Doi: 10.58414/SCIENTIFICTEMPER.2025.16.11.10

E-ISSN: 2231-6396, ISSN: 0976-8653

https://scientifictemper.com/

RESEARCH ARTICLE

Bilevel Fractional/Quadratic Green Transshipment Problem by Implementing AI traffic control system with Multi Choice Parameters Under Fuzzy Environment

U. Johns Praveena, J. Merline Vinotha*

Abstract

Objectives: To investigate the effectiveness of AI based traffic control system on reducing the economic and environmental cost in the context of the transshipment problem.

Methods: The mathematical model of bilevel fractional/quadratic green transshipment problem by implementing AI traffic control system is formulated and numerical example is provided to emphasize the nature of this model. Due to inherent uncertainty, fermatean fuzzy parameters are incorporated in this model. Also, Supply and demand are considered as fermatean fuzzy multi choice. Existing fermatean fuzzy programming is used to find the solutions for proposed transshipment model.

Findings: Comparative study has been made for bilevel fractional/quadratic transshipment problem with and without implementation of AI traffic control system. Optimum Solutions obtained for the proposed model by using prescribed method reveals that the bilevel fractional/quadratic green transshipment problem gives the minimum transportation cost, deterioration cost, carbon emission cost than the transshipment problem with traditional traffic control system. Obtained solutions for bilevel fractional/quadratic green transshipment problem with implementation of AI traffic control system shows a reduction of 7.8% in transportation cost, 4% in cost of carbon emission than the traditional transshipment problem. Meanwhile, obtained solutions for bilevel fractional/quadratic green transshipment problem shows a reduction of 14% in cost and 14.4% in time than bilevel fractional/quadratic green transportation problem.

Novelty: The efficiency of bilevel fractional/quadratic green transshipment problem by implementing AI traffic control system with multi choice parameters under Fermatean fuzzy environment is not yet investigated in literature.

Keywords: Transshipment problem, fractional transshipment problem, quadratic transshipment problem, bilevel programming, triangular fermatean fuzzy number, fermatean fuzzy programming.

关键词中转问题、分数中转问题、二次中转问题、双层规划、三角费马模糊数、费马模糊规划。

PG & Research Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirappalli-620002, India.

*Corresponding Author: Author, PG & Research Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirappalli-620002, India, E-Mail: merlinevinotha@gmail.com

How to cite this article: Praveena, U.J., Vinotha, J.M. (2025). Bilevel Fractional/Quadratic Green Transshipment Problem by Implementing Al traffic control system with Multi Choice Parameters Under Fuzzy Environment. The Scientific Temper, **16**(11):5048-5057.

Doi: 10.58414/SCIENTIFICTEMPER.2025.16.11.10

Source of support: Nil **Conflict of interest:** None.

Introduction

Transportation problem was first introduced by Hitchcock (1941) involves transporting goods from source to destination in such a way that a specific objective is optimized. To improve the overall efficiency, flexibility, and cost-effectiveness, transshipment problem can be used instead of transportation when goods are need to pass through intermediate point before reaching their destination. Various authors have explored discussed different approaches to solve the transshipment problem. Kumar, A et.al (2020) studied new efficient algorithm to solve the transshipment problem. Additionally, Fractional transshipment problem become pertinent as these models are aim to minimize costs while maximizing the efficiency. Garg, H. et. al (2021) discussed a fractional two stage

Received: 25/10/2025 **Accepted:** 12/11/2025 **Published:** 22/11/2025

transshipment problem focused on maximize the profit. In many real-world transshipment scenarios, logistic costs can be non-linear, and in some cases, they follow a quadratic nature. This type of transshipment problem is referred to as the quadratic transshipment problem. As compared with the linear fractional transshipment problem, the quadratic fractional transshipment problem gives a superior representation of real-life distribution problem. Arya, N. V. et.al (2019) proposed an algorithm for solving fractional quadratic transportation problem.

Bilevel Linear Programming (BLP) is an important method for solving two-level managerial problems, where decision-makers at each level aim to maximize their own objectives, but their decisions are also influenced by the decisions made at the other level. Qiu et al. (2020) introduced the BLP approach in the aviation industry to reduce carbon emissions and fossil fuel consumption. Similarly, Zhu et al. (2022) applied the BLP methodology to minimize both carbon emissions and transportation time for goods. Kaushal et al. developed a bilevel fixed-charge fractional transportation problem. In their model, they divided the multiple objectives into two levels: the upper level, referred to as the leader, and the lower level, referred to as the follower. The upper level involves a fractional transportation problem, while the lower level deals with a fixed-charge transportation problem. Arora et al. (2021) introduced a bilevel indefinite quadratic transportation problem, focusing on minimizing both transportation and deterioration costs. Jaggi et al. discussed a bilevel fractional/quadratic green transportation problem. They have constructed two levels where first level typically minimizes the transportation cost with the carbon emission cost for non-perishable items and lower level minimizes the cost of damage along with cost of carbon emission for perishable items.

In the fast-paced world, environmental safety has become more crucial than ever, as increasing traffic congestion and pollution are major contributors to climate change and deteriorating air quality. Al traffic control systems play a vital role in addressing these issues by optimizing traffic flow, reducing idle times, and minimizing fuel consumption, all of which helps decrease carbon emissions. By adapting to real-time traffic conditions and offering smarter route management, Al can not only improve traffic efficiency but also contribute to sustainable urban mobility, ensuring a cleaner and healthier environment for future generations. Dikshit, S. et al. (2023) explored the use of artificial intelligence to optimize vehicle routing and alleviate traffic congestion in urban areas. Pillai, A. S. et al. (2024) examined how Al-driven traffic management can significantly enhance traffic flow, reduce carbon emissions, and provide a scalable solution for modern urban planning.

To the best of our knowledge, most of the existing literature has focused on the transshipment problem in the context of traditional traffic control systems. During the transshipment process, goods may be damaged due

to handling or delays, and increased traffic congestion can lead to higher carbon emissions from vehicles, contributing to environmental pollution. Therefore, research on the transshipment problem is essential to achieve sustainability by optimizing processes and reducing environmental impacts. Most studies have examined the transshipment problem with traditional traffic management. But Al traffic control system has also significantly reduced the carbon emissions, transportation cost, and damage cost. Consequently, this leads to the existence of a research gap that does not investigates transshipment problem by implementing AI traffic control system. So, in this study, mathematical model for bilevel fractional/quadratic transshipment problem has been investigated. Al traffic control system is incorporated simultaneously to reduce the economic and environment cost. Due to market becomes more competitive with the emergence of new industries, cost, supply, and demand parameters are not fixed. To address this fluctuating market behaviour, the concept of multi-choice programming with uncertainty was introduced. In this paper supply and demand are treated as multi choice triangular fermatean fuzzy parameters while economic and environmental costs are represented as triangular fermatean fuzzy number. It is first introduced by T Senapati which is the extension of Pythagorean fuzzy number. In fermatean fuzzy set, sum of cube of membership and non-membership lies between 0 and 1. The proposed model is solved using fermatean fuzzy programming to obtain the pareto optimal solution after converting deterministic model with the ranking function and interpolating polynomial. The remainder of this research is classified below. In Section 2, basis definition of fermatean and multi choice programming has been defined. Section 3 represented Notations and Assumptions. The suggested model is formulated and methodology has been described in Section 3. In Section 4, Numerical illustrations are performed. Finally, conclusion has been stated in Section

