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RESEARCH ARTICLE

A Deterministic Inventory Model with Automation-Enabled Processes for Defective Item Management

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Abstract

The increasing demand for sustainable practices has highlighted the role of automation in enhancing production and inventory management systems. Although inventory models with imperfect items, rework process, and unmet demand have been extensively studied, they frequently ignore the capability of small-scale automation to improve performance and cut expenses. This study proposes a sustainable production inventory model by incorporating machine enabled techniques while capturing their effect on storage, rework, and backordering costs. The model is structured based on two conditions: (i) an inventory model with defective items, rework and shortages backordered and (ii) an inventory model with automated systems for defective item management. The optimum production lot size, backorder quantity, and the overall system costs are derived analytically and a numerical example is used to validate the suggested models. By comparing both the models, results demonstrate that the incorporation of automation significantly decreases the overall costs from \$8922.65 to \$8874.81. This research offers a decision-making framework for academics as well as real-world application by incorporating automation into production systems to improve efficiency and promote sustainability.

Keywords: Inventory model, Defective items, Automated systems, Inspection, Stock Management, Sustainability.

关键词:库存模型、缺陷品、自动化系统、检验、库存管理、可持续性。

Introduction

In the contemporary production as well as distribution systems, ensuring operational efficiency while safeguarding sustainability has emerged as a key concern. Inventory models provide a basic foundation for controlling imperfect items, optimizing production quantities, and minimizing costs. However, in modern technological-driven environment, the majority of the traditional models are restricted in their applicability due to their reliance on human inspection processes and static cost patterns.

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Sustainability refers to tactics that require fewer materials, improve product quality and minimize the ecological impacts. In order to accomplish these goals, businesses are progressively implementing automation-enabled techniques which improve product availability, reduce fault misclassification and enhance operational efficiency.

Numerous researchers have investigated inventory models with imperfect items, shortages and rework processes. Harun Ozturk (2020) constructed a deterministic production inventory model that includes imperfect items, rework procedures, and delayed shortages. Jain et al. (2023) developed a sustainable inventory approach which involves random defects and rework, illustrating how mathematical optimization can improve economic and environmental effects. Yadav et al. (2023) looked into smart manufacturing systems with random flawed processes and partial backordering. Gautam et al. (2022) showed how sustainable approaches may enhance cost effectiveness and customer satisfaction by developing inventory plans for an imperfect manufacturing model with faulty items, rework and demand based on price and advertisement. Kohlmann et al. (2024) proposed a versatile planning strategy for stock management systems which integrates lot sizing and rework. Nobil et al. (2024) suggested a sustainabilityfocused inventory handling method with reworking and warm-up time. According to this review, it is clear that

existing production inventory models either focus on lost sales, price and advertisement, carbon emissions or broader environmental issues but they rarely assess the financial benefits from automation.

Recently, researchers have also delved into the application of automation in identifying defects and inventory control. Dey et al. (2024) demonstrated how automation can improve efficiency by introducing intelligent stock management techniques with machine-based verification. Mohandas et al. (2025) employed automated visual inspection for finding errors and sorting in remanufacturing, showing its efficacy in sustainable flaw prevention. Muralidhar (2025) reviewed improvements in automated vision technologies for industrial defect identification, emphasizing gains in rapidity and dependability. Although these kinds of innovations are widely recognized in industry, their incorporation into deterministic inventory models has yet to be investigated. Furthermore, there is a lack of mathematical frameworks that account for amortized automation expenditures and specifically predict the impact of automation on storage, rework and backordering costs. Without this modification, conventional models risk overvaluing total expenses while undervaluing sustainable gains which can be attained by affordable automation systems. To address these challenges, this study proposes a deterministic production inventory model integrating automation tools like barcode scanning, vision inspection and conveyors into manufacturing and holding processes to improve operational performance and environmental sustainability. The model constructs and evaluates two operational circumstances, one representing the fundamental framework and the other representing the integration of automation. A numerical example is provided to evaluate the effectiveness of the suggested model. The results indicate that implementing automation lowers overall costs, proving that efficiency and environmental stewardship can be accomplished simultaneously.

The main contributions of this research are as follows:

- Incorporating amortized automation costs in inventory operations, ensuring low-cost automation can be systematically described and embedded within a deterministic inventory model.
- Applying automation efficiency parameters (μ_h , μ_r , μ_b) to measure the savings in holding, rework and backordering costs.
- Enhancing sustainability by illustrating how automation minimizes waste, and improves total cost efficiency.

