



RESEARCH ARTICLE

The Study on Plithogenic Fuzzy Sets & its Properties

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Abstract

Numerous mathematical tools, including Crisp Set (CS), Fuzzy Set (FS), Intuitionistic Fuzzy Sets (IFS) and Neutrosophic Sets (NS), are available to measure the degree of accuracy. Moreover, Plithogenic Sets (PS) is a generalization of these four sets. Plithogenic sets have become an effective tool for handling vagueness and imprecision in various fields, including decision-making. This research aims to investigate the novel properties of Plithogenic Fuzzy Sets, focusing on the extension of Plithogenic Set operations and their implications in real-world problems. The results provide a theoretical basis for enhanced Plithogenic Fuzzy Systems and contribute to a deep understanding of Plithogenic Fuzzy Sets.

Keywords: Properties of Fuzzy Sets, Plithogenic Fuzzy Sets, Plithogenic fuzzy set operations, Properties of plithogenic Fuzzy Sets. 模糊集的性质 · 多属性模糊集 · 多属性模糊集运算 · 多属性模糊集的性质。

Introduction

In real existence, there might be some uncertainty over the membership degree in the variable assumption. Zadeh's Fuzzy set and logic, introduced in 1965 (Zadeh, 1965), are effective tools for describing ambiguous conditions. Fuzzy Sets (FS) are an extension of Crisp set (CS). FS theory addresses inaccuracies and uncertainty. The element of FS has a degree of membership function. FS could be represented as a set of ordered pairs $\tilde{P} = \{(u, \varphi_{\tilde{P}}(u))\}$. FS are further extended to Intuitionistic Fuzzy Set (IFS) by Atanassov & Baruah. Atanassov proposed the properties & operations on IFS. The elements of IFS have both Membership and Non-Membership functions. Then, Florentine Smarandache developed the next evaluation of IFS, known as the Neutrosophic set (NS), an extension of IFS and FS. It is an

effective tool for processing with some degree of ambiguity, inconsistency, and incomplete data that occurs in daily life. The elements of NS have membership, indeterminacy & non membership functions. NS is an extremely useful application for dealing with issues in real life. To enhance the accuracy, it is believed that evaluating data uncertainty should be given more attribute value at the advanced research phase.

To improve the accuracy Florentine Smarandache proposed Plithogenic Sets (PS) which is an extension of NS, IFS, FS & CS. "Plithogenic sets (PS), are distinguished by having four or more values of attributes, whereas Crisp sets are defined with a single value (membership), Fuzzy sets with two values (membership, non-membership), and intuitionistic & Neutrosophic sets with three values (membership, indeterminate, and non-membership)". Plithogenic aggregation operator is a *linear combination involving fuzzy operators*, such as *t-norm* and *t-conorm*. PS aggregation operators are associated based on the degree of conflict in order to increase accuracy levels. The initial research has focussed on fundamental operations of PS including union, intersection, complement as well as their applications in decision making. This article aims to investigate the novel properties of plithogenic fuzzy sets (PFS) such as Commutative, Identity, Idempotency, Involution & Demorgan's by using the plithogenic set operations.

Smarandache was the first person to utilise the term "plithogeny" (Smarandache, 2017). PS is a generalisations of former sets such as Crisp, Fuzzy, Intuitionistic fuzzy, Neutrosophic sets. The significance of Plithogenic set is

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the incorporation of the appurtenance degree & degree of contradiction with regard to the elements of decision-making component. In 2020, Gomathy S, proposed the applications of PS in decision-making (Gomathy, 2020). Nivetha Martin and Florentine invented the Plithogenic Cognitive Map (PCM). In 2023, Priyadharshini and Irudhayam developed Single valued Plithogenic Fuzzy Set for the research of obesity on school children during the pandemic (Priyadharshini et al., 2023), additionally an application on insurance field was proposed by utilizing Intuitionistic plithogenic sets (Priyadharshini et al., 2023), moreover an advanced approach of Multi-Dimensional Plithogenic Neutrosophic sets in agriculture field was invented (Priyadharshini et al., 2021).

