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RESEARCH ARTICLE

Multi-objective Multi-route Soft Rough Sustainable Transportation Problem based on Various Road Maintenance Conditions

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Abstract

A heap of transportation problems is communicated and sorted out everyday yet are not prone to hybrid ambiguity tools like soft rough environment. Soft rough transportation parameters amplify the impreciseness particularly with reference to each decision alternative in the supply chain. The intent of this chapter is to conceive a multi-objective soft rough transportation model with multiple distribution routes. To promote green transportation, a sustainability influencing parameter set namely 'various maintenance condition of roads' which contain parameters namely good, moderate and no maintenance is chosen. Meanwhile, transportation cost, International Roughness Index (IRI) of road and carbon emission are contemplated as objectives. Each unique element in the parameter set propounds as a soft rough model that is made deterministic using expected operators and then solved using fuzzy goal programming approach in LINGO (19.0). Numerical examples are furnished to evaluate the soft rough models that look up to the preference of decision makers.

Mathematics Subject Classsifications (2020): 90B06, 90C08

Keywords: Soft rough transportation problem, Multi-objective, Multi-route, Road maintenance condition, Sustainability.

Introduction

Transportation experts and researchers have suggested and discovered numerous tools for handling and optimizing the flaws in and around movement of goods from number of sources to destinations. The primary ambition of the linear programming problem designed for transportation is to choose the optimum quantity transported to the demand area under the consideration of total availability at each supplier point. The insufficiency of single objective optimization established multiple objectives in transportation problem. Certain other factors like multiple conveyance and their carrying capacity, multiple routes, multiple commodities etc. were taken into consideration to magnify and resolve transportation problem. Kacher Y and Singh P (2021) and Malacký P and Madleňák R (2023) provided a literature review of transportation problem and the methods used to optimize them. Futuristic scope and extension are also recommended by them.

Uncertainty is the thief behind the improper sorting of issues in real life problems. Due to the ignorance of uncertainty, the backup strategies and desired outcomes are never made realistic and it is mandatory to evaluate all possible reason of concern before optimizing a decision-making problem including the transportation problem. Fuzzy set by L.A. Zadeh (1965) and rough set by Z. Pawlak (1982) are the most renown imprecision tools primarily used by decision makers of transportation problem. Though prominent, these uncertainty measures lack the usage of parameters to represent objects which is overcomed by soft set theory of Molodstov (1999). Soft set is a criteria-based analysis of real-world problems which excludes

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membership values and equivalence relations. Sharma G, Gaurav V, Pardasani KR and Alshehri M (2020) developed a soft set based mathematical model for transportation problem with multiple objectives and modes. Soft sets are adjoined with fuzziness, roughness to emphasize two- or three-fold uncertainty. They render the best parameter to be focused along with their bounds to upgrade difficulties in decision making.

Two-fold uncertainty is already initiated in transportation problem but are not more prevalent. Roy SK, Midya S and Weber GW (2019) analyzed a multi-objective multi-item fixed-charge solid transportation problem with fuzzy rough variables. Midya S, Roy SK (2021) considered and solved rough and fuzzy rough multi-objective fixed charge solid transportation problem in industry. Similarly, fuzzy soft set-based transportation problem is elaborated by Vinotha JM, Gladys LB, Ritha W and Vinoline IA (2021) subject to parameter 'transportation mode'. Soft rough transportation model is not discussed by researchers and so this paper proposes a soft rough set based multi-objective multi-route transportation model under the parameter 'road maintenance'. As environmental sustainability is a progressive goal in transportation sector, three objectives namely transportation cost, International Roughness Index and carbon emission are optimized.

Methodology

Preliminaries

Soft set

Molodstov's soft set contains subsets of the universe of discourse U. The pair (G, A) is said to be a soft set when $G: A \to \wp(U)\, and\, G(A) = \{G(e_1), G(e_2), ..., G(e_p)\}$. Here, $A \subset E$ is the set of parameters and $G(e_p)$ is the value corresponding to parameter e_p.