Model Formulation

This study focuses on a deterministic economic production quantity model with defective items, rework, shortages and machine-assisted processes. Every cycle commences with a verification process, in which a fraction of defective products is identified. Among these, reworkable defects are handled and put back into stock whereas non-reworkable products

are discarded. Customer demand is fulfilled from on-hand stock whenever feasible, whereas shortages are allowed and completely backordered. At the end, the stock level drops back to zero and the entire procedure repeats. To improve operational efficiency and environmental sustainability, automated techniques like barcode recognition, vision scrutiny and conveyors are included in the model. These strategies help to reduce inspection errors, prevent rework, enhance stock visibility and permit quicker handling of items as indicated by efficiency parameters which lessen storing, rework and backordering costs. An amortized technological cost is incorporated to reflect the expenses of investing and managing these advancements. The system's overall cost consists of setup cost, production cost for both flawless and defective items, rework cost, scrap cost, holding cost, backordering cost and automation cost. The model is formulated under two scenarios: (i) an inventory model with defective items, rework and shortages backordered and (ii) an inventory model with automated systems for defective item management. The goal is to formulate the overall cost and find the optimum quantity and backorder level while encouraging sustainability via waste reduction, fewer stocks, and enhance operational performance.

Notations

 D_a : Demand rate (units/year)

 Q_{l} : Manufacturing lot size (units/cycle)

 B_m : Maximum number of backorders in a cycle (units)

 P^{m} : Production rate (units/year)

 P_r : Rework rate (units/year)

r: Production rate of faulty products (units/year)

 r_1 : Production rate of waste products during rework (units/year)

 r_2 : Production rate of substandard products during rework (units/year)

 r^0 : The total amount of waste and imperfect products generated throughout the rework process.

 T_1 : Time span when shortage exists.

 T_2 : Production runtime when backorder is refilled.

 T_3 : Production runtime when stock accumulates.

 $\tilde{T}_{\scriptscriptstyle A}$: Time span required to rework.

 T_s : Time span when stock runs out.

 $\tilde{T_0}$: Cycle duration.

x: Fraction of faulty products.

 α : Fraction of waste products in substandard items.

 β : Fraction of imperfect quality items in substandard items.

 γ : Fraction of reworkable products in substandard items.

 α_1 : Fraction of waste products generated during rework process.

 β_1 : Fraction of imperfect quality items generated during rework process.

 G_1 : Maximum level of available stocks of flawless items, when normal production ends.

G: Maximum level of available stocks of flawless items,

when rework process ceases.

S: Setup cost per production (\$)

 C_n : Production cost (\$/unit)

 C_d : Disposal cost for each unit of scrap items(\$/unit)

 C_b : Backordering cost (\$/unit)

 \vec{C}_h : Holding cost (\$/unit/year)

 C_{hr} : Holding cost per reworkable products (\$/unit/year)

 $C_{\it r}$: Rework cost per reworkable products (\$/unit)

 C_h , C_{hr} , C_r , C_b : Holding, rework and backordering costs using automation.

 μ_h , μ_r , μ_b : Automation efficiency parameters $0 \le \mu < 1$ I_c : Investment cost for automation (\$)

 $\check{T_i}$: Lifespan of automation devices (years)

 \dot{M}_e : Annual maintenance and operational expenditures for automation (\$/year)

Assumptions

- · Only one product is taken into account.
- Throughout the entire period, the rate of demand is predictable and consistent.
- Both the manufacturing and rework are done in the same cycle.
- Shortages are acceptable and fully backordered.
- Automation enhances identification and handling of defects but does not modify their distribution.
- Annual maintenance and operational expenditures for automation are considered as a fixed percentage of investment cost.

The cycle time is the sum of the shortage time $\,T_{\!_1}$, operating time ($T_{\!_2}+T_{\!_3}$), reworking time $\,T_{\!_4}$ and non-operating time $\,T_{\!_5}$.

$$T_0 = T_1 + T_2 + T_3 + T_4 + T_5 \tag{1}$$

$$T_{1} = \frac{B_{m}}{D_{a}} \tag{2}$$

$$T_2 = \frac{B_m}{P - r - D_a} \tag{3}$$

$$T_3 = \frac{G_1}{P - r - D_a} \tag{4}$$

$$T_4 = \frac{G - G_1}{P_r - r^0 - D_a} \tag{5}$$

$$T_5 = \frac{G}{D_a} \tag{6}$$

$$T_4 = \frac{\gamma r Q_l}{P_r P} = \frac{\gamma x Q_l}{P_r} \tag{7}$$

Also, T_1 is the non-operating time and T_2 is the time period required to fulfill all the backorders for the upcoming production.