Methodology

Preliminaries

Definition

Consider S be a non-empty set. Let \tilde{A}^F of S is a fermatean fuzzy set and is defined as $\tilde{A}^F = \{\langle s, \mu_{\chi^c}(s), \nu_{\chi^c}(s) \rangle : s \in S \}$ where $\mu_{\tilde{A}^F}(s), \nu_{\tilde{A}^C}(s) : S \to [0,1]$ are membership and non-membership functions respectively. Furthermore, fermatean fuzzy number satisfies the condition $0 \le (\mu_{\tilde{A}^C}(s))^3 + (\nu_{\tilde{A}^C}(s))^3 \le 1$ for all $s \in S$. Additionally, degree of hesitation is defined as $\pi_{\tilde{A}^C}(s) = \sqrt{1 - (\mu_{\tilde{A}^C}(s))^3 - (\nu_{\tilde{A}^C}(s))^3}$ for all $s \in S$. (Akram, M. et.al 2023)

Definition

Let \tilde{A}^F be a triangular fermatean fuzzy number (TFFN) and it is defined as $\tilde{A}^F = \langle (a^l, a^m, a^n), \alpha, \beta \rangle$ whose membership and non-membership function can be described as

$$\mu_{\tilde{A}^{F}}(s) = \begin{cases} \frac{(s-a^{l})}{a^{m}-a^{l}} & \text{if } a^{l} \leq s \leq a^{m} \\ \alpha & \text{if } s = a^{m} \end{cases}$$

$$\frac{(a^{r}-s)\beta}{a^{r}-a^{m}} & \text{if } a^{m} \leq s < a^{r} \\ 0 & \text{if } s < a^{l} \text{ or } s > a^{r} \end{cases}$$

$$v_{\tilde{A}^{F}}(s) = \begin{cases} \frac{[s-a^{m}+\alpha(s-a^{l})]}{a^{m}-a^{l}} & \text{if } a^{l} \leq s \leq a^{m} \\ \beta & \text{if } s = a^{m} \end{cases}$$

$$\frac{[s-a^{m}+\beta(a^{r}-s)\beta}{a^{r}-a^{m}} & \text{if } a^{m} \leq s < a^{r} \\ 1 & \text{if } s < a^{l} \text{ or } s > a^{r} \end{cases}$$

Here α represents the maximum value of $MF(\mu_{\tilde{A}'})$ and β represents the minimum value of $MF(\nu_{\tilde{A}'})$ respectively, such that $\alpha \in [0,1], \beta \in [0,1]$ and $0 \le \alpha^3 + \beta^3 \le 1$. Assume $\alpha = 1$ and $\beta = 0$. Then TFFNbecomes $\tilde{A}^F = \langle (a', a^*, a^*), (a', a^*, a') \rangle$ whose $MF(\mu_{\tilde{A}^F})$ and $MF(\nu_{\tilde{A}^F})$ can be defined as follows

$$\mu_{\tilde{A}^{F}}(s) = \begin{cases} \frac{(s-a^{l})}{a^{m}-a^{l}} & \text{if } a^{l} \leq s \leq a^{m} \\ 1 & \text{if } s = a^{m} \\ \frac{(a^{r}-s)}{a^{r}-a^{m}} & \text{if } a^{m} \leq s < a^{r} \\ 0 & \text{if } s < a^{l} \text{ or } s > a^{r} \end{cases}$$

$$\nu_{\tilde{A}^{F}}(s) = \begin{cases} \frac{a^{m}-s}{a^{m}-a^{l}} & \text{if } a^{l'} \leq s \leq a^{m} \\ 0 & \text{if } a = e^{m} \\ \frac{s-a^{m}}{a^{r'}-a^{m}} & \text{if } a^{m} \leq s < a^{r'} \\ 1 & \text{if } s < a^{l'} \text{ or } s > a^{r'} \end{cases}$$

Where $a^{l'} \le a^{l} \le a^{m} \le a^{r} \le a^{r'}$. (Akram, M. *et.al* 2023)

Definition

Let $\tilde{A}^F = \langle (a^i, a^m, a^n), (a^r, a^{m'}, a^{n'}) \rangle$ be a triangular fermatean fuzzy number. Then the ranking function for triangular fermatean fuzzy number is defined as $\Re(\tilde{A}^F) = \frac{(a^l + 4a^m + a^n) + (a^r + 4a^m + a^{n'})}{12}$ (Akram, M. *et.al* 2023)

Definition

The Lagrange's interpolation is one of the numerical approximation methods which is used to convert the muti

choice parameter into the optimal choice. An integer variable is introduced for each multi-choice parameter to formulate the interpolating polynomial. Since above proposed model has w_{k_i} number of choices for both the supply and demand parameters, integer variables are introduced accordingly

The multi choice supply and demand parameters $s_{k_i}^{w_{k_i}}, d_{k_i}^{w_{k_i}}(k_i \in K_1)$ are replaced by assigning the integer variables α_{k_i} . it determines w_{k_i} values; $\alpha_{k_i} = 0, 1, ..., w_{k_i} - 1$. The corresponding polynomial is determined as follows:

$$\begin{split} z_{s_{k_{1}}}^{l}(\alpha_{k_{1}}) &= \frac{(\alpha_{k_{1}}-1)(\alpha_{k_{1}}-2)...(\alpha_{k_{1}}-w_{k_{1}}+1)}{(-1)^{w_{k_{1}}-1}(w_{k_{1}}-1)!} s_{k1}^{(1)} + \\ &\frac{(\alpha_{k_{1}})(\alpha_{k_{1}}-1)...(\alpha_{k_{1}}-w_{k_{1}}+1)}{(-1)^{w_{k_{1}}-2}(w_{k_{1}}-2)!} s_{k1}^{(2)} \\ &+ \frac{(\alpha_{k_{1}})(\alpha_{k_{1}}-1)...(\alpha_{k_{1}}-w_{k_{1}}+1)}{(-1)^{w_{k_{1}}-3}(w_{k_{1}}-3)!2!} s_{k1}^{(3)} + ... \\ &+ \frac{(\alpha_{k_{1}})(\alpha_{k_{1}}-1)...(\alpha_{k_{1}}-w_{k_{1}}+2)}{(w_{k_{1}}-1)!} s_{k1}^{(k)} \\ &z_{d_{k_{1}}}^{l}(\alpha_{k_{1}}) &= \frac{(\alpha_{k_{1}}-1)(\alpha_{k_{1}}-2)...(\alpha_{k_{1}}-w_{k_{1}}+1)}{(-1)^{w_{k_{1}}-1}(w_{k_{1}}-1)!} d_{k1}^{(1)} + \\ &\frac{(\alpha_{k_{1}})(\alpha_{k_{1}}-1)...(\alpha_{k_{1}}-w_{k_{1}}+1)}{(-1)^{w_{k_{1}}-2}(w_{k_{1}}-2)!} d_{k1}^{(2)} \\ &+ \frac{(\alpha_{k_{1}})(\alpha_{k_{1}}-1)...(\alpha_{k_{1}}-w_{k_{1}}+1)}{(-1)^{w_{k_{1}}-3}(w_{k_{1}}-3)!2!} d_{k1}^{(k)} + ... \\ &+ \frac{(\alpha_{k_{1}})(\alpha_{k_{1}}-1)...(\alpha_{k_{1}}-w_{k_{1}}+2)}{(w_{k_{1}}-1)!} d_{k1}^{(k)} \end{split}$$
 (EI Sayed et.al 2023)

Mathematical Model for Bilevel Fractional/Quadratic Green Transshipment Problem by Implementing Al traffic control system with Multi Choice Parameters Under Fuzzy Environment

This section provides list of notations along with their intended meaning and assumptions are made in this proposed model. Subsequently, a mathematical formulation is constructed for bilevel fractional/quadratic transshipment problem by implementing AI traffic signal control system with multi choice under fermatean fuzzy environment

Notations and Assumptions

$$D_1^{'}$$
 = sources at upper level $(m = 1, 2, 3, ...s_1)$

$$S_{1}^{'}=$$
 destinations at upper level $(n=1,2,3,...t_{1})$

$$D_2^{'}$$
 = sources at lower level $(i = 1, 2, 3, ...s_2)$

$$S_2'$$
 = destinations at lower level $(j = 1, 2, 3, ..., t_2)$

 ρ -Al reduction parameter in percentage

$$\begin{split} \tilde{h}_{1} &= \left[\tilde{h}'_{mn} \right], \quad \tilde{\mathcal{S}}_{1} &= \left[\tilde{\mathcal{S}}'_{mn} \right], \quad \tilde{\mathcal{V}}_{1} &= \left[\tilde{\mathcal{V}}'_{mn} \right], \quad \tilde{k}_{1} &= \left[\tilde{k}'_{mn} \right], \\ \tilde{\gamma}_{1} &= \left[\tilde{\gamma}'_{mn} \right], \quad \tilde{u}_{1} &= \left[\tilde{u}'_{mn} \right], \quad \tilde{m}, \tilde{n} \in D_{1} \cup S_{1} \end{split}$$

$$\begin{split} \tilde{h}_2 = & \left[\tilde{h}'_{ij} \right] \quad \tilde{\delta}_2 = \left[\tilde{\delta}'_{ij} \right] \quad \tilde{v}_2 = \left[\tilde{v}'_{ij} \right] \quad \tilde{k}_2 = \left[\tilde{k}'_{ij} \right] \quad \\ \tilde{\gamma}_2 = & \left[\tilde{\gamma}'_{ij} \right] \quad \tilde{u}_1 = \left[\tilde{u}'_{ij} \right] \quad i, \, j \in M'' \cup N'' \end{split}$$

 $\tilde{h}'_{\it mn} > 0$, $\tilde{\delta}'_{\it mn} > 0$, $\tilde{v}'_{\it mn} > 0$, $\tilde{k}'_{\it mn} > 0$, $\tilde{\gamma}'_{\it mn} > 0$, $\tilde{u}'_{\it mn} > 0$ are the fermatean cost parameters at upper levels respectively.

 $\tilde{h}'_{ij} > 0$, $\tilde{\delta}'_{ij} > 0$, $\tilde{v}'_{ij} > 0$, $\tilde{k}'_{ij} > 0$, $\tilde{\gamma}'_{ij} > 0$, $\tilde{u}'_{ij} > 0$ are the cost parameters at lower level respectively

 $D_1' = [d'_{mn}], D_2' = [d'_{ij}]$ where $d'_{mn}, d'_{ij} \ge 0$ are decision variables denoting the amount of goods are transported at lower and upper level.

 $(\tilde{o}_m'^{(1)}, \tilde{o}_m'^{(2)}, ..., \tilde{o}_m'^{(g_m)}), (\tilde{o}_n'^{(1)}, \tilde{o}_n'^{(2)}, ..., \tilde{o}_n'^{(g_n)})$ are multi choice parameter for supply and demand in the upper-level problem. $o_m^{(g_n)}, o_n'^{(g_n)} \ge 0, m \in D_1, n \in D_2$

 $(\tilde{o}_i'^{(l)}, \tilde{o}_i'^{(2)}, ..., \tilde{o}_i'^{(x_i)}), (\tilde{o}_j'^{(l)}, \tilde{o}_j'^{(2)}, ..., \tilde{o}_j'^{(y_j)})$ are multi choice parameter for supply and demand in the lower-level problem. $o_i'^{(x_i)}, o_i'^{(y_j)} \ge 0, i \in D_j', j \in S_2'$

Feasibility condition for BTrPMCP: $\sum_{m \in D_i} o'_m \ge \sum_{n \in S_i} o'_n; \sum_{i \in D_i} o'_i \ge \sum_{j \in D_i} o'_j$

Mathematical Formulation

Bilevel Fractional/Quadratic Green Transshipment Problem by Implementing AI traffic control system with Multi Choice Parameters Under Fuzzy Environment

The proposed BFQGTrP with multi choice under fermatean fuzzy environment model consists of two levels: an upper level and a lower level. In this defined model, the demand and supply parameters are multi-choice. The cost coefficients of the objective functions at both levels are represented as fermatean triangular fuzzy numbers. The main goal of the upper level is to minimize transportation costs and cost of carbon emissions while transferring the maximum number of units from origins to destinations through transshipment points. To achieve this, fractional green transshipment problem is defined at the upper level. During transshipment process, some units of goods may be damaged, or perishable goods may spoil due to unfavourable climatic conditions or poor road conditions. Additionally, carbon emissions will occur during the distribution process. To minimize the costs associated with deterioration and carbon emissions, the Quadratic green Transshipment Problem is discussed at the lower level.