Preliminaries

Fuzzy Set (Zadeh., 1965)

Let $\dot{U}$ be any universal set $\forall u \in U^*$, then the fuzzy set $\tilde{P}$ defined on $\dot{U}$ may be written as a collection of ordered pairs

$$\tilde{P} = \{(u, \varphi_{\tilde{P}}(u)) / u \in \dot{U}\}$$

A fuzzy set $\tilde{P}$ have the following membership function:

$$\varphi_{\tilde{P}}(u) : \dot{U} \rightarrow [0, 1]$$

Operations on Fuzzy Set (Zadeh., 1965)

Let $\tilde{P}$ and $\tilde{Q}$ are two fuzzy sets defined on the Universal Set $\dot{U}$, the operations Union, Intersection and complement are defined as

- Union: $\varphi_{\tilde{P} \cup \tilde{Q}}(x) = \max\{\varphi_{\tilde{P}}(x), \varphi_{\tilde{Q}}(x)\}$
- Intersection: $\varphi_{\tilde{P} \cap \tilde{Q}}(x) = \min\{\varphi_{\tilde{P}}(x), \varphi_{\tilde{Q}}(x)\}$
- Complement: $\varphi_{\tilde{P}^c}(x) = 1 - \varphi_{\tilde{P}}(x)$

Plithogenic Set (Smarandache., 2018)

Let $\dot{U}$ represent a universal set and $P \subset \dot{U}$ & $v \in \tilde{N}$ be the generic element, P is a Plithogenic set of wthe form (P, a, V, d, C) ,

where, "a" – "attributes"

"V" – "range of attributes values"

"d" – "degree of appurtenance of each element v's attribute value to the set P"

"d" implies $/"d_{IF}"/"d_N"$, while coping with "fuzzy"/ "intuitionistic fuzzy"/ "neutrosophic degree of appurtenance" correspondingly from the PS to the element v .

"C" implies $"C_F"/"C_{IF}"/"C_N"$ while coping with "fuzzy"/ "intuitionistic fuzzy"/ "neutrosophic degree of contradiction" with relation to attribute values, accordingly

Operations on Plithogenic Fuzzy Set (Smarandache., 2018)

- Union: $\varphi_{\vee_{\mathcal{P}}} \sigma = (1 - \mathcal{C}_0) \cdot (\varphi_{\vee_{\mathcal{F}}} \sigma) + \mathcal{C}_0 \cdot (\varphi_{\wedge_{\mathcal{F}}} \sigma)$
- Intersection: $\varphi_{\wedge_{\mathcal{P}}} \sigma = (1 - \mathcal{C}_0) \cdot (\varphi_{\wedge_{\mathcal{F}}} \sigma) + \mathcal{C}_0 \cdot (\varphi_{\vee_{\mathcal{F}}} \sigma)$
- Complement: $\neg_{\mathcal{P}} \varphi = 1 - \varphi$ (or) $d_A^{\mathcal{F}}(v) = 1 - \varphi$

Properties of Fuzzy Sets (George J et al., 2008)

Let $\dot{U}$ be the Universal Sets. Let $\tilde{R}, \tilde{S}$ & $\tilde{T}$ are three fuzzy sets with membership function $[0, 1]$, the fuzzy set properties are

Commutative Property

$$\tilde{R} \cup \tilde{S} = \tilde{S} \cup \tilde{R}$$

$$\tilde{R} \cap \tilde{S} = \tilde{S} \cap \tilde{R}$$

Associative Property

$$(\tilde{R} \cup (\tilde{S} \cap \tilde{T})) = (\tilde{R} \cup \tilde{S}) \cap \tilde{T}$$

$$\tilde{R} \cap (\tilde{S} \cup \tilde{T}) = (\tilde{R} \cap \tilde{S}) \cup \tilde{T}$$

Distributive Property

$$(\tilde{R} \cup (\tilde{S} \cap \tilde{T})) = (\tilde{R} \cup \tilde{S}) \cap (\tilde{R} \cup \tilde{T})$$

$$\tilde{R} \cap (\tilde{S} \cup \tilde{T}) = (\tilde{R} \cap \tilde{S}) \cup (\tilde{R} \cap \tilde{T})$$

Idempotency Property

$$\tilde{R} \cup \tilde{R} = \tilde{R} \text{ \& } \tilde{R} \cap \tilde{R} = \tilde{R}$$

Identity Property

$$\tilde{R} \cup \emptyset = \tilde{R} \text{ \& } \tilde{R} \cap X = \tilde{R}$$

$$\tilde{R} \cap \emptyset = \emptyset \text{ \& } \tilde{R} \cup X = X$$

Transitive Property

$$\text{If } \tilde{R} \subseteq \tilde{S} \text{ \& } \tilde{S} \subseteq \tilde{T} \text{ then } \tilde{R} \subseteq \tilde{T}$$