The available inventory of $\,G_{\scriptscriptstyle 1}\,$ and $\,G\,$ are

$$G_1 = (P - r - D_a)T_3 \tag{8}$$

$$G = G_1 + (P_r - r^0 - D_a)T_4 (9)$$

During the regular manufacturing uptime, the imperfect items produced is

$$I = r(T_2 + T_3) = xQ_l (10)$$

whereas
$$T_2 + T_3 = \frac{Q_l}{P}$$
 (11)

The quantity of reworkable items generated throughout the normal production process is

$$I_{1} = r\gamma(T_{2} + T_{3}) = P_{r}T_{4} = \gamma x Q_{l}$$
(12)

The quantity of imperfect items generated throughout the normal production process is

$$I_2 = r\beta(T_2 + T_3) = \beta x Q_l \tag{13}$$

The quantity of scrap items generated throughout the normal production process is

$$I_3 = r\alpha(T_2 + T_3) = \alpha x Q_I \tag{14}$$

Let $\,r\,$ represent the production rate of faulty items produced during the regular manufacturing process, then it can be calculated as the product of the production rate $\,P\,$ and the fraction of the faulty products $\,x\,$.

$$r = Px \tag{15}$$

During the rework process, the rate of scrap and defective items produced are

$$r_1 = P_r \alpha_1 \tag{16}$$

$$r_2 = P_r \beta_1 \tag{17}$$

Therefore, the overall scrap and defective items produced are determined as

$$I_{4} = I_{2} + r_{2}T_{4} = (\beta + \beta_{1}\gamma)xQ_{1}$$
(18)

$$I_{5} = I_{3} + r_{1}T_{4} = (\alpha + \alpha_{1}\gamma)xQ_{1}$$
(19)

Also, r^0 is the sum of the scrap rate and defective items rate produced during the rework process.

$$r^{0} = r_{1} + r_{2} = P_{r}(\alpha_{1} + \beta_{1})$$
 (20)

Then, the available inventory of G_1 and G are determined as follows:

$$G_1 = (P - r - D_a)T_3 = F_1Q_l - B_m$$
 (21)

$$G = G_1 + (P_r - r^0 - D_a) \frac{\gamma x Q_l}{P_r} = F_1 Q_l - B_m + F_2 \gamma x Q_l$$

Then,
$$G = (F_1 + F_2 \gamma x)Q_1 - B_m$$
 (22)

where
$$P-r-D_a = P\left(1-x-\frac{D_a}{P}\right) = PF_1$$
 and

$$P_r - r^0 - D_a = P_r \left(1 - \alpha_1 - \beta_1 - \frac{D_a}{P_r} \right) = P_r F_2$$

Therefore, the cycle time T_0 is given by

$$T_{0} = T_{1} + T_{2} + T_{3} + T_{4} + T_{5}$$

$$T_{0} = \frac{\left[1 - x(1 - \gamma(1 - \alpha_{1} - \beta_{1}))\right]Q_{l}}{D_{a}}$$
(23)

Model I: An inventory model with defective items, rework and shortages backordered

The system's total cost consists of setup cost, production cost, rework cost, disposal cost, holding cost, and backordering cost. Each cost components are defined below.

Setup Cost

The fixed setup cost that is paid at the beginning of every production cycle.

$$SC = S$$

Production Cost

This cost represents the cost of producing items, whether it is flawed or flawless, that are needed to fulfill customer demand.

$$PC = C_p Q_l$$

Disposal Cost

A proportion of faulty items that cannot be reworked are discarded, which results in disposing expenses.

$$DC = C_d \alpha x Q_I + C_d \alpha_1 \gamma x Q_I$$

Rework Cost

It represents the cost of reworking the portion of damaged items, that can be handled again.

