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RESEARCH ARTICLE

A Study on an Optimal Four Echelon Inventory Model for Growing Items with Imperfect Quality and Trade Credit Financing

I. Francina Nishandhi*

Abstract

Giving quality products to the customers is the desire to every manufactures. And this strategy plays a vital role to retain the customers. Another important strategy to retain the customers is to practicing the trade credit financing. In this customer can buy the products from the vendor and repay the amount later. So this proposed work develops an integrated inventory model for a four echelons in the form of farming, processing, screening and retail operations along with the trade credit financing. The farmer grows new born items and then delivers them to a processor once the items are mature enough. During the processing time the items are slaughtered, processed, packaged and screened for quality. The processor then delivers good quality items to the retailer and sells the processed poorer quality items to the secondary markets. The main objective is to retain the customers as well as maximize the overall supply chain profit. Mathematical Model and Numerical example illustrates the proposed model in a clear understanding.

Keywords: supply chain, four echelon, imperfect quality, Trade credit
关键词：供应链，四层级，不完美质量，贸易信用

Introduction

Inventory management is vital for smooth functioning of every business scenario to fulfill the customer's desire at the right time. The first EOQ model was developed by Harris in the year 1913 with the base of this model Taft established an economic production quantity model in the year 1918. Based on the EOQ and EPQ model many researchers have developed their model under various assumptions. (Rosenblatt *et al*., 1986) he explained the inventory model under the concept of imperfect quality items. (Salameh and Jaber 2000) developed an EOQ model with defective items in which the products are sold in a single batch at the end of 100% screening process. (Razaei 2014) was the first researcher who has developed his inventory model under the concept of growing items. In this model he explained the inventory system had distinct periods, namely growth and consumption periods. Various researchers have started to extend the work on inventory control for growing items into diverse other areas. (Nobil *et al*., 2019) extended the growing items inventory under concept of permissible shortages and fully back ordered. (Sebatjane M. *et al*., 2018) has explained the Economic order quantity model for growing items with imperfect quality. Based on the previous results on growing items, (Sebatjane. M. *et al*., 2020) was the first person who has introduced four echelons inventory model in growing items. (Mittal et.al., 2021) was introduced trade credit financing in the economic order quantity model for growing items. This proposed model is the extension work of (Sebatjane. M. *et al*., 2020) under trade credit financing. The supplier send their product to the retailer on credit, the retailer can sell the item and earn revenue from it and settle back his amount to the seller along with the interest. So this proposed work concentrates to maximize the overall profit of the supply chain in the four echelon inventory under trade credit financing.

The remainder of the session organized as follows, section 2 develops the notation and assumptions of the proposed model. Section 3 illustrates the mathematical model of the proposed model, section 4 explains the numerical example and section 5 concludes the proposed model.

Assistant Professor, Department of Mathematics, Holy Cross College (Autonomous), Trichy-2, Tamil Nadu, India.

***Corresponding Author:** I. Francina Nishandhi, Assistant Professor, Department of Mathematics, Holy Cross College (Autonomous), Trichy-2, Tamil Nadu, India, E-Mail: francinajude@gmail.com

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Materials and Methods

This proposed work develops an integrated inventory model for a four echelons in the form of farming, processing, screening and retail operations along with the trade credit financing. The main objective is to retain the customers as well as maximize the overall supply chain profit.

Notations and Assumptions:

The following notations and assumptions is used for this proposed model

D: Demand rate for good quality processed items in weight units per unit time

R: Processing rate in weight units per unit time

$w(t)$: The weight of an item at time t

: Newborn weight of each item

: Maturity weight of each item

y: Farmer's order quantity for live newborn items per cycle

: Fraction of the live items which survive throughout the growth period

: Procurement cost per weight unit of live new born items

: Farmer's setup cost per cycle

: Farmer's feeding cost per weight unit per unit time

: Farmer's mortality cost per weight unit per unit time

: Duration of the farmer's growth period

: Farmer's selling price per weight unit of live items

: Processor's processing facility setup cost per cycle

: Processor's processing facility holding cost per weight unit per unit time

: Time required to process the entire lot size

: Processors cost of sending single batch of good quality processed inventory to the retailer

: Processor's screening facility holding cot per weight unit per unit time

: Time between consecutive deliveries of good quality batches from the screening facility to the retailer

n: Number of batches of good quality processed inventory delivered to the retailer during a single screening run

: Time required screening the entire lot size

/p: Processor's selling price per weight unit of good quality inventory

: Processor's selling price per weight unit of poorer quality inventory

a : Fraction of processed inventory that are poorer quality

s: Screening rate in weight units per unit time

: Screening cost per weight unit

z: Numbers of items per batch of good quality processed items sent to the retailer

T: Replenishment cycle time for all echelons

: Retailer's ordering cost per cycle

: Retailer's holding cost per weight unit per unit time

/s : Retailer's selling price per weight unit of good quality processed inventory