Involution Property

$$\overline{\overline{\tilde{R}}} = \tilde{R}$$

De Morgan's Law

$$\overline{\tilde{\mathcal{R}} \cup \tilde{\mathcal{S}}} = \overline{\tilde{\mathcal{R}}} \cap \overline{\tilde{\mathcal{S}}} \quad \overline{\tilde{\mathcal{R}} \cap \tilde{\mathcal{S}}} = \overline{\tilde{\mathcal{R}}} \cup \overline{\tilde{\mathcal{S}}}$$

Absorption Property

$$\tilde{\mathcal{R}} \cup (\tilde{\mathcal{R}} \cap \tilde{\mathcal{S}}) = \tilde{\mathcal{R}}$$

$$\tilde{\mathcal{R}} \cap (\tilde{\mathcal{R}} \cup \tilde{\mathcal{S}}) = \tilde{\mathcal{R}}$$

Results and Discussion

Properties of Plithogenic Fuzzy Sets

Let $\mathcal{U}$ denote the universal set. Let $D_A^{\mathcal{F}}(v) = \varphi \in [0,1]$; $D_B^{\mathcal{F}}(v) = \sigma \in [0,1]$ be the degree of fuzzy appurtenance of Plithogenic Sets A & B by the experts. Then the PS properties are

Commutative Property

- Union: $\varphi \vee_{\varphi} \sigma = \sigma \vee_{\varphi} \varphi$
- Intersection: $\varphi \wedge_{\varphi} \sigma = \sigma \wedge_{\varphi} \varphi$

Proof

$$\begin{aligned} \varphi \vee_{\varphi} \sigma &= C_0(\varphi \wedge_{\mathcal{F}} \sigma) + (1 - \mathcal{C}_0)(\varphi \vee_{\mathcal{F}} \sigma) \\ &= \mathcal{C}_0(\varphi \sigma) + (1 - \mathcal{C}_0)(\varphi + \sigma - \varphi \sigma) \\ &= \mathcal{C}_0 \cdot \varphi \sigma + \varphi + \sigma - \varphi \sigma - \mathcal{C}_0 \cdot \varphi - \mathcal{C}_0 \cdot \sigma + \mathcal{C}_0 \cdot \varphi \sigma \\ &= \varphi(1 - \mathcal{C}_0) + \sigma(1 - \mathcal{C}_0) + \varphi \sigma(2\mathcal{C}_0 - 1) \\ \varphi \vee_{\varphi} \sigma &= (\varphi + \sigma)(1 - \mathcal{C}_0) + \varphi \sigma(2\mathcal{C}_0 - 1) \end{aligned} \quad (1)$$

$$\begin{aligned} \sigma \vee_{\varphi} \varphi &= \mathcal{C}_0(\sigma \wedge_{\mathcal{F}} \varphi) + (1 - \mathcal{C}_0)(\sigma \vee_{\mathcal{F}} \varphi) \\ &= C_0(\sigma \varphi) + (1 - C_0)(\sigma + \varphi - \sigma \varphi) \\ &= C_0 \cdot \sigma \varphi + \sigma + \varphi - \sigma \varphi - C_0 \cdot \sigma - C_0 \cdot \varphi + C_0 \cdot \sigma \varphi \\ &= \varphi(1 - \mathcal{C}_0) + \sigma(1 - \mathcal{C}_0) + \sigma \varphi(2\mathcal{C}_0 - 1) \\ &= (\varphi + \sigma)(1 - C_0) + \varphi \sigma(2C_0 - 1) \end{aligned} \quad (2)$$

$$[\because \varphi \sigma = \sigma \varphi]$$

Therefore, $\mathcal{L.H.S} = \mathcal{R.H.S}$

Commutative Property for Union is Satisfied

$$\begin{aligned} \varphi \wedge_{\varphi} \sigma &= \mathcal{C}_0(\varphi \vee_{\mathcal{F}} \sigma) + (1 - \mathcal{C}_0)(\varphi \wedge_{\mathcal{F}} \sigma) \\ &= \mathcal{C}_0(\varphi + \sigma - \varphi \sigma) + (1 - \mathcal{C}_0)(\varphi \sigma) \\ &= \mathcal{C}_0 \cdot \varphi + \mathcal{C}_0 \cdot \sigma - \mathcal{C}_0 \cdot \varphi \sigma + \varphi \sigma - \mathcal{C}_0 \cdot \varphi \sigma \end{aligned}$$