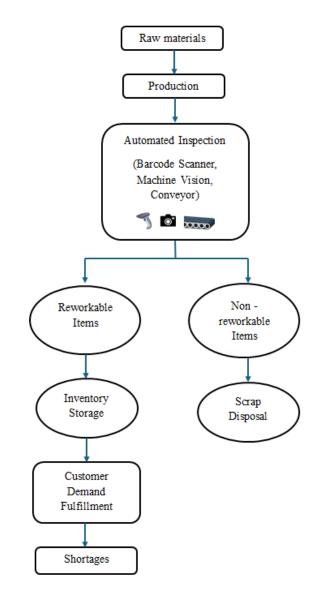


Figure 1: Flowchart of the automation-enabled deterministic inventory model

$$RC = C_r \gamma x Q_I$$

Holding Cost

The expense of holding perfect and reworkable items throughout the manufacturing process.

$$HC = C_h \left(\frac{G_1(T_3)}{2} + \frac{(G_1 + G)(T_4)}{2} + \frac{G(T_5)}{2} + \frac{I(T_2 + T_3)}{2} \right) + C_{hr} \frac{I_1(T_4)}{2}$$

Backorder Cost

Shortage cost reflects the cost associated with short-term insufficiency that happen during the production cycle.

$$BC = C_b \frac{B_m (T_1 + T_2)}{2}$$

Therefore, the total cost is given by

$$TC_{1} = SC + PC + DC + RC + HC + BC$$

$$TC_{1} = S + C_{p}Q_{l} + C_{d}\alpha xQ_{l} + C_{d}\alpha_{1}\gamma xQ_{l}$$

$$+ C_{r}\gamma xQ_{l} + C_{h} \left(\frac{G_{1}(T_{3})}{2} + \frac{(G_{1} + G)(T_{4})}{2} + \frac{G(T_{5})}{2} + \frac{I(T_{2} + T_{3})}{2} \right)$$

$$+ C_{hr} \frac{I_{1}(T_{4})}{2} + C_{b} \frac{B_{m}(T_{1} + T_{2})}{2}$$

$$(25)$$

Substituting $T_{\rm 1}$, $T_{\rm 2}$, $T_{\rm 3}$, $T_{\rm 4}$, $T_{\rm 5}$, G, $G_{\rm 1}$, I, $I_{\rm 1}$ in the above equation, we get,

$$TC_{1} = S + C_{p}Q_{l} + C_{d}(\alpha + \alpha_{1}\gamma)xQ_{l} + C_{r}\gamma xQ_{l}$$

$$+ \frac{C_{h}Q_{l}^{2}}{2P_{r}} \left\{ +2\left(1 + \frac{P_{r}F_{2}}{D_{a}}\right)\gamma xF_{1} + \left(1 + \frac{PF_{1}}{D_{a}} + \frac{x}{F_{1}}\right)\frac{P_{r}F_{1}}{P} \right\}$$

$$- \frac{C_{h}B_{m}Q_{l}}{D_{a}} \left[1 - x(1 - \gamma(1 - \alpha_{1} - \beta_{1}))\right]$$

$$+ \frac{(C_{h} + C_{b})B_{m}^{2}}{2D_{a}} \left(\frac{1 - x}{F_{1}}\right)$$
(26)

The expected cycle time is

$$E(T_0) = \frac{\left[1 - E(x)(1 - \gamma(1 - \alpha_1 - \beta_1))\right]Q_l}{D_a} = \frac{(1 - E_1)Q_l}{D_a}$$
(27)

The expected total cost is determined as follows:

$$E(TCU_{1}) = \frac{E(TC_{1})}{E(T_{0})}$$

$$E(TCU_{1}) = \frac{SD_{a}}{(1 - E_{1})Q_{l}} + \frac{(C_{p} + E(x)[C_{d}(\alpha + \alpha_{1}\gamma) + C_{r}\gamma])D_{a}}{(1 - E_{1})}$$

$$+ \frac{C_{h}D_{a}Q_{l}}{2(1 - E_{1})P_{r}} \begin{cases} \left[\left(1 + \frac{P_{r}F_{2}}{D_{a}}\right)F_{2} + \frac{C_{hr}}{C_{h}}\right]\gamma^{2}E(x^{2}) \\ + 2\left(1 + \frac{P_{r}F_{2}}{D_{a}}\right)\gamma E(x)E_{2} \\ + \left(1 + \frac{PE_{2}}{D_{a}} + E_{3}\right)\frac{P_{r}E_{2}}{P} \end{cases}$$

$$- C_{h}B_{m} + \frac{(E_{4} - E_{3})(C_{h} + C_{b})B_{m}^{2}}{2(1 - E_{1})Q_{l}}$$
(28)

where,

$$E_{1} = E(x)(1 - \gamma(1 - \alpha_{1} - \beta_{1})), E_{2} = 1 - E(x) - \frac{D_{a}}{P},$$

$$E_{3} = E\left(\frac{x}{1 - x - \frac{D_{a}}{P}}\right), E_{4} = E\left(\frac{1}{1 - x - \frac{D_{a}}{P}}\right), E_{5} = 1 - \alpha_{1} - \frac{D_{a}}{P_{r}}$$