Rough set

Rough set is introduced by Pawlak to endow the best and worst approximations of subsets of universe U with the help of indiscernibility relation R of x on U.

Lower approximation, $\underline{R}X = \{x \in U \mid R(x) \subseteq X\}, where\, X \subseteq U$ and Upper approximation, $\bar{R}X = \{x \in U \mid R(x) \cap X \neq \phi\}$.

Rough Intervals

Rebolledo's (2006) concept of rough intervals is found by annexing rough sets with intervals. Rough intervals accommodate two closed intervals in which, one is contained in the other. It is represented by $\Re(a) = [a_1, a_2][a_3, a_4] \ni [a_1, a_2] \subseteq [a_3, a_4]\, where\, a_1, a_2, a_3, a_4 \in \mathbb{R}$.

Soft Rough Approximation

In soft rough set, rough approximations concept is extended for each set $G(e_p)$ so that Lower approximation, $\underline{G}(e_p) = \{x \in U \mid R(x) \subseteq G(e_p)\}, where\, X \subseteq U$ and Upper approximation, $\bar{G}(e_p) = \{x \in U \mid R(x) \cap G(e_p) \neq \phi\}$.

Expected Operator of Rough interval

For a rough interval $\Re(a) = [a_1, a_2][a_3, a_4]$, the expected value is found as follows.

$E[\Re(a)] = \frac{1}{2}[\eta(a_1 + a_2) + (1-\eta)(a_3 + a_4)], \eta \in (0,1)$ is a parameter rendered by the decision maker.

Mathematical Model

Notations

I: Set of sources (i=1, 2, ..., I),

J: Set of sinks (j=1, 2, ..., J),

R: Set of routes (r=1, 2, ..., R),

p: Set of parameters, $p \in \mathbb{N}$,

q: Set of objectives, $q \in \mathbb{N}$,

d^{ijr}: Distance between i and j via route r,

$\Re(H^{ijr}_{pq})$: Rough qth objective value associated with parameter p from i to j via r,

$\Re(c^{ijr}_p)$: Rough transportation cost for parameter p from i to j using r,

$\Re(s^{ijr}_p)$: Rough IRI with respect to parameter p from i to j via r,

$\Re(e^{ijr}_p)$: Rough carbon emission from i to j in r corresponding to parameter p,

$\Re(a^i)$: Rough availability at i,

$\Re(b^j)$: Rough demand at j,

x^{ijr}_{pq}: units transported from i to j via r for objective q subject to parameter p,

y^{ijr}_{pq}: binary variable based on x^{ijr}_{pq}.

Assumptions

Products are transported completely on roads with no maintenance (or) good maintenance (or) moderate maintenance condition.

Limitations

- Various combination of road condition parameters is not discussed in the model.
- The damage on the vehicle used for transit and products while operating on such road conditions are excluded in the model.

Mathematical formulation for soft rough multi-objective multi-route transportation problem

$$Min\,(or)\,Max\, Z_{pq} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R} \Re(H^{ijr}_{pq}) x^{ijr}_{pq}, p \in \mathbb{N} \tag{1a}$$

$$Min\,(or)\,Max\, Z_{pq} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R} \Re(H^{ijr}_{pq}) y^{ijr}_{pq}, p \in \mathbb{N} \tag{1b}$$

$$subject\ to \sum_{j=1}^{J} x^{ijr}_{pq} \geq \Re(a^i), i = 1, 2, ..., I \tag{2}$$

$$\sum_{i=1}^{I} x^{ijr}_{pq} \leq \Re(b^j), j = 1, 2, ..., J \tag{3}$$

$$x^{ijr}_{pq} \geq 0, \forall\, i, j, k \tag{4}$$

$$y_{pq}^{ijr} = \begin{cases} 0 \ if \ x_{pq}^{ijr} = 0 \\ 1 \ if \ x_{pq}^{ijr} > 0 \end{cases} \qquad (5)$$

Here, parameter 'p' decides the number of multi-objective problems to be discussed which is aided by soft sets. (2) and (3) are the rough availability and demand constraints.