$$\begin{aligned} &= \mathcal{C}_0(\varphi + \sigma) + \varphi \sigma(1 - 2\mathcal{C}_0) \\ \varphi \wedge_{\varphi} \sigma &= \mathcal{C}_0(\varphi + \sigma) + \varphi \sigma(1 - 2\mathcal{C}_0) \end{aligned} \quad (1)$$

$$\begin{aligned} \sigma \wedge_{\varphi} \varphi &= \mathcal{C}_0(\sigma \vee_{\mathcal{F}} \varphi) + (1 - \mathcal{C}_0)(\sigma \wedge_{\mathcal{F}} \varphi) \\ &= \mathcal{C}_0(\sigma + \varphi - \sigma \varphi) + (1 - \mathcal{C}_0)(\sigma \varphi) \\ &= \mathcal{C}_0 \cdot \sigma + \mathcal{C}_0 \cdot \varphi - \mathcal{C}_0 \cdot \sigma \varphi + \sigma \varphi - \mathcal{C}_0 \cdot \sigma \varphi \\ &= (\sigma + \varphi)\mathcal{C}_0 + \sigma \varphi(1 - 2\mathcal{C}_0) \\ \sigma \wedge_{\varphi} \varphi &= \mathcal{C}_0(\sigma + \varphi) + \varphi \sigma(1 - 2C_0) \end{aligned} \quad (2)$$

$$[\because \varphi \sigma = \sigma \varphi; \varphi + \sigma = \sigma + \varphi]$$

$$\therefore \mathcal{L.H.S} = \mathcal{R.H.S}$$

Commutative Property for Intersection is Satisfied.

Identity Property

$$\begin{aligned} A \vee_{\varphi} \emptyset &= A & A \wedge_{\varphi} X &= A \\ A \wedge_{\varphi} \emptyset &= \emptyset & A \vee_{\varphi} X &= X \end{aligned}$$

Proof

Let $\mathcal{U}$ be the Universal Set, $\mu_{\mathcal{U}}(\varphi) = 1$, for all φ belongs to $\mathcal{U}$ &

$\emptyset$ - Empty Set, $\mu_{\emptyset}(\varphi) = 0 \quad \forall \varphi \in \emptyset$

$$\begin{aligned} (i) \quad \varphi \vee_{\varphi} \emptyset &= \\ &= \mathcal{C}_0(\varphi \wedge_{\mathcal{F}} 0) + (1 - \mathcal{C}_0)(\varphi \vee_{\mathcal{F}} 0) \\ &= \mathcal{C}_0(0) + (1 - \mathcal{C}_0)(\varphi) \\ &= (1 - \mathcal{C}_0) \cdot \varphi \\ &= \varphi - \varphi \cdot \mathcal{C}_0 \end{aligned}$$

$$\begin{aligned} (ii) \quad \varphi \wedge_{\varphi} \emptyset &= \\ &= C_0(\varphi \vee_{\mathcal{F}} 0) + (1 - \mathcal{C}_0)(\varphi \wedge_{\mathcal{F}} 0) \\ &= \mathcal{C}_0(\varphi) + (1 - \mathcal{C}_0)(0) \\ &= \mathcal{C}_0 \cdot \varphi \end{aligned}$$

$$(iii) \quad \varphi \wedge_{\wp} \dot{\mathcal{A}} =$$

$$\begin{aligned} & \mathcal{L}_0 (\varphi \vee_{\mathcal{F}} 1) + (1 - \mathcal{L}_0) (\varphi \wedge_{\mathcal{F}} 1) \\ &= \mathcal{L}_0 (1) + (1 - \mathcal{L}_0) (\varphi) \\ &= \mathcal{L}_0 + (1 - \mathcal{L}_0) \cdot \varphi \end{aligned}$$

$$(iv) \quad \varphi \vee_{\wp} \dot{\mathcal{A}} =$$

$$\begin{aligned} & \mathcal{L}_0 \cdot (\varphi \wedge_{\mathcal{F}} 1) + (1 - \mathcal{L}_0) (\varphi \vee_{\mathcal{F}} 1) \\ &= \mathcal{L}_0 (\varphi) + (1 - \mathcal{L}_0) (1) \\ &= \mathcal{L}_0 \cdot \varphi + (1 - \mathcal{L}_0) \end{aligned}$$

If $\mathcal{L}_0 = 0$, then Identity property is satisfied

If $0 < \mathcal{L}_0 \leq 1$, then Identity property is not satisfied

Involution Property

$$\overline{\overline{A}} = A$$

Proof: Plithogenic Fuzzy Negation $\neg_{\wp}(\varphi) = 1 - \varphi$

$$\neg_{\wp}(\neg_{\wp}(\varphi)) = \neg_{\wp}(1 - \varphi) = 1 - (1 - \varphi) = \varphi$$

Therefore, Involution Property is satisfied.