Model II: An inventory model with automated systems for defective item management

This model extends Model I by incorporating automationbased techniques such as barcode scanner, vision inspection and conveyors. An automation cost is integrated along with efficiency variables to reflect practical applications. This represents how automation can directly impact storage, rework and shortage costs. While Model I replicates the classic deterministic paradigm, Model II addresses the effect of technology investment on cost fluctuations along with ecological outcomes. This highlights how mechanization reshapes the optimization decision parameters and cost formulation without altering the core production process. The system's total cost consists of setup cost, production cost, disposal cost, rework cost, holding cost, backordering cost and automation cost. The setup, production, and disposal costs remain same as in Model I. Therefore, the other costs are

Rework Cost

Through early identification of issues and sorting, automation lowers production waste and reprocessing effort.

Then the rework rate is adjusted as $C_r = (1 - \mu_r)C_r$ Therefore, $RC = C_r \gamma x Q_l$

Holding Cost

Holding costs are reduced due to automation-based enhancements in inventory visibility and output rate.

Both the holding rates are adjusted as $C_h = (1 - \mu_h)C_h$, $C_{hr} = (1 - \mu_h)C_{hr}$ Therefore, $HC = C_h \left(\frac{G_1(T_3)}{2} + \frac{(G_1 + G)(T_4)}{2} + \frac{G(T_5)}{2} + \frac{I(T_2 + T_3)}{2}\right) + C_{hr} \frac{I_1(T_4)}{2}$

Backorder Cost

Automation improves quickness and lowers the backorder cost.

Then the backorder rate is adjusted as $C_b^{'} = (1 - \mu_b)C_b$

Therefore,
$$BC = C_b' \frac{B_m(T_1 + T_2)}{2}$$

Automation Cost

The annually automation expense comprises of amortized capital expenditure (I_c) as well as operating and maintenance charges (M_e), which were estimated to be 5% of the entire investment cost. The amortization (T_l) was computed over a 5-year lifespan of the automated machinery.

$$AC = \frac{I_c}{T_l} + M_e \left(\frac{D_a}{(1 - E_1)Q_l} \right)$$

$$AC = \frac{I_c}{T_l} + 5\% (I_c) \left(\frac{D_a}{(1 - E_1)Q_l} \right)$$

Therefore, the total cost is given by

$$TC_2 = SC + PC + DC + RC + HC + BC + AC$$

$$TC_{2} = S + C_{p}Q_{l} + C_{d}\alpha x Q_{l} + C_{d}\alpha_{1}\gamma x Q_{l}$$

$$+ C'_{r}\gamma x Q_{l} + C'_{h} \left(\frac{G_{1}(T_{3})}{2} + \frac{(G_{1} + G)(T_{4})}{2} + \frac{G(T_{5})}{2} + \frac{I(T_{2} + T_{3})}{2} \right)$$

$$+ C'_{hr} \frac{I_{1}(T_{4})}{2} + C'_{b} \frac{B_{m}(T_{1} + T_{2})}{2}$$

$$+ \frac{D_{a}}{(1 - E_{1})Q_{l}} \left(\frac{I_{c}}{T_{l}} + 5\%(I_{c}) \right)$$
(29)

$$\begin{split} TC_{2} &= S + C_{p}Q_{l} + C_{d}(\alpha + \alpha_{1}\gamma)xQ_{l} + C_{r}^{'}\gamma xQ_{l} \\ &+ \frac{C_{h}^{'}Q_{l}^{2}}{2P_{r}} \begin{cases} \left[\left(1 + \frac{P_{r}F_{2}}{D_{a}} \right)F_{2} + \frac{C_{hr}^{'}}{C_{h}^{'}} \right]\gamma^{2}x^{2} \\ + 2\left(1 + \frac{P_{r}F_{2}}{D_{a}} \right)\gamma xF_{1} \\ + \left(1 + \frac{PF_{1}}{D_{r}} + \frac{x}{F_{1}} \right)\frac{P_{r}F_{1}}{P} \end{cases} \end{split}$$