Fuzzy Goal Programming Approach for Multiobjective Multi-route Soft Rough Optimization

Fuzzy goal programming is a long-standing multiobjective solving mechanism constructed by Zangiabadi M and Maleki H (2007). It diminishes the worst-case scenario with the help of over deviation and under deviation variables which are defined separately for each objective function. The method also ensures a balance between deviations and the membership functions of objectives. Fuzzy goal programming based mathematical description of the multiobjective multi-route problem with 'q' objectives and 'p' parameters is provided below.

Step 1

Use expected operator to get the deterministic model of rough multi-objective multi-route transportation problem with respect to parameter p=1.

Step 2

Solve the 'q' individual objectives of the multi-objective problem corresponding to p=1 separately to obtain the solutions $X_{11}, X_{12}, ..., X_{1Q}$ using LINGO (19.0).

Step 3

Construct the pay-off matrix of the obtained solutions subject to $Z_{1q}(x)$.

Step 4

Find $Z_{1q}^{\max} = \max(Z_{11,} Z_{12,} ..., Z_{1q,})$ *and* $Z_{1q}^{\min} = \min(Z_{11,} Z_{12,} ..., Z_{1q,})$ and solve the following in LINGO (19.0) to get the compromise solution subject to p = 1.

Minimize φ

Subject to $\mu_{1q}(x) + d_{1q}^{-} + d_{1q}^{+} = 1,$

$$where \ \mu_{1q}(x) = \begin{cases} 1 & if \ Z_{1q}(x) < L_{1q} \\ \dfrac{Z_{1q}^{\max} - Z_{1q}(x)}{Z_{1q}^{\max} - Z_{1q}^{\min}} & if \ L_{1q} \le Z_{1q}(x) \le U_{1q} \\ 0 & if \ Z_{1q}(x) > U_{1q} \end{cases}$$

And d_{1q}^{-} *and* d_{1q}^{+} are the under deviation and over deviation variables of the qth objective function for parameter p=1.

$\varphi \ge d_{1q}^{-}, q = 1, 2, ..., Q$,

$d_{1q}^{-} d_{1q}^{+} = 0 \ \& \varphi \in [0,1]$ and (2)-(4).

The constraint (5) is included only if the objective function depends upon binary decision variables as in (1b).

Step 5

Repeat step-1 to step-4 for $p = 2, 3, ..., \mathbb{N}$ to get its corresponding compromise solution.

Results and Discussion

Numerical Example

The data for the formulated multi-objective multi-route problem is traced from Shivani and Rani (2024) and converted into rough intervals for easier understanding. Rough road condition parameters related to maintenance are affixed to the problem of study from Prafulla S, Gupta S, Landge VS and Hokam VS (2017) which used a machine for evaluating road roughness index and emission incurred around an industrial area in India.

$a^1 = [60,100][40,120]$; $a^2 = [80,120][60,140]$;
$b^1 = [40,80][20,100]$; $b^2 = [70,110][50,130]$

Mathematical Expansion and Solution

The multi-objective problem corresponding to no road maintenance condition is taken as p=1.

$$Min\, Z_{11} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R} d^{ijr} E[c_1^{ijr}] y_1^{ijr} = 27.625\text{y}111 + 33.15\text{y}112$$
$$+38.675\text{y}121 + 41.4375\text{y}122 + 55.25\text{y}211 + 49.725\text{y}212$$
$$+44.2\text{y}221 + 41.4375\text{y}222;$$

$$Min\, Z_{12} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R} E[s_1^{ijr}] y_1^{ijr} = 3779.8475\text{y}111 + 4535.8095\text{y}112$$
$$+5291.7865\text{y}121 + 5669.7713\text{y}122 + 7559.695\text{y}211$$
$$+6803.7255\text{y}212 + 6047.756\text{y}221 + 5669.7713\text{y}222;$$

$$\text{Min } Z_{13} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{r=1}^{R} E[e_1^{ijr}] x_1^{ijr} = 45\text{x}111 + 43.5\text{x}112 + 61.25\text{x}121$$
$$+97.5\text{x}122 + 82.5\text{x}211 + 83.25\text{x}212 + 82\text{x}221 + 58.125\text{x}222;$$