The mathematic model for bilevel fractional/quadratic green transshipment problem with multi choice under fermatean fuzzy environment is expressed as follows

$$\min_{D_1} Z_{11}(D_1, D_2) = \frac{(\tilde{h}_1 D_1 + \tilde{h}_2 D_2) + (\tilde{\delta}_1 D_1 + \tilde{\delta}_2 D_2)}{(\tilde{v}_1 D_1 + \tilde{v}_2 D_2)}$$

$$\min_{D} Z_{12}(D_1, D_2) == ((\tilde{k_1}D_1 + \tilde{k_2}D_2)) + (\tilde{\gamma}_1D_1 + \tilde{\gamma}_2D_2))(\tilde{u}_1D_1 + \tilde{u}_2D_2)$$

Subject to the constraints

$$\sum_{m=1}^{t_1} d_{mn} - \sum_{m=1}^{t_1} d_{nm} \le (\tilde{o}_m^{\prime(1)}, \tilde{o}_m^{\prime(2)}, ..., \tilde{o}_m^{\prime(g_m)}), m = 1, 2, 3, 4, ..., s_1$$

$$\sum_{m=1}^{s_1} d_{mn} - \sum_{m=1}^{s_1} d_{nm} \ge (\tilde{o}_n'^{(1)}, \tilde{o}_n'^{(2)}, ..., \tilde{o}_n'^{(g_n)}), n = 1, 2, ..., t_1$$

$$\sum_{j=1}^{t_2} d_{ij} - \sum_{j=1}^{t_2} d_{ji} \le (\tilde{o}_i^{\prime(1)}, \tilde{o}_i^{\prime(2)}, ..., \tilde{o}_i^{\prime(x_i)}), i = 1, 2, 3, 4, ..., s_2$$

$$\sum_{i=1}^{s_2} d_{ij} - \sum_{i=1}^{s_2} d_{ji} \ge (\tilde{o}_j^{\prime(1)}, \tilde{o}_j^{\prime(2)}, ..., \tilde{o}_j^{\prime(x_j)}), j = 1, 2, ..., t_2$$

$$d_{mn} \ge 0 \quad \forall m = 1, 2, 3, ..., s_1; n = 1, 2, 3, ..., t_1$$

$$d_{ii} \ge 0 \quad \forall i = 1, 2, 3, ..., s_2; j = 1, 2, 3, ..., t_2$$

At upper level, $(\tilde{h}_1D_1 + \tilde{h}_2D_2)$, $(\tilde{\delta}_1D_1 + \tilde{\delta}_2D_2)$ and $(\tilde{v}_1D_1 + \tilde{v}_2D_2)$ are represent the delivery cost, carbon emission during the transportation of commodities and the maximum amount of goods transported. At lower level, $(\tilde{k}_1D_1 + \tilde{k}_2D_2)$, $(\tilde{y}_1D_1 + \tilde{y}_2D_2)$ $(\tilde{u}_1D_1 + \tilde{u}_2D_2)$ denotes the transportation cost, carbon emission cost, deterioration cost during the transportation of products.

The mathematic model for bilevel fractional/quadratic green transshipment problem by implementing Al traffic control system with multi choice under fermatean fuzzy environment is expressed as follows

$$\min_{D_1} Z_{11}(D_1, D_2) = \frac{(\tilde{h}_1 D_1 + \tilde{h}_2 D_2) + (\tilde{\delta}_1 D_1 + \tilde{\delta}_2 D_2)(1 - \omega)}{(\tilde{v}_1 D_1 + \tilde{v}_2 D_2)}$$

Where D_{γ} solves

$$\min_{D_2} Z_{12}(D_1, D_2) = (1 - \omega)((\tilde{k_1}D_1 + \tilde{k_2}D_2)) + (\tilde{\gamma_1}D_1 + \tilde{\gamma_2}D_2))(\tilde{u_1}D_1 + \tilde{u_2}D_2)$$
 (for a given D_1)

Subject to the constraints

$$\sum_{m=1}^{t_1} d_{mn} - \sum_{n=1}^{t_1} d_{nm} \le (\tilde{o}'_m^{(1)}, \tilde{o}'_m^{(2)}, ..., \tilde{o}'_m^{(g_m)}), m = 1, 2, 3, 4, ..., s_1$$

$$\sum_{m=1}^{s_1} d_{mn} - \sum_{m=1}^{s_1} d_{nm} \ge (\tilde{o}_n'^{(1)}, \tilde{o}_n'^{(2)}, ..., \tilde{o}_n'^{(g_n)}), n = 1, 2, ..., t_1$$

$$\sum_{j=1}^{t_2} d_{ij} - \sum_{j=1}^{t_2} d_{ji} \le (\tilde{o}_i^{\prime(1)}, \tilde{o}_i^{\prime(2)}, ..., \tilde{o}_i^{\prime(x_i)}), i = 1, 2, 3, 4, ..., s_2$$

$$\sum_{j=1}^{s_2} d_{ij} - \sum_{i=1}^{s_2} d_{ji} \ge (\tilde{o}_j^{\prime(1)}, \tilde{o}_j^{\prime(2)}, ..., \tilde{o}_j^{\prime(x_j)}), j = 1, 2, ..., t_2$$

$$d_{mn} \ge 0 \quad \forall m = 1, 2, 3, ..., s_1; n = 1, 2, 3, ..., t_1$$

$$d_{ii} \ge 0 \quad \forall i = 1, 2, 3, ..., s_2; j = 1, 2, 3, ..., t_2$$

Identical Deterministic Model

In this bilevel fractional/quadratic transshipment problem with multi choice underfermatean fuzzy environment model, the parameters such that cost of delivery commodities, cost of carbon emission cost, and deterioration cost are considered as triangular Fermatean fuzzy numbers while supply and demand are considered as multi choice triangular fermatean fuzzy numbers to resolve the prevailing uncertainty in the real-life situations. So, this model becomes complex to solve directly. Therefore, this model is transformed into deterministic model by utilizing the ranking function and Lagrange interpolating polynomial as follows:

Deterministic Model for Bilevel Fractional/Quadratic Green Transshipment Problem by Implementing AI traffic control system with Multi Choice Parameters Under Fuzzy Environment

$$\min_{D_1} Z_{11}(D_1, D_2) = \frac{(\Re(\tilde{h}_1)D_1 + \Re(\tilde{h}_2)D_2) + (\Re(\tilde{\delta}_1)D_1 + \Re(\tilde{\delta}_2)D_2)}{(\Re(\tilde{v}_1)D_1 + \Re(\tilde{v}_2)D_2)}$$

Where D_{γ} solves

$$\begin{split} \min_{D_2} Z_{12}(D_1,D_2) &= ((\Re(\tilde{k}_1)D_1 + \Re(\tilde{k}_2)D_2)) + (\Re(\tilde{\gamma}_1)D_1 + \Re(\tilde{\gamma}_2)D_2)) \\ \Re(\tilde{\gamma}_2)D_2))(\Re(\tilde{u}_1)D_1 + \Re(\tilde{u}_2)D_2) \end{split}$$
 (for a given D_1)