Idempotency Property :

$$A \vee_{\wp} A = A$$

$$A \wedge_{\wp} A = A$$

Proof:

$$\begin{aligned} \varphi \vee_{\wp} \varphi &= (1 - \mathcal{L}_0) (\varphi \vee_{\mathcal{F}} \varphi) + \mathcal{L}_0 (\varphi \wedge_{\mathcal{F}} \varphi) \\ &= (1 - \mathcal{L}_0) (2\varphi - \varphi^2) + \mathcal{L}_0 \cdot \varphi^2 \\ &= 2\varphi - 2\varphi \cdot \mathcal{L}_0 - \varphi^2 + \varphi^2 \cdot \mathcal{L}_0 + \varphi^2 \cdot \mathcal{L}_0 \\ \varphi \vee_{\wp} \varphi &= 2\varphi (1 - \mathcal{L}_0) + \varphi^2 (2\mathcal{L}_0 - 1) \end{aligned}$$

$$\varphi \wedge_{\wp} \varphi = (1 - \mathcal{L}_0) (\varphi \wedge_{\mathcal{F}} \varphi) + \mathcal{L}_0 (\varphi \vee_{\mathcal{F}} \varphi)$$

$$= (1 - \mathcal{L}_0) (\varphi^2) + \mathcal{L}_0 (2\varphi - \varphi^2)$$

$$= \varphi^2 - \varphi^2 \cdot \mathcal{L}_0 + 2\mathcal{L}_0 \cdot \varphi - \varphi^2 \cdot \mathcal{L}_0$$

$$\varphi \wedge_{\wp} \varphi = 2\mathcal{L}_0 \cdot \varphi + \varphi^2 (1 - 2\mathcal{L}_0)$$

If $\mathcal{L}_0 = 1/2$, then Idempotency property is Satisfied.

De morgan's Property:

$$\begin{aligned} \overline{A \vee_{\wp} B} &= \overline{A} \wedge_{\wp} \overline{B} \\ \overline{A \wedge_{\wp} B} &= \overline{A} \vee_{\wp} \overline{B} \end{aligned}$$

Proof:

Let $\varphi \in A$ and $\sigma \in B$,

$$\text{T.P: } (i) \overline{\varphi \vee_{\wp} \sigma} = \overline{\varphi} \wedge_{\wp} \overline{\sigma}$$

$$(ii) \overline{\varphi \wedge_{\wp} \sigma} = \overline{\varphi} \vee_{\wp} \overline{\sigma}$$

$$\overline{\varphi \vee_{\wp} \sigma} = 1 - (\varphi \vee_{\wp} \sigma)$$

$$= 1 - [(1 - \mathcal{L}_0) (\varphi \vee_{\mathcal{F}} \sigma) + \mathcal{L}_0 (\varphi \wedge_{\mathcal{F}} \sigma)]$$

$$\overline{\varphi \vee_{\wp} \sigma} = 1 - [(1 - \mathcal{L}_0) (\varphi + \sigma - \varphi\sigma) + \mathcal{L}_0 (\varphi\sigma)]$$

$$= 1 - (\varphi + \sigma - \varphi\sigma - \varphi \cdot \mathcal{L}_0 - \sigma \cdot \mathcal{L}_0 + \varphi\sigma \cdot \mathcal{L}_0 + \varphi\sigma \cdot \mathcal{L}_0)$$

$$= 1 - [\varphi(1 - \mathcal{L}_0) + \sigma(1 - \mathcal{L}_0) + \varphi\sigma(2\mathcal{L}_0 - 1)]$$

$$\overline{\varphi \vee_{\wp} \sigma} = 1 - ((\varphi + \sigma)(1 - \mathcal{L}_0) + \varphi\sigma(2\mathcal{L}_0 - 1)) \dots (1)$$

$$\overline{\varphi} \wedge_{\wp} \overline{\sigma} = (1 - \varphi) \wedge_{\wp} (1 - \sigma)$$

$$= (1 - \mathcal{L}_0) ((1 - \varphi) \wedge_{\mathcal{F}} (1 - \sigma)) + \mathcal{L}_0 ((1 - \varphi) \vee_{\mathcal{F}} (1 - \sigma))$$