$\text{x}111+\text{x}112+\text{x}121+\text{x}122 \le 80$; $\text{x}211+\text{x}212+\text{x}221+\text{x}222 \le 100$;
$\text{x}111+\text{x}112+\text{x}211+\text{x}212 \le 60$; $\text{x}121+\text{x}122+\text{x}221+\text{x}222 \le 90$;

$\text{x}111 \ge 0$; $\text{x}112 \ge 0$; $\text{x}121 \ge 0$; $\text{x}122 \ge 0$; $\text{x}211 \ge 0$;
$\text{x}212 \ge 0$; $\text{x}221 \ge 0$; $\text{x}222 \ge 0$;

and constraint (5).

Using Fuzzy goal programming approach, the pay-off matrix subject to p=1 is

331.5	45358.16	10538.44
331.5	45358.16	10538.44
74.5875	10205.58	7841.25

Compromise solution using LINGO (19.0) is

$$\varphi = 0.2580646, d_{11}^{-} = 0.2580645,$$

Table 1: Distance between sources and destination along multiple routes

i	*j*	*1*		*2*	
	r	*1*	*2*	*1*	*2*
1		5	6	7	7.5
2		10	9	8	7.5

Table 2: Per unit per km Rough Transportation cost in no maintenance condition

i	*j*	*1*		*2*	
	r	*1*	*2*	*1*	*2*
1		[8,12] [3,13]	[6,10] [3,11]	[7,13] [2,13]	[10,19] [3,20]
2		[7,11] [2,13]	[6,13] [4,14]	[7,14] [5,15]	[5,10] [3,13]

Table 3: Per unit per km Rough Transportation cost in good maintenance condition

i	*j*	*1*		*2*	
	r	*1*	*2*	*1*	*2*
1		[4,8] [3,10]	[4,7] [2,9]	[3,7] [2,8]	[6,10] [5,15]
2		[5,9] [2,11]	[7,9] [3,12]	[7,10] [4,12]	[5,8] [3,10]

Table 4: Per unit per km Rough Transportation cost in moderate maintenance condition

i	*j*	*1*		*2*	
	r	*1*	*2*	*1*	*2*
1		[6,10] [3,11.5]	[5,8.5] [2.5,10]	[5,10] [2,10.5]	[8,14.5] [4,17.5]
2		[6,10] [2,12]	[6.5,11] [3.5,13]	[7,12] [4.5,13.5]	[5,9] [3,11.5]

Table 5: IRI in no maintenance condition

i	*j*	*1*		*2*	
	r	*1*	*2*	*1*	*2*
1		[26.5,28] [26,30]	[31.8,33.6] [31.2,36]	[37.1,39.2] [36.4,42]	[39.75,41.4375] [39,45]
2		[53,56] [52,60]	[47.7,50.4] [46.8,54]	[42.4,44.8] [41.6,48]	[39.75,42] [39,45]

Table 6: IRI in good maintenance condition

i	*j*	*1*		*2*	
	r	*1*	*2*	*1*	*2*
1		[16.5,19.5] [15,20]	[19.8,23.4] [18,24]	[23.1,27.3] [21,28]	[24.75,29.25] [22.5,30]
2		[33,39] [30,40]	[29.7,35.1] [27,36]	[26.4,31.2] [24,32]	[24.75,29.25] [22.5,30]

Table 7: IRI in moderate maintenance condition

i	*j*	*1*		*2*	
	r	*1*	*2*	*1*	*2*
1		[25,26.8] [24.475,28.75]	[30,32.16] [29.37,34.5]	[35,37.52] [34.265, 40.25]	[37.5.40.2] [36.7125, 43.125]
2		[50,53.6] [48.95,57.5]	[45,48.24] [44.055,51.75]	[40,42.88] [39.16,46]	[37.5,40.2] [36.7125, 43.125]

