m=1}^{s_1} d_{mn} \ge (\tilde{o}'^{(1)}_n, \tilde{o}'^{(2)}_n, ..., \tilde{o}'^{(g_n)}_n), n = 1,2,\ldots,t_1$$

$$\sum_{j=1}^{t_2} d_{ij} \le (\tilde{o}'^{(1)}_i, \tilde{o}'^{(2)}_i, ..., \tilde{o}'^{(x_i)}_i), i = 1,2,3,4,\ldots,s_2$$

$$\sum_{i=1}^{s_2} d_{ij} \ge (\tilde{o}'^{(1)}_j, \tilde{o}'^{(2)}_j, ..., \tilde{o}'^{(x_j)}_j), j = 1,2,\ldots,t_2$$

$$d_{mn} \ge 0 \quad \forall m = 1,2,3,\ldots,s_1; n = 1,2,3,\ldots,t_1$$

$$d_{ij} \ge 0 \quad \forall i = 1,2,3,\ldots,s_2; j = 1,2,3,\ldots,t_2$$

Identical Deterministic Model

In this bilevel fractional/quadratic transportation problem with multi choice under fermatean fuzzy environment model, the parameters such that cost of delivery commodities, cost of carbon emission cost, and deterioration cost are considered as triangular Fermatean fuzzy numbers while supply and demand are considered as multi choice triangular fermatean fuzzy numbers to resolve the prevailing uncertainty in the real-life situations. So, this model becomes complex to solve directly. Therefore, this model is transformed into deterministic model by utilizing the ranking function and Lagrange interpolating polynomial as follows:

Deterministic Model for Bilevel Fractional/Quadratic Green Transportation Problem by Implementing AI traffic control system with Multi Choice Parameters Under Fuzzy Environment

$$\min_{D_1} Z_{11}(D_1, D_2) = \frac{(\Re(\tilde{h}_1) D_1 + \Re(\tilde{h}_2) D_2) + (\Re(\tilde{\delta}_1) D_1 + \Re(\tilde{\delta}_2) D_2)}{(\Re(\tilde{v}_1) D_1 + \Re(\tilde{v}_2) D_2)}$$

Where D_2 solves

$$\min_{D_2} Z_{12}(D_1, D_2) = ((\Re(\tilde{k}_1) D_1 + \Re(\tilde{k}_2) D_2)) + (\Re(\tilde{\gamma}_1) D_1 + \Re(\tilde{\gamma}_2) D_2))(\Re(\tilde{u}_1) D_1 + \Re(\tilde{u}_2) D_2)$$

(for a given D_1)

Subject to the constraints

$$\sum_{n=1}^{t_1} d_{mn} \le Q^1_{\Re(o'_m)}(\phi_m), m = 1,2,3,4,\ldots,s_1$$

$$\sum_{m=1}^{s_1} d_{mn} \ge Q^1_{\Re(o'_n)}(\theta_n), n = 1,2,\ldots,t_1$$

$$\sum_{j=1}^{t_2} d_{ij} \le Q^1_{\Re(o'_i)}(\phi_i), i = 1,2,3,4,\ldots,s_2$$

$$\sum_{i=1}^{s_2} d_{ij} \ge Q^1_{\Re(o'_j)}(\phi_j), j = 1,2,\ldots,t_2$$

$$d_{mn} \ge 0 \quad \forall m = 1,2,3,\ldots,s_1; n = 1,2,3,\ldots,t_1$$

$$d_{ij} \ge 0 \quad \forall i = 1,2,3,\ldots,s_2; j = 1,2,3,\ldots,t_2$$

The deterministic model for bilevel fractional/quadratic green transportation problem by implementing AI traffic control system with multi choice under fermatean fuzzy environment is expressed as follows

$$\min_{D_1} Z_{11}(D_1, D_2) = \frac{(\Re(\tilde{h}_1) D_1 + \Re(\tilde{h}_2) D_2) + (\Re(\tilde{\delta}_1) D_1 + \Re(\tilde{\delta}_2) D_2)(1-\omega)}{(\Re(\tilde{v}_1) D_1 + \Re(\tilde{v}_2) D_2)}$$

Where D_2 solves

$$\min_{D_2} Z_{12}(D_1, D_2) = (1-\omega)((\Re(\tilde{k}_1) D_1 + \Re(\tilde{k}_2) D_2)) + (\Re(\tilde{\gamma}_1) D_1 + \Re(\tilde{\gamma}_2) D_2))(\Re(\tilde{u}_1) D_1 + \Re(\tilde{u}_2) D_2)$$