$$-\frac{C_{h}B_{m}Q_{l}}{D_{a}}\left[1-x(1-\gamma(1-\alpha_{1}-\beta_{1}))\right] + \frac{(C_{h}'+C_{b}')B_{m}^{2}}{2D_{a}}\left(\frac{1-x}{F_{1}}\right) + \frac{D_{a}}{(1-E_{1})Q_{l}}\left(\frac{I_{c}}{T_{l}}+5\%(I_{c})\right)$$
(30)

The expected cycle time is

$$E(T_0) = \frac{\left[1 - E(x)(1 - \gamma(1 - \alpha_1 - \beta_1))\right]Q_l}{D_a} = \frac{(1 - E_1)Q_l}{D_a}$$
(31)

The expected total cost is determined as follows:

$$E(TCU_2) = \frac{E(TC_2)}{E(T_0)}$$

$$E(TCU_{2}) = \frac{SD_{a}}{(1 - E_{1})Q_{l}} + \frac{(C_{p} + E(x)[C_{d}(\alpha + \alpha_{1}\gamma) + C_{r}'\gamma])D_{a}}{(1 - E_{1})} + \frac{C_{h}'D_{a}Q_{l}}{2(1 - E_{1})P_{r}} \left\{ \left[\left(1 + \frac{P_{r}F_{2}}{D_{a}}\right)F_{2} + \frac{C_{hr}'}{C_{h}'}\right]\gamma^{2}E(x^{2}) + 2\left(1 + \frac{P_{r}F_{2}}{D_{a}}\right)\gamma E(x)E_{2} + \left(1 + \frac{PE_{2}}{D_{a}} + E_{3}\right)\frac{P_{r}E_{2}}{P} - C_{h}'B_{m} + \frac{(E_{4} - E_{3})(C_{h}' + C_{b}')B_{m}^{2}}{2(1 - E_{1})Q_{l}} + \left(\frac{I_{c}}{T_{l}} + 5\%(I_{c})\right)$$
(32)

Solution Methodology

By differentiating the estimated overall cost $E(TCU_1)$ and $E(TCU_2)$ with respect to Q_l and B_m , and setting the partial derivatives equal to zero, the ideal manufacturing lot size and the maximum backorders are determined.

Model I

$$Q_{l} = \frac{2SD_{a}}{\begin{bmatrix} \left[\left(1 + \frac{P_{r}F_{2}}{D_{a}} \right) F_{2} + \frac{C_{hr}}{C_{h}} \right) \gamma^{2} E(x^{2}) \right] D_{a}} \\ C_{h} \begin{cases} \left[\left(1 + \frac{P_{r}F_{2}}{D_{a}} \right) F_{2} + \frac{C_{hr}}{C_{h}} \right) \gamma^{2} E(x^{2}) \right] D_{a}} \\ + \left(1 + \frac{P_{r}F_{2}}{D_{a}} + E_{3} \right) \frac{E_{2}D_{a}}{P} \\ - \frac{C_{h}^{2} (1 - E_{1})^{2}}{(E_{4} - E_{3})(C_{h} + C_{b})} \end{bmatrix}$$
(33)

$$B_{m} = \frac{C_{h}(1 - E_{1})Q_{l}}{(E_{4} - E_{3})(C_{h} + C_{b})}$$
(34)

Model II

$$Q_{l} = \frac{2SD_{a}}{\begin{bmatrix} \left(\left(1 + \frac{P_{r}F_{2}}{D_{a}}\right)F_{2} + \frac{C_{hr}^{'}}{C_{h}^{'}}\right)\gamma^{2}E(x^{2}) \\ + 2\left(1 + \frac{P_{r}F_{2}}{D_{a}}\right)\gamma E(x)E_{2} \end{bmatrix}} \frac{D_{a}}{P_{r}} \\ + \left(1 + \frac{PE_{2}}{D_{a}} + E_{3}\right)\frac{E_{2}D_{a}}{P} \\ - \frac{C_{h}^{'2}(1 - E_{1})^{2}}{(E_{4} - E_{3})(C_{h}^{'} + C_{h}^{'})} \end{aligned}$$