Table 8: Carbon emission during no maintenance condition

i	*j*	*1*		*2*	
	r	*1*	*2*	*1*	*2*
1		[3606.65, 3838.52] [3522.22, 4152]	[4327.98, 4606.224] [4226.664, 4982.4]	[5049.31, 5373.928] [4931.108, 5812.8]	[5409.975, 5757.78] [5283.33, 6228]
2		[7213.3, 7677.04] [7044.44, 8304]	[6491.97, 6909.336] [6339.996, 7473.6]	[5770.64, 6141.632] [5635.552, 6643.2]	[5409.975, 5757.78] [5283.33, 6228]

Table 9: Carbon emission during moderate maintenance condition

i	*j*	*1*		*2*	
	r	*1*	*2*	*1*	*2*
1		[3362.5, 3628.586] [3248.68 91,3927.25]	[4035, 4354.3032] [3898.427, 4712.7]	[4707.5, 5080.0204] [4548.1648, 5498.15]	[5043.75, 5442.879] [4873.0337, 5890.87]
2		[6725, 7257.172] [6497.3783, 7854.5]	[6052.5, 6531.4548] [5847.6404, 7069.05]	[5380, 5805.7376] [5197.9026, 6283.6]	[5043.75, 5442.879] [4873.0337, 5890.875]

Table 10: Carbon emission during good maintenance condition

i	*j*	*1*		*2*	
	r	*1*	*2*	*1*	*2*
1		[2192.85, 2607.15] [1950, 2696]	[2631.42, 3128.58] [2340, 3235.2]	[3069.99, 3650.01] [2730, 3774.4]	[3289.275, 3910.725] [2925, 4044]
2		[4385.7, 5214.3] [3900, 5392]	[3947.13, 4692.87] [3510, 4852.8]	[3508.56, 4171.44] [3120, 4313.6]	[3289.275, 3910.725] [2925, 4044]

Table 11: Comparative analysis of solutions from no, moderate and good maintenance parameters (p = 1,2,3)

p	*Transportation cost*	*IRI*	*Carbon emission*
1	Rs.7974.25	140.8875 m	19277.21 kgs
2	Rs. 9805.625	133.9082 m	18062.96 kgs
3	Rs. 6351.75	118.925 m	15822.05 kgs

$d_{12}^{-} = 0.2580645, d_{13}^{-} = 0.04931058, \quad d_{11}^{+} = d_{12}^{+} = d_{12}^{+} = 0,$

$x111 = 47, x112 = 13, x121 = 20, x222 = 70,$

$y111 = y112 = y121 = y222 = 1.$

Similarly, the pay-off matrix and compromise solution for moderate and good maintenance road (p=2 & 3) are found using LINGO (19.0).

From Table 11, the importance of good maintenance condition along transportation network is emphasized. It is also observed that transportation cost, roughness index and emission are lesser in good maintenance road.

Conclusion

This paper proposed an initiative model for studying multi-objective multi-route rough transportation problem with multiple alternatives subject to a parameter. The model also optimized the conflicting objectives by simultaneously accessing each and every possible alternative corresponding to various road maintenance conditions in rough environment. The solutions from good road maintenance bestowed a road roughness index of 118.925 metres, carbon emission of 15822.05 kgs and an expense of Rs. 6351.75 which is comparatively lower than roads with no or moderate maintenance. Also, an increase in carbon emission is noted with increase in roughness index of road. This reflects the significance of maintaining roads which is essential for sustainable development across supply chain and management. This model can be extended to various other variants of transportation problem including multi-item, multi-vehicles etc. Distribution time may also be included as an additional goal in the same problem which is essential due to increasing thirst for faster delivery among people. Roads with combination of maintenance parameters can also be evaluated using some other methods of multi-objective optimization. Constraints related to restrictions on overall road roughness index may be an alternative talk for futuristic research in transportation problem.

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