(for a given D_1)

Subject to the constraints

$$\sum_{n=1}^{t_1} d_{mn} \le Q^1_{\Re(o'_m)}(\phi_m), m = 1,2,3,4,\ldots,s_1$$

$$\sum_{m=1}^{s_1} d_{mn} \geq Q^1_{\Re(o'_n)}(\theta_n), n = 1, 2, \ldots, t_1$$

$$\sum_{j=1}^{t_2} d_{ij} \leq Q^1_{\Re(o'_i)}(\phi_i), i = 1, 2, 3, 4, \ldots, s_2$$

$$\sum_{i=1}^{s_2} d_{ij} \geq Q^1_{\Re(o'_j)}(\phi_j), j = 1, 2, \ldots, t_2$$

$$d_{mn} \geq 0 \quad \forall m = 1, 2, 3, \ldots, s_1; n = 1, 2, 3, \ldots, t_1$$

$$d_{ij} \geq 0 \quad \forall i = 1, 2, 3, \ldots, s_2; j = 1, 2, 3, \ldots, t_2$$

Solution Procedure

To obtain the optimum solution for this above deterministic model, fermatean fuzzy programming is applied in this paper as follows:

Fermatean Fuzzy Programming

To handle uncertain situations, Zadeh introduced the concept of the fuzzy set. This was followed by the development of the intuitionistic and Pythagorean fuzzy sets. The Fermatean fuzzy set is considered more realistic and can handle greater uncertainty than both the intuitionistic and Pythagorean fuzzy sets. In a Fermatean fuzzy set, the sum of the cubes of the truth and false membership grades can exceed 1. Senapati and Yager was first introduced the concept of fermatean fuzzy set. In this environment, Zimmermann [6] proposed a fuzzy programming approach for multi-objective decision-making problems, utilizing a min-max operator. This approach incorporates linear, exponential, or hyperbolic truth functions to find compromised optimal solutions. Intuitionistic fuzzy programming and Pythagorean fuzzy programming are further developed for multi-objective problems in an intuitionistic fuzzy environment, where the truth and false grades are represented by linear, exponential, or hyperbolic functions. In this paper, a non-linear programming called as fermatean fuzzy programming has been applied to obtain optimum solutions for bilevel fractional/quadratic transportation problem.

Consider U_k and L_k are upper and lower bound of objective functions U_{21} and U_{22}. Then the membership and non-membership for the objective functions are represented as $\mu(Z_k)$ and $\nu(Z_k)$ respectively. Then the proposed model for fermatean fuzzy programming is described as follows

$$\max \xi^3 - \psi^3$$

Where $\mu(Z_k)^3 \geq \xi^3, \nu(Z_k)^3 \geq \psi^3 \forall k$

$$\mu(U_{kl}) = \begin{cases} 1 & \text{if} Z_k \leq L_k \\ \dfrac{U_k - Z_k}{U_k - L_k} & \text{if} L \leq U_{kl} \leq U_k \\ 0 & \text{if} Z_k \geq U_k \end{cases}$$

$$\nu(Z_k) = \begin{cases} 0 & \text{if} Z_k \leq L_k \\ \dfrac{Z_k - L_k}{U_k - L_k} & \text{if} L \leq U_{kl} \leq U_k \\ 1 & \text{if} Z_k \geq U_k \end{cases}$$

i.e., $(U_k - Z_k)^3 \geq b_k^3 \xi^3, \forall k$

$(Z_k - L_k)^3 \geq b_k^3 \psi^3 \forall k$ where $b_k = U_k - L_k$

Subject to the constraints,

$$\sum_{n=1}^{t_1} d_{mn} - \sum_{n=1}^{t_1} d_{nm} \leq Q^1_{\Re(o'_m)}(\phi_m), m = 1, 2, 3, 4, \ldots, s_1$$

$$\sum_{m=1}^{s_1} d_{mn} - \sum_{m=1}^{s_1} d_{nm} \geq Q^1_{\Re(o'_n)}(\theta_n), n = 1, 2, \ldots, t_1$$

$$\sum_{j=1}^{t_2} d_{ij} - \sum_{j=1}^{t_2} d_{ji} \leq Q^1_{\Re(o'_i)}(\phi_i), i = 1, 2, 3, 4, \ldots, s_2$$

$$\sum_{i=1}^{s_2} d_{ij} - \sum_{i=1}^{s_2} d_{ji} \geq Q^1_{\Re(o'_j)}(\phi_j), j = 1, 2, \ldots, t_2$$

$$d_{mn} \geq 0 \quad \forall m = 1, 2, 3, \ldots, s_1; n = 1, 2, 3, \ldots, t_1$$

$$d_{ij} \geq 0 \quad \forall i = 1, 2, 3, \ldots, s_2; j = 1, 2, 3, \ldots, t_2$$