: Asymptotic weight of the items

: Integration constant

: Exponential growth rate of the items

I_e: Interest earned

I_p : Interest paid

The following assumptions are used to develop proposed inventory model

- There is only one farmer one processor and one retailer in the supply chain involved, respectively in the growing, the processing with screening and the selling of a single type of item.
- A fraction of the live inventory item does not survive until the end of the growth period.
- As soon as processing is complete, the entire processed lot is transferred toa screening warehouse where a 100% screening process takes place in which the items are classified as either being of good quality or poorer quality.
- The retailer uses the good quality processed inventory to meet consumer demand assoon as the first batch is delivered.
- The poorer quality processed inventories are allowed to accumulate at the processor'sscreening warehouse from where they are sold when a single screening run ends.
- Trade credit financing is considered in this proposed Model.

Model Formation

In this section we elaborated the four echelons inventory model under trade credit financing.

Farmer's profit

The farmer's total profit per cycle is the farmer's revenue less the farmer's cyclic total cost,(that is sum of the farmer's purchase cost, setup cost, feeding cost and mortality cost and it is represent as

$$TP_f = p_f xyw_1 - p_v yw_0 - K_f - y\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}\left[c_f x + m_f\left(1-x\right)\right] \quad (1)$$

Processor's profit

Processor's cyclic profit is computed as the revenue from both good and poorer quality inventory less the total cost of processor's setup cost, procurement cost, holding cost at the processing facility, and the total cost incurred by the processor at the screening facility, it will show as given below,

$$TP_p = P_p xyw_1(1-a) + P_q xyw_1 a - K_p - p_f xyw_1 - h_p\left(\frac{x^2y^2w_1^2}{2R}\right) - nK_s - lxyw_1 - h_s\left[\frac{x^2y^2w_1^2}{s} - \frac{(n-1)x^2y^2w_1^2(1-a)}{2ns}\right] \quad (2)$$

Retailer's profit

The retailer faces a demand rate (for good quality processed inventory) of D. To meet this demand, the retailer receives n batches of size zw_1 from the processor after the items have been screened for quality. The time duration of time between successive order cycles is therefore,

$$T = \frac{xyw_1(1-a)}{D} \tag{3}$$

The retailer's total profit per cycle is calculated as the revenue earned from the sales of good quality processed inventory minus the sum of the ordering cost, procurement cost and holding costs, it follows that,

$$TP_r = P_r xyw_1(1-a) - K_r - p_p xyw_1(1-a) - h_r\left[\frac{xyw_1(1-a)T}{2} - \frac{(n-1)x^2y^2w_1^2(1-a)}{2ns}\right] \tag{4}$$

Supply chain Profit

The supply chain profit is computed as the sum of the profits generated at each supply chain echelon as given in equations (1), (2) and (3).

$$TP_{sc} =$$
$$p_f xyw_1 - p_v yw_0 - K_f - y\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln\left(1+\beta\right)\right]\right\}\left[c_f x + m_f(1-x)\right]$$
$$+ P_p xyw_1(1-a) + P_q xyw_1 a - K_p - p_f xyw_1 - h_p\left(\frac{x^2y^2w_1^2}{2R}\right) - nK_s - lxyw_1$$
$$- h_s\left[\frac{x^2y^2w_1^2}{s} - \frac{(n-1)x^2y^2w_1^2(1-a)}{2ns}\right] + P_r xyw(1-a) - K_r - p_p xyw(1-a) -$$
$$h_r\left[\frac{xyw_1(1-a)T}{2} - \frac{(n-1)x^2y^2w_1^2(1-a)}{2ns}\right] \tag{5}$$

To assures that the retailer never runs out of processed inventory, the farmer and the processor start new replenishment cycles every *T* time units which is the amount of time it takes for the processed inventory at the retailer's facility to reach zero. The total supply chain profit per unit time TPU_{sc} is determined by dividing the equation (5) by the replenishment cycle time T as given in equation (3) and the result is,

$$TPU_{sc} = p_r D + \frac{p_q aD}{(1-a)} - \frac{K_r D}{xyw_1(1-a)} - h_r y\left[\frac{xw_1(1-a)}{2} - \frac{(n-1)xw_1 D}{2ns}\right] - \frac{K_p D}{xyw_1(1-a)} -$$
$$h_p\left(\frac{xyw_1 D}{2R(1-a)}\right) - \frac{nK_s D}{xyw_1(1-a)} - \frac{lD}{x(1-a)} - h_s y\left[\frac{xw_1 D}{(1-a)s} - \frac{(n-1)xw_1 D}{2ns}\right] - \frac{p_v w_0 D}{xw_1(1-a)} -$$
$$\frac{K_f D}{xyw_1(1-a)} - \frac{D}{xw_1(1-a)}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln(1+\beta)\right]\right\}\left(c_f x + m_f(1-x)\right)$$

Now we concentrate the total profit of the supply chain under the concept trade credit financing. In trade credit financing two cases is considered. In both cases the total profit of the supply chain will be calculated.