(35)

$$B_{m} = \frac{C_{h}'(1 - E_{1})Q_{l}}{(E_{4} - E_{3})(C_{h}' + C_{b}')}$$
(36)

Result

To validate the proposed model in practical situations, a comparative analysis is performed between model I and II. The data's employed in this study were taken from Öztürk, H. (2020) research, industrial case studies and manufacturer market prices.

$$\begin{split} D_{a} &= 4,000 \, units \, / \, year; P = 10,000 \, units \, / \, year; \\ P_{r} &= 5,000 \, units \, / \, year; C_{p} = \$2 \, / \, unit; \\ S &= \$450 \, per \, production; C_{d} = \$0.3 \, / \, unit; \\ C_{r} &= \$0.5 \, / \, unit; C_{b} = \$0.2 \, / \, unit; \\ C_{h} &= \$0.6 \, / \, unit \, / \, year; C_{hr} = \$0.8 \, / \, unit \, / \, year; \\ \alpha &= 0.1; \beta &= 0.2; \gamma &= 0.7; \alpha_{1} &= 0.1; \beta_{1} &= 0.3; \\ \mu_{h} &= 0.30; \mu_{r} &= 0.35; \mu_{b} &= 0.40; I_{c} &= \$753.49; \\ T_{l} &= 5 \, years; E(x) &= 0.05; E(x^{2}) &= 0.003333 \end{split}$$

Then,
$$E_1 = 0.029$$
; $E_2 = 0.55$; $E_3 = 0.090909$; $E_4 = 1.818181$; $E_5 = 0.1$

By applying model I, production lot size, backordered quantity and the total cost without automation are as follows,

From equations (33), (34), (28), we obtain

$$Q_l = 6,167.16$$
; $B_m = 2,600.19$; $TC_1 = 8922.65

By applying model II, production lot size, backordered quantity and the total cost adopting automation are as follows,

From equations (35), (36), (32), we obtain

$$B_m = 3,389.36$$
; $Q_l = 7,752.43$; $TC_2 = 8874.81

The efficiency of the proposed model is displayed in Table 1. The evaluation of Model I and Model II shows that the total cost reduced from \$8922.65 to \$8874.81. This cost savings proves that incorporating small-scale automation in production inventory model leads to financial gains that encourage sustainability.

Discussion

The existing study has mainly concentrated on faulty products, rework procedures and inadequate supplies; nevertheless, they usually assume fixed costs and manual scrutiny, which restricts their capacity to reflect contemporary manufacturing practices. (Öztürk. 2020, Jain et al. 2023). In real world, firms are increasingly depending on automation techniques like scanning barcodes and vision-based inspection to prevent scrutiny errors. Saberironaghi et al.

Table 1: Displays the efficiency of the proposed model

Model	Production lot size	Maximum backorder quantity	Total cost
Model I	6,167.16	2,600.19	\$8922.65
Model II	7,752.43	3,389.36	\$8874.81

(2023) examined defect identification techniques based on deep learning and machine vision; Tong (2023) constructed a partially automated warehouse stocktaking method including barcode scanners along with data analytics; Zhang et al. (2022) employed IoT and machine learning in storage sector to boost logistics and stock visibility. Despite their widespread adoption, the monetary costs of this kind of technology are not frequently incorporated into computational inventory models, which leads to a gap between mathematical formulation and real operations. In order to bridge this gap, this research incorporates automation into the cost framework, integrating an amortized yearly automation fee as well as efficiency aspects that minimize storing, rework and backordering costs. This modification makes the scenario more feasible and sustainable by measuring both automation cost and subsequent savings in trash, inefficiencies and shortfalls.

Conclusions

In today's production processes, ongoing issues such as defect formation, rework and shortages remain significant barriers to both expenses and sustainable development. So, this study proposes a deterministic inventory model that integrates automation-based operations for better handling defective products and shortages. The implementation of barcode scanning, visual assessment and conveyors established a measurable connection between technology investment and inventory cost. The findings indicate that adopting amortized automation charges and efficiency-oriented cost modifications leads to beneficial advancements in the overall performance of the system. Aside from its analytical contribution, this study suggests that sustainable management can be accomplished via effective process planning instead of large-scale technical upgrades. By combining operational efficiency with ecological stock management, the suggested model provides a pathway for enterprises to attain both cost savings and sustainable responsibility. Future investigation might broaden this model's application to unpredictable demand circumstances, multi-product or multi-stage manufacturing systems and fluctuating automation levels.

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