Subject to the constraints

$$\begin{split} &\sum_{n=1}^{t_1} d_{mn} - \sum_{n=1}^{t_1} d_{nm} \leq Q_{\Re(o'_m)}^1(\phi_m), m = 1, 2, 3, 4, \dots, s_1 \\ &\sum_{m=1}^{s_1} d_{mn} - \sum_{m=1}^{s_1} d_{nm} \geq Q_{\Re(o'_n)}^1(\theta_n), n = 1, 2, \dots, t_1 \\ &\sum_{j=1}^{t_2} d_{ij} - \sum_{j=1}^{t_2} d_{ji} \leq Q_{\Re(o'_j)}^1(\phi_i), i = 1, 2, 3, 4, \dots, s_2 \\ &\sum_{i=1}^{s_2} d_{ij} - \sum_{i=1}^{s_2} d_{ji} \geq Q_{\Re(o'_j)}^1(\phi_j), j = 1, 2, \dots, t_2 \\ &d_{mn} \geq 0 \quad \forall m = 1, 2, 3, \dots, s_1; n = 1, 2, 3, \dots, t_1 \\ &d_{ij} \geq 0 \quad \forall i = 1, 2, 3, \dots, s_2; j = 1, 2, 3, \dots, t_2 \end{split}$$

The deterministic model for bilevel fractional/quadratic green transshipment problem by implementing AI traffic control system with multi choice under fermatean fuzzy environment is expressed as follows

$$\min_{D_1} Z_{11}(D_1,D_2) = \frac{(\Re(\tilde{h}_1)D_1 + \Re(\tilde{h}_2)D_2) + (\Re(\tilde{\delta}_1)D_1 + \Re(\tilde{\delta}_2)D_2)(1-\omega)}{(\Re(\tilde{v}_1)D_1 + \Re(\tilde{v}_2)D_2)}$$

Where D_2 solves

$$\begin{split} \min_{D_2} Z_{12}(D_1,D_2) &= (1-\omega)((\Re(\tilde{k}_1)D_1 + \Re(\tilde{k}_2)D_2)) + (\Re(\tilde{\gamma}_1)D_1 + \Re(\tilde{\gamma}_2)D_2))(\Re(\tilde{u}_1)D_1 + \Re(\tilde{u}_2)D_2) \end{split}$$

(for a given D_1)

Subject to the constraints

$$\begin{split} &\sum_{n=1}^{t_1} d_{mn} - \sum_{n=1}^{t_1} d_{nm} \leq \mathcal{Q}_{\Re(o'_n)}^1(\phi_m), m = 1, 2, 3, 4, \dots, s_1 \\ &\sum_{m=1}^{s_1} d_{mn} - \sum_{m=1}^{s_1} d_{nm} \geq \mathcal{Q}_{\Re(o'_n)}^1(\theta_n), n = 1, 2, \dots, t_1 \\ &\sum_{j=1}^{t_2} d_{ij} - \sum_{j=1}^{t_2} d_{ji} \leq \mathcal{Q}_{\Re(o'_j)}^1(\phi_i), i = 1, 2, 3, 4, \dots, s_2 \\ &\sum_{i=1}^{s_2} d_{ij} - \sum_{i=1}^{s_2} d_{ji} \geq \mathcal{Q}_{\Re(o'_j)}^1(\phi_j), j = 1, 2, \dots, t_2 \\ &d_{mn} \geq 0 \quad \forall m = 1, 2, 3, \dots, s_1; n = 1, 2, 3, \dots, t_1 \\ &d_{ii} \geq 0 \quad \forall i = 1, 2, 3, \dots, s_2; j = 1, 2, 3, \dots, t_2 \end{split}$$

Solution Procedure

To obtain the optimum solution for this above deterministic model, fermatean fuzzy programming is applied in this paper as follows:

Fermatean Fuzzy Programming

To handle uncertain situations, Zadeh introduced the concept of the fuzzy set. This was followed by the development of the intuitionistic and Pythagorean fuzzy sets. The Fermatean fuzzy set is considered more realistic and can handle greater uncertainty than both the intuitionistic and Pythagorean fuzzy sets. In a Fermatean fuzzy set, the sum of the cubes of the truth and false membership grades can exceed 1. Senapati and Yager was first introduced the concept of fermatean fuzzy set. In this environment, Zimmermann [6] proposed a fuzzy programming approach for multiobjective decision-making problems, utilizing a min-max operator. This approach incorporates linear, exponential, or hyperbolic truth functions to find compromised optimal solutions. Intuitionistic fuzzy programming and Pythagorean fuzzy programming are further developed for multiobjective problems in an intuitionistic fuzzy environment, where the truth and false grades are represented by linear, exponential, or hyperbolic functions. In this paper, a nonlinear programming called as fermatean fuzzy programming has been applied to obtain optimum solutions for bilevel fractional/quadratic transshipment problem.

Consider U_k and L_k are upper and lower bound of objective functions U_{21} and U_{22} . Then the membership and non-membership for the objective functions are represented as $\mu(Z_k)$ and $\nu(Z_k)$ respectively. Then the proposed model for fermatean fuzzy programming is described as follows

$$\max \xi^3 - \psi^3$$

Where $\mu(Z_k)^3 \ge \xi^3, \nu(Z_k)^3 \ge \psi^3 \forall k$

$$\mu(U_{kl}) = \begin{cases} 1 & \text{if } Z_k \leq L_k \\ \frac{U_k - Z_k}{U_k - L_k} & \text{if } L \leq U_{kl} \leq U_k \\ 0 & \text{if } Z_k \geq U_k \end{cases}$$

$$v(Z_k) = \begin{cases} 0 & \text{if } Z_k \le L_k \\ \frac{Z_k - L_k}{U_k - L_k} & \text{if } L \le U_{kl} \le U_k \\ 1 & \text{if } Z_k \ge U_k \end{cases}$$

i.e.,
$$(U_{k} - Z_{k})^{3} \ge b_{k}^{3} \xi^{3}, \forall k$$

 $(Z_k - L_k)^3 \ge b_k^3 \psi^3 \forall k$ where $b_k = U_k - L_k$ Subject to the constraints,

$$\begin{split} &\sum_{n=1}^{t_1} d_{mn} - \sum_{n=1}^{t_1} d_{nm} \leq Q_{\Re(o'_m)}^1(\phi_m), m = 1, 2, 3, 4, \dots, s_1 \\ &\sum_{m=1}^{s_1} d_{mn} - \sum_{m=1}^{s_1} d_{nm} \geq Q_{\Re(o'_n)}^1(\theta_n), n = 1, 2, \dots, t_1 \\ &\sum_{j=1}^{t_2} d_{ij} - \sum_{j=1}^{t_2} d_{ji} \leq Q_{\Re(o'_j)}^1(\phi_i), i = 1, 2, 3, 4, \dots, s_2 \\ &\sum_{i=1}^{s_2} d_{ij} - \sum_{i=1}^{s_2} d_{ji} \geq Q_{\Re(o'_j)}^1(\phi_j), j = 1, 2, \dots, t_2 \\ &d_{mn} \geq 0 \quad \forall m = 1, 2, 3, \dots, s_1; n = 1, 2, 3, \dots, t_1 \\ &d_{ji} \geq 0 \quad \forall i = 1, 2, 3, \dots, s_2; j = 1, 2, 3, \dots, t_2 \end{split}$$

The prescribed method has been solved with the help of Lingo Global Solver (20.0) to find the optimum solutions for proposed model.