$$= (1 - \mathcal{L}_0) ((1 - \varphi)(1 - \sigma)) + \mathcal{L}_0 ((1 - \varphi) + (1 - \sigma) - (1 - \varphi)(1 - \sigma))$$

$$= (1 - \mathcal{L}_0) (1 - \sigma - \varphi + \varphi\sigma) + \mathcal{L}_0 (1 - \varphi + 1 - \sigma - 1 + \sigma + \varphi - \varphi\sigma)$$

$$= 1 - \varphi(1 - \mathcal{L}_0) - \sigma(1 - \mathcal{L}_0) - \varphi\sigma(2\mathcal{L}_0 - 1)$$

$$\overline{\varphi} \wedge_{\wp} \overline{\sigma}$$

$$= 1 - ((\varphi + \sigma)(1 - \mathcal{L}_0) + \varphi\sigma(2\mathcal{L}_0 - 1)) \dots (2)$$

Therefore, $\mathcal{L}.H.S = \mathcal{R}.H.S$

In a similar way, we prove $\overline{\varphi \wedge_{\wp} \sigma} = \overline{\varphi} \vee_{\wp} \overline{\sigma}$

$$\begin{aligned}
 \overline{\varphi \wedge_{\wp} \sigma} &= 1 - (\varphi \wedge_{\wp} \sigma) \\
 &= 1 - [(1 - \zeta_0)(\varphi \wedge_{\mathcal{F}} \sigma) + C_0(\varphi \vee_{\mathcal{F}} \sigma)] \\
 \overline{\varphi \wedge_{\wp} \sigma} &= 1 - [(1 - \zeta_0)(\varphi \sigma) + \zeta_0(\varphi + \sigma - \varphi \sigma)] \\
 &= 1 - (\varphi \sigma - \zeta_0 \cdot \varphi \sigma + \zeta_0 \cdot \varphi + \zeta_0 \cdot \sigma - \zeta_0 \cdot \varphi \sigma) \\
 &= 1 - [\varphi \cdot \zeta_0 + \sigma \cdot \zeta_0 - \varphi \sigma (2\zeta_0 - 1)] \\
 \overline{\varphi \wedge_{\wp} \sigma} &= 1 - ((\varphi + \sigma)\zeta_0 - \varphi \sigma (2\zeta_0 - 1)) \dots (1) \\
 \overline{\varphi} \vee_p \overline{\sigma} &= (1 - \varphi) \vee_p (1 - \sigma) \\
 \overline{\varphi} \vee_p \overline{\sigma} &= (1 - \zeta_0)((1 - \varphi) \vee_f (1 - \sigma)) + \zeta_0((1 - \varphi) \wedge_f (1 - \sigma)) \\
 &= (1 - \zeta_0)(1 - \varphi + 1 - \sigma - (1 - \varphi)(1 - \sigma)) + \zeta_0[(1 - \varphi)(1 - \sigma)] \\
 &= (1 - \zeta_0)(1 - \varphi + 1 - \sigma - (1 - \sigma - \varphi + \varphi \sigma)) + \zeta_0(1 - \sigma - \varphi + \varphi \sigma) \\
 &= (1 - \zeta_0)(1 - \varphi + 1 - \sigma - 1 + \sigma + \varphi - \varphi \sigma) + C_0(1 - \sigma - \varphi + \varphi \sigma) \\
 &= (1 - \zeta_0)(1 - \varphi \sigma) + \zeta_0(1 - \sigma - \varphi + \varphi \sigma) \\
 &= 1 + \varphi \sigma (2\zeta_0 - 1) - (\varphi + \sigma)\zeta_0 \\
 &= 1 - (\varphi + \sigma)\zeta_0 + \varphi \sigma (2\zeta_0 - 1) \dots (2)
 \end{aligned}$$

Therefore, $\mathcal{L}.H.S = \mathcal{R}.H.S$

Hence, Demorgan's Property is Satisfied .

Conclusion

In this article, we examined some properties of Plithogenic Fuzzy Sets (PFS) by incorporating the concept of attribute value contradiction degrees and plithogenic set operators. The study focused on analysing and formalising properties such as Commutative, Identity, Involution, Idempotency &

Demorgan's in PFS to demonstrate the behaviour under different conditions. These PFS properties will enhance the potential of plithogenic set for real-life decision-making applications. The introduction of these new PFS properties lays the groundwork for future advancement in uncertainty modelling.

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