The prescribed method has been solved with the help of Lingo Global Solver (20.0) to find the optimum solutions for proposed model.

Result

In this section, the Bilevel Fractional/Quadratic Green Transportation Model integrated with an AI-based traffic control system under a fuzzy multi-choice environment is applied to a real-world scenario. The model focuses on optimizing the transportation of perishable goods while minimizing cost, time, and carbon emissions. Unlike existing studies that mainly address preservation technologies in transportation, this work considers the dynamic role of transshipment during uncertain traffic conditions. By incorporating artificial intelligence, the system efficiently controls traffic flow to reduce delays and spoilage. The fuzzy multi-choice parameters further help to manage uncertainty, making the model more realistic and effective for sustainable logistics operations.

Numerical Example

Consider Gujarati company called MLN that produces unique products for resorts in Agra, Jaipur, Mussoorie, and Shimla [14]. Depending on the type of goods, the company has three storage inventories. Non-perishable goods like tea leaves, sugar, salt, and so forth are kept in Inventory I, Perishable goods including meat, fruits, and other items are kept in Inventory II, Products for cutlery are in Inventory III. This ABC Company faces numerous challenges related to economic efficiency and environmental sustainability during product transportation. To mitigate these issues, the company has implemented an AI-driven traffic control system. This system utilizes real-time data such as traffic patterns, weather conditions, and road disruptions. The company aims to maximize profits by minimizing AI-driven economic and environmental costs. However, variables such as transportation costs, product deterioration, carbon emissions, and fixed operational costs are inherently uncertain due to factors like social activities, weather conditions, and multiple choice. Consequently, these parameters are modeled as triangular Fermatean fuzzy numbers, while supply and demand are represented as triangular Fermatean fuzzy multi-choice values. To handle these uncertainties, a ranking function and interpolating polynomial are applied to convert the fuzzy parameters into crisp values. The cost parameters for both upper and lower-level objective functions are presented in Tables 1 and 2, while the supply and demand data are detailed in Table 3. The upper-level objective focuses on minimizing transportation costs and carbon emissions from inventories to resorts, while simultaneously maximizing the quantity of goods delivered. The lower-level objective aims to minimize deterioration-related costs and associated emissions during transit.

Table 1: The upper cost matrix

	Y_1	Y_2	Y_3	Y_4
X_1	$\{(8,9,10),(7,9,12)\}$ $\{(0.6,0.9,1.2),(0.4,0.9,1.6)\}$	$\{(5,8,8),(4,8,9)\}$ $\{(2,3,4),(1,3,4)\}$	$\{(6,7,8),(5,7,10)\}$ $\{(1,3,3),(0,3,4)\}$	$\{(4,7,7),(3,7,8)\}$ $\{(1,2,3),(1,2,5)\}$
X_2	$\{(5,6,7),(4,6,9)\}$ $\{(2,4,4),(1,4,5)\}$	$\{(3,6,7),(2,6,8)\}$ $\{(2,5,5),(1,5,6)\}$	$\{(4,5,6),(3,5,9)\}$ $\{(3,6,8),(2,6,9)\}$	$\{(2,5,6),(3,5,9)\}$ $\{(4,7,7),(3,7,8)\}$
X_3	$\{(3,4,5),(2,4,7)\}$ $\{(4,7,9),(3,7,10)\}$	$\{(2,3,4),(1,3,6)\}$ $\{(6,7,9),(6,7,11)\}$	$\{(1,2,2),(1,2,3)\}$ $\{(6,9,9),(5,9,12)\}$	$\{(0,2,2),(0,2,2)\}$ $\{(7,10,10),(6,10,11)\}$