Result

In this section, the Bilevel Fractional/Quadratic Green Transshipment Model integrated with an Al-based traffic control system under a fuzzy multi-choice environment is applied to a real-world scenario. The model focuses on optimizing the transportation of perishable goods while minimizing cost, time, and carbon emissions. Unlike existing

studies that mainly address preservation technologies in transportation, this work considers the dynamic role of transshipment during uncertain traffic conditions. By incorporating artificial intelligence, the system efficiently controls traffic flow to reduce delays and spoilage. The fuzzy multi-choice parameters further help to manage uncertainty, making the model more realistic and effective for sustainable logistics operations.

Numerical example

Consider ABC company in Gujarat, manufactures distinct products three manufacturing intended for resorts across four different regions of the country such that Agra, Jaipur, Mussorie, and Shimla [12]. The company maintains three different types of inventories each catering to specific kinds of products. Inventory I stores non-perishable items like tea leaves, sugar, salt, and other similar goods. Inventory II stores perishable products such as fruits, meat, and other items that have a limited shelf life. Inventory III holds cutlery and other related products. The demand for these products in the resorts is influenced by the number of guests staying at the resorts at any given time. The duration of guest stays varies depending on the season and specific circumstances. To handle fluctuations in demand and supply, both the inventories and resorts act as transshipment points, allowing the redistribution of products between locations as needed. While transporting these products, ABC Company faces significant challenges related to both economic factors and environmental conditions. To overcome these challenges in transshipment, ABC Company has implemented an Al-driven traffic control system. This system leverages real-time data, including traffic conditions, weather forecasts, and potential road disruptions, to dynamically adjust delivery routes and schedules. Company wants to maximize the profit by minimization of Al implanted economic and environmental costs. The actual values of AI implemented transportation cost, deterioration cost, carbon emission cost, fixed cost, supply, and demand are often unpredictable due to factors like social activities, weather condition, multiple choices. As a result, Al implemented transportation cost, deterioration cost, carbon emission cost, fixed cost are act as triangular fermatean fuzzy numbers while supply and demand are modelled as triangular fermatean fuzzy multi choice. Using ranking function and interpolating polynomial, fermatean and multi choice fermatean fuzzy parameters are converted into crisp values. The cost for upper and lower level of objective functions are shown in table 1,2. The supply and demand are presented in table 3. The primary goal at the upper level is to minimize transportation costs from inventories to resorts, as well as reduce carbon emissions, while maximizing the number of goods delivered. At the lower level, the objective is to minimize the costs associated with goods or food damage during transit, along with minimizing carbon emissions.

Table 1: Th	e upper	cost matrix
-------------	---------	-------------

	X_1	X_2	X_3	Y_1	Y_2	Y_3	Y_4
X_1	0	{(2,5,5),	{(4,5,6),	{(8,9,10),	{(5,8,8),	{(6,7,8),	{(4,7,7),
	·	(1,5,6)} {(0.4,0.7,0.9),	$(3,5,7)$ }	$(7,9,12)$ }	$(4,8,9)$ }	(5,7,10)	(3,7,8)
		{(0.4, 0.7, 0.9), (0.2, 0.7, 1.4)}	{(0.5, 0.8, 1),	{(0.6, 0.9, 1.2),	$\{(2,3,4),$	$\{(1,3,3),$	$\{(1,2,3),$
			$(0.3, 0.8, 1.4)$ }	(0.4, 0.9, 1.6)}	$(1,3,4)$ }	$(0,3,4)$ }	$(1,2,5)$ }
X_2	{(2,5,5),	0	{(1,4,4),	{(5,6,7),	{(3,6,7),	{(4,5,6),	{(2,5,6),
	$(1,5,6)$ }		$(1,4,5)$ }	$(4,6,9)$ }	$(2,6,8)$ }	(3,5,9)	(3,5,9)
	{(0.4, 0.7, 0.9),		{(1,1.5,2),	$\{(2,4,4),$	$\{(2,5,5),$	$\{(3,6,8),$	$\{(4,7,7),$
	(0.2, 0.7, 1.4)}		(0,1.5,3)	$(1,4,5)$ }	$(1,5,6)$ }	$(2,6,9)$ }	$(3,7,8)$ }
X_3	{(4,5,6),	{(1, 4, 4),	0	{(3,4,5),	{(2,3,4),	{(1,2,2),	{(0,2,2),
	(3,5,7)	$(1,4,5)$ }	-	$(2,4,7)$ }	$(1,3,6)$ }	$(1,2,3)$ }	$(0,2,2)$ }
	{(0.5, 0.8, 1),	$\{(1,1.5,2),$		{(4,7,9),	$\{(6,7,9),$	$\{(6,9,9),$	{(7,10,10),
	(0.3, 0.8, 1.4)}	$(0,1.5,3)$ }		(3,7,10)}	(6,7,11)}	(5,9,12)}	(6,10,11)}
Y_1	{(8,9,10),	{(5,6,7),	{(3,4,5),	0	{(1,3,3),	{(1, 2, 4),	{(1, 2, 3),
	$(7,9,12)$ }	$(4,6,9)$ }	$(2,4,7)$ }		(0,3,3)	$(1,2,5)$ }	$(1,2,3)$ }
	{(0.6, 0.9, 1.2),	$\{(2,4,4),$	{(4,7,9),		$\{(2,3,4),$	$\{(1,4,4),$	$\{(3,4,5),$
	(0.4, 0.9, 1.6)}	$(1,4,5)$ }	(3,7,10)}		$(1,3,5)$ }	$(1,4,5)$ }	$(2,4,6)$ }
Y_2	{(5,8,8),	{(3,6,7),	{(2,3,4),	{(1,3,3),	0	{(2,3,4),	{(1,3,4),
	$(4,8,9)$ }	$(2,6,8)$ }	$(1,3,6)$ }	(0,3,3)		$(1,3,5)$ }	$(0,3,4)$ }
	$\{(2,3,4),$	$\{(2,5,5),$	{(6,7,9),	{(2,3,4),		$\{(1,4,4),$	$\{(3,4,5),$
	$(1,3,4)$ }	$(1,5,6)$ }	$(6,7,11)$ }	$(1,3,5)$ }		$(1,4,5)$ }	$(2,4,6)$ }
Y_3	{(6,7,8),	{(4,5,6),	{(1,2,2),	{(1, 2, 4),	{(2,3,4),	0	{(1, 2, 3),
	(5,7,10)	(3,5,9)	$(1,2,3)$ }	$(1,2,5)$ }	$(1,3,5)$ }	· ·	(1,2,3)
	$\{(1,3,3),$	$\{(3,6,8),$	{(6,9,9),	{(1, 4, 4),	{(1, 4, 4),		$\{(1,2,3),$
	$(0,3,4)$ }	$(2,6,9)$ }	(5,9,12)}	$(1,4,5)$ }	$(1,4,5)$ }		$(0,2,4)$ }
Y_4	{(4,7,7),	{(2,5,6),	{(0,2,2),	{(1,2,3),	{(1,3,4),	{(1, 2, 3),	0 0
•	(3,7,8)	(3,5,9)	$(0,2,2)$ }	(1,2,3)	(0,3,4)	(1,2,3)	J
	$\{(1,2,3),$	$\{(4,7,7),$	{(7,10,10),	{(3,4,5),	{(3,4,5),	$\{(1,2,3),$	
	$(1,2,5)$ }	$(3,7,8)$ }	(6,10,11)}	$(2,4,6)$ }	$(2,4,6)$ }	$(0,2,4)$ }	

Table	2. The	lower	cost	matrix

	X_1	X_2	X_3	Y_1	Y_2	Y_3	Y_4
X_1	0	{(4,5,7),	{(4,5,7),	{(7,10,10),	{(6,7,9),	{(6,7,9),	{(4,7,9),
	· ·	$(4,5,9)$ }	$(3,5,9)$ }	(6,10,11)}	$(6,7,11)$ }	$(6,7,11)$ }	(3,7,10)
		$\{(3,4,6),$	$\{(3,4,5),$	{(9,10,11),	{(6,9,10),	$\{(7,8,9),$	$\{(6,7,8),$
		$(3,4,8)$ }	$(2,4,5)$ }	(8,10,13)}	(5,9,13)}	(6,8,12)}	$(5,7,8)$ }
X_2	{(4,5,7),	0	{(2,5,5),	{(4,7,7),	{(3,6,8),	{(2,5,5),	{(2,4,4),
	$(4,5,9)$ }		$(1,5,6)$ }	(3,7,8)	$(2,6,9)$ }	$(1,5,6)$ }	$(1,4,5)$ }
	{(3,4,6),		$\{(1,1.3,2),$	$\{(4,5,6),$	$\{(4,5,6),$	$\{(3,4,5),$	$\{(1,2,4),$
	$(3,4,8)$ }		$(0,1.3,2.5)$ }	$(3,5,6)$ }	$(3,5,6)$ }	$(1,4,5)$ }	$(1,2,5)$ }
X_3	{(4,5,7),	{(2,5,5),	0	{(2,3,4),	{(1,3,3),	{(1, 2, 3),	{(0.6, 0.9, 1.2),
	(3,5,9)	$(1,5,6)$ }	v	$(1,3,4)$ }	$(0,3,4)$ }	$(1,2,5)$ }	(0.4, 0.9, 1.6)
	{(3,4,5),	{(1,1.3,2),		$\{(3,4,5),$	{(1,1.3,2),	{(1,1.1,2),	$\{(0.3, 0.5, 0.7), (0.2, 0.5, 0.9)\}$
	$(2,4,5)$ }	(0,1.3,2.5)}		$(1,4,5)$ }	$(0,1.3,2.5)$ }	(0,1.1,2.2)}	(**-,***,***))
Y_1	{(7,10,10),	{(4,7,7),	{(2,3,4),	0	{(1,3,3),	{(2,3,4),	{(2,3,4),
	(6,10,11)	(3,7,8)	$(1,3,4)$ }		$(0,3,3)$ }	$(1,3,7)$ }	$(1,3,5)$ }
	{(9,10,11),	{(4,5,6),	$\{(3,4,5),$		$\{(2,5,5),$	$\{(4,5,6),$	$\{(3,4,5),$
	(8,10,13)}	$(3,5,6)$ }	$(1,4,5)$ }		$(1,5,6)$ }	$(3,5,7)$ }	$(1,4,5)$ }
Y_2	{(6,7,9),	{(3,6,8),	{(1,3,3),	{(1,3,3),	0	{(1,3,4),	{(1,2,3),
	$(6,7,11)$ }	$(2,6,9)$ }	$(0,3,4)$ }	(0,3,3)		$(0,3,4)$ }	$(1,2,4)$ }
	{(6,9,10),	{(4,5,6),	$\{(1,1.3,2),$	$\{(2,5,5),$		$\{(3,6,7),$	$\{(5,6,7),$
	(5,9,13)}	$(3,5,6)$ }	$(0,1.3,2.5)$ }	$(1,5,6)$ }		$(2,6,7)$ }	$(4,6,8)$ }
Y_3	{(6,7,9),	{(2,5,5),	{(1, 2, 3),	{(2,3,4),	{(1,3,4),	0	{(1,1.5,2),
	$(6,7,11)$ }	$(1,5,6)$ }	$(1,2,5)$ }	$(1,3,7)$ }	$(0,3,4)$ }		(1,1.5,3)
	{(7,8,9),	$\{(3,4,5),$	$\{(1,1.1,2),$	{(4,5,6),	$\{(3,6,7),$		$\{(6,7,8),$
	(6,8,12)}	$(1,4,5)$ }	$(0,1.1,2.2)$ }	$(3,5,7)$ }	$(2,6,7)$ }		(5,7,9)}
Y_4	{(4,7,9),	{(2,4,4),	{(0.6, 0.9, 1.2),	{(2,3,4),	{(1,2,3),	{(1,1.5,2),	0
•	(3,7,10)	$(1,4,5)$ }	(0.4, 0.9, 1.6)}	$(1,3,5)$ }	$(1,2,4)$ }	(1,1.5,3)	U
	{(6,7,8),	{(1, 2, 4),	$\{(0.3, 0.5, 0.7),$	$\{(3,4,5),$	{(5,6,7),	$\{(6,7,8),$	
	$(5,7,8)$ }	$(1,2,5)$ }	(0.2, 0.5, 0.9)}	(1,4,5)}	(4,6,8)}	(5,7,9)}	

Table 3: Supply and demand

$\tilde{O}_1' = (40, 42, 46, 48)$	$\tilde{O}_1'' = (9,14,10,12)$
$\tilde{O}_2' = (35, 33, 40)$	$\tilde{O}_2'' = (24, 23, 28)$
$\tilde{O}_1' = (48, 50, 52, 53)$	$\tilde{O}_3'' = (13,11,14)$
	$\tilde{O}_{4}'' = (8, 7, 6, 9)$