Table 2: The lower cost matrix

	Y_1	Y_2	Y_3	Y_4
X_1	$\{(7,10,10),(6,10,11)\}$ $\{(9,10,11),(8,10,13)\}$	$\{(6,7,9),(6,7,11)\}$ $\{(6,9,10),(5,9,13)\}$	$\{(6,7,9),(6,7,11)\}$ $\{(7,8,9),(6,8,12)\}$	$\{(4,7,9),(3,7,10)\}$ $\{(6,7,8),(5,7,8)\}$
X_2	$\{(4,7,7),(3,7,8)\}$ $\{(4,5,6),(3,5,6)\}$	$\{(3,6,8),(2,6,9)\}$ $\{(4,5,6),(3,5,6)\}$	$\{(2,5,5),(1,5,6)\}$ $\{(3,4,5),(1,4,5)\}$	$\{(2,4,4),(1,4,5)\}$ $\{(1,2,4),(1,2,5)\}$
X_3	$\{(2,3,4),(1,3,4)\}$ $\{(3,4,5),(1,4,5)\}$	$\{(1,3,3),(0,3,4)\}$ $\{(1,1.3,2),(0,1.3,2.5)\}$	$\{(1,2,3),(1,2,5)\}$ $\{(1,1.1,2),(0,1.1,2.2)\}$	$\{(0.6,0.9,1.2),(0.4,0.9,1.6)\}$ $\{(0.3,0.5,0.7),(0.2,0.5,0.9)\}$

Optimal solution and comparative study

By using method prescribed in 2.3.1, the obtained optimum solutions of deterministic bilevel fractional/quadratic green transportation problem by implementing AI traffic control system with multi choice using Lingo Global Solver (20.0) are presented in [Table 4.]. Consequently, the comparison between deterministic bilevel fractional/quadratic green transportation problem with and without implementing traffic control system is detailed in [Table 4.].

Discussion

Jaggi et al. (2024) proposed a Bilevel Fractional/Quadratic Green Transportation Problem (BFQGMCTP) incorporating multi-choice supply and demand parameters under uncertainty. The model addresses real-world challenges such as rising fuel costs, carbon emissions, and product deterioration during transit. The main objective is to maximize the quantity of goods delivered while minimizing

Table 3: Supply and demand

$\tilde{O}_1' = (40,42,46,48)$	$\tilde{O}_1'' = (9,14,10,12)$
$\tilde{O}_2' = (35,33,40)$	$\tilde{O}_2'' = (24,23,28)$
$\tilde{O}_1' = (48,50,52,53)$	$\tilde{O}_3'' = (13,11,14)$
	$\tilde{O}_4'' = (8,7,6,9)$

Table 4: comparison for transshipment problem with and without AI implementation

Model	*Fermatean fuzzy programming*
Bilevel fractional/quadratic transshipment problem without implementing AI traffic control system	$d_{36} = 10.16$, $d_{37} = 41.17$, $d_{74} = 9$, $d_{75} = 23$, $Z_1 = 0.28665$, $Z_2 = 5888.58$
Bilevel fractional/quadratic transshipment problem with implementing AI traffic control system	$d_{36} = 10.17$, $d_{37} = 41.22$, $d_{74} = 9$, $d_{75} = 23$, $Z_1 = 0.2839$, $Z_2 = 2654.2$

transportation cost, deterioration cost, and carbon emission cost simultaneously. The authors introduced two distinct solution techniques and conducted a comparative analysis to evaluate their performance.

Conclusion

Traffic management is a significant challenge in the transportation problem, as it greatly affects environmental outcomes during the distribution process. Implementing an AI-based traffic control system within the transportation model can effectively reduce these environmental impacts. Given the uncertainty inherent in real-world scenarios, the proposed model has been developed within a Fermatean fuzzy environment to better handle these complexities. In this model, supply and demand are treated as triangular Fermatean fuzzy multi-choice parameters, while cost coefficients are represented as triangular Fermatean fuzzy numbers. The multi-choice Fermatean fuzzy model is then transformed into a deterministic model using a ranking function and Lagrange interpolating polynomial. Pareto-optimal solutions for the resulting deterministic model are obtained through Fermatean fuzzy programming, implemented using the LINGO software package. The numerical example has been discussed for transportation problem with and without implementing AI traffic control system. Obtained solutions for bilevel fractional/quadratic green transportation problem with AI traffic control system shows a reduction of 7.8% in cost and 11.19% than without AI traffic control system. The proposed model could be extended by incorporating key social factors to further enhance sustainability in future developments.

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