Table 4: Comparison for transshipment and transportation problem

Model	Fermatean fuzzy programming
Bilevel fractional/ quadratic transportation problem without implementing Al traffic control system	$d_{31} = 9$, $d_{32} = 23$, $d_{33} = 10.17$, $d_{34} = 8.35$, $Z_1 = 0.3537$, $Z_2 = 7439.53$
Bilevel fractional/ quadratic transshipment problem without implementing Al traffic control system	$d_{36} = 10.16$, $d_{37} = 41.17$, $d_{74} = 9$, $d_{75} = 23$, $Z_1 = 0.28665$, $Z_2 = 5888.58$

Table 5: comparison for transshipment with and without Al implementation

Model	Fermatean fuzzy programming
Bilevel fractional/ quadratic transshipment problem without implementing AI traffic control system	$d_{36} = 10.16 d_{37} = 41.17 d_{74} = 9 d_{75} = 23 Z_1 = 0.28665 d_{75}$ $Z_2 = 5888.58$
Bilevel fractional/ quadratic transshipment problem with implementing AI traffic control system	$d_{36} = 10.17$, $d_{37} = 41.22$, $d_{74} = 9$, $d_{75} = 23$, $Z_1 = 0.2839$, $Z_2 = 2654.2$

Optimal solution and comparative study

By using two methods prescribed in 2.3.1 & 2.3.2, the obtained optimum solutions of deterministic bilevel fractional/quadratic green transshipment problem by implementing Al traffic control system with multi choice using Lingo Global Solver (20.0) are presented in [Table 4.]. Consequently, the comparison between deterministic bilevel fractional/quadratic green transshipment problem with and without implementing traffic control system is detailed in [Table 5.]. Also, comparison between bilevel fractional/quadratic green transshipment and transportation problem is shown in in [Table 5.].

Discussion

Jaggi et al. (2024) proposed a Bilevel Fractional/Quadratic Green Transportation Problem (BFQGMCTP) incorporating

multi-choice supply and demand parameters under uncertainty. The model addresses real-world challenges such as rising fuel costs, carbon emissions, and product deterioration during transit. The main objective is to maximize the quantity of goods delivered while minimizing transportation cost, deterioration cost, and carbon emission cost simultaneously. The authors introduced two distinct solution techniques and conducted a comparative analysis to evaluate their performance.

Conclusion

Traffic management is a major challenge in the transshipment problem, as it significantly impacts the environment during the distribution process. Implementing an Al-based traffic control system in the transshipment model can effectively mitigate these environmental impacts. Given the inherent uncertainty in real-world scenarios, the proposed model has been developed under a fermatean fuzzy environment to better address these complexities. Then multi choice fermatean proposed is converted into deterministic model using ranking function and Lagrange interpolating polynomial. Pareto optimum solutions have been obtained for the proposed deterministic model applying Fermatean fuzzy programming using Lingo Software package. The numerical example has been discussed for transshipment problem with and without implementing AI traffic control system. Obtained solutions for bilevel fractional/quadratic transshipment problem with AI traffic control system shows a reduction of 7.8% in cost and 11.19% than without AI traffic control system. Consequently, obtained solutions also shows that the bilevel fractional/quadratic transshipment problem shows that 14.4% reduction in time and 14% reduction in cost than the bilevel fractional/quadratic transportation problem. The proposed model could be extended by incorporating key social factors to further enhance sustainability in future developments.

ACKNOWLEDGEMENTS

We thank the Department of Science and Technology, Government of India, for providing support through the Fund for Improvement of S&T Infrastructure in Universities and Higher Educational Institutions (FIST) program (Grant No. SR/FIST/College-/2020/943).

References

Akram, M., Shah, S. M. U., Al-Shamiri, M. M. A., & Edalatpanah, S. A. (2023). Extended DEA method for solving multi-objective transportation problem with Fermatean fuzzy sets. AIMS mathematics, 8(1), 924-961. https://www.aimspress.com/aimspress-data/math/2023/1/PDF/math-08-01-045.pdf

Ali, W., & Javaid, S. (2025). A solution of mathematical multiobjective transportation problems using the fermatean fuzzy programming approach. International Journal of System Assurance Engineering and Management, 1-19. https://doi. org/10.1007/s13198-025-02716-5

Arora, R., Singh, A., & Arora, S. (2021). An aspect of bilevel indefinite

- quadratic transportation problem under intuitionistic fuzzy environment. In Fuzzy Systems and Data Mining VII (pp. 264-276). IOS Press. 10.3233/FAIA210198
- Arya, N. V., & Singh, P. (2019). An optimization procedure for quadratic fractional transportation problem. In Computational Network Application Tools for Performance Management (pp. 9-15). Singapore: Springer Singapore. https://doi.org/10.1007/978-981-32-9585-8_2
- Dikshit, S., Atiq, A., Shahid, M., Dwivedi, V., & Thusu, A. (2023). The use of artificial intelligence to optimize the routing of vehicles and reduce traffic congestion in urban areas. EAI Endorsed Transactions on Energy Web, 10, 1-13. 10.4108/ew.4613
- El Sayed MA, Baky IA. Multi-choice fractional stochastic multiobjective transportation problem. Soft Computing. 2023 Aug;27(16):11551-67.
- https://doi.org/10.1007/s00500-023-08101-3
- Garg, H., Mahmoodirad, A., & Niroomand, S. (2021). Fractional two-stage transshipment problem under uncertainty: application of the extension principle approach. Complex & Intelligent Systems, 7(2), 807-822. https://doi.org/10.1007/s40747-020-00236-2
- Jaggi, C. K., Gautam, P., & Arora, R. (2024). A NEW APPROACH

- FOR SOLVING BILEVEL FRACTIONAL/QUADRATIC GREEN TRANSPORTATION PROBLEM WITH MULTI-CHOICE PARAMETERS UNDER UNCERTAINTY. Pesquisa Operacional, 44, e286992. https://doi.org/10.1590/0101-7438.2023.043.00286992
- Kumar, A., Chopra, R., & Saxena, R. R. (2020). An efficient algorithm to solve transshipment problem in uncertain environment. International Journal of Fuzzy Systems, 22(8), 2613-2624. https://doi.org/10.1007/s40815-020-00923-9
- PILLAI, A. S. (2024). TRAFFIC MANAGEMENT: IMPLEMENTING AI TO OPTIMIZE TRAFFIC FLOW AND REDUCE CONGESTION. Journal of Emerging Technologies and Innovative Research, 11(7). https://dx.doi.org/10.2139/ssrn.4916398
- Qiu, R., Xu, J., Xie, H., Zeng, Z., & Lv, C. (2020). Carbon tax incentive policy towards air passenger transport carbon emissions reduction. Transportation Research Part D: Transport and Environment, 85, 102441. https://doi.org/10.1016/j. trd.2020.102441
- Zhu, H., Liu, C., & Song, Y. (2022). A bi-level programming model for the integrated problem of low carbon supplier selection and transportation. Sustainability, 14(16), 10446. https://doi.org/10.3390/su141610446