Case 1: $M \le T$

Interest earned from the revenue generated from the average sales during the time period 0 to M.

Interest earned per cycle $= \dfrac{DM^2 I_e s}{2}$

Interest paid per cycle $= \dfrac{D(T-M)^2 I_p p}{2}$

Therefore the total profit will be,

$$TPU_{sc} = p_r D + \frac{p_q aD}{(1-a)} - \frac{K_r D}{xyw_1(1-a)} - h_r y\left[\frac{xw_1(1-a)}{2} - \frac{(n-1)xw_1 D}{2ns}\right] - \frac{K_p D}{xyw_1(1-a)} -$$
$$h_p\left(\frac{xyw_1 D}{2R(1-a)}\right) - \frac{nK_s D}{xyw_1(1-a)} - \frac{lD}{x(1-a)} - h_s y\left[\frac{xw_1 D}{(1-a)s} - \frac{(n-1)xw_1 D}{2ns}\right] - \frac{p_v w_0 D}{xw_1(1-a)} -$$
$$\frac{K_f D}{xyw_1(1-a)} - \frac{D}{xw_1(1-a)}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln(1+\beta)\right]\right\}\left(c_f x + m_f(1-x)\right) +$$
$$\frac{D^2M^2 I_e s}{2xyw_1} - \frac{D^2\left(\frac{xyw_1(1-a)}{D} - M\right)^2 I_p p}{2xyw_1(1-a)} \quad - (5)$$

The optimal order quantity is,

$$y^* = \sqrt{\frac{2D\left[K_r + K_p + nK_s + K_f\right] - M^2D^2(I_e s - I_p p)}{x^2w_1^2\left\{h_r\left[(1-a)^2 - \frac{D(n-1)(1-a)}{ns}\right] + h_p\left[\frac{D}{R}\right] + h_s\left[\frac{2D}{s} - \frac{D(n-1)(1-a)}{ns}\right] + (1-a)^2 I_p p\right\}}}$$

Case 2: $M \ge T$

Interest earned from the revenue generated from the average sales after the time period T

Interest earned per cycle $= \dfrac{DT^2 I_e s}{2} + DTs(M-T)I_e$

Interest paid per cycle $= 0$

Therefore the total profit will be,

$$TPU_{sc} = p_r D + \frac{p_q aD}{(1-a)} - \frac{K_r D}{xyw_1(1-a)} - h_r y\left[\frac{xw_1(1-a)}{2} - \frac{(n-1)xw_1 D}{2ns}\right] - \frac{K_p D}{xyw_1(1-a)} -$$
$$h_p\left(\frac{xyw_1 D}{2R(1-a)}\right) - \frac{nK_s D}{xyw_1(1-a)} - \frac{lD}{x(1-a)} - h_s y\left[\frac{xw_1 D}{(1-a)s} - \frac{(n-1)xw_1 D}{2ns}\right] - \frac{p_v w_0 D}{xw_1(1-a)} -$$
$$\frac{K_f D}{xyw_1(1-a)} - \frac{D}{xw_1(1-a)}\left\{\alpha T_f + \frac{\alpha}{\lambda}\left[\ln\left(1+\beta e^{-\lambda T_f}\right) - \ln(1+\beta)\right]\right\}\left(c_f x + m_f(1-x)\right)$$
$$+ \frac{xyw_1(1-a)I_e s}{2} + DsI_e\left(M - \frac{xyw_1(1-a)}{D}\right) \quad - (6)$$

The optimal order quantity is,

$$y^* = \sqrt{\frac{2D\left[K_r + K_p + nK_s + K_f\right]}{x^2w_1^2\left\{h_r\left[(1-a)^2 - \frac{D(n-1)(1-a)}{ns}\right] + h_p\left[\frac{D}{R}\right] + h_s\left[\frac{2D}{s} - \frac{D(n-1)(1-a)}{ns}\right] + (1-a)^2 I_e s\right\}}}$$

Results and Discussion

The example makes use of the following parameters: *D*= 250 kg/week; *R*= 300 kg/week; w_0= 8.5 kg; w_1= 30 kg; K_r= 2500 ZAR; h_r= 1 ZAR/kg/week; p_r= 50 ZAR/kg; K_p= 25000 ZAR; h_p= 0.5 ZAR/kg/week; p_p= 30 ZAR/kg; K_s= 200 ZAR; h_s= 0.5 ZAR/kg/week; p_q= 20 ZAR/kg; *l*= 0.5 ZAR/kg; *s*= 1000 kg/week; K_f= 30000 ZAR; p_f= 15 ZAR/kg; c_f= 1 ZAR/kg/week; m_f= 2 ZAR/ kg/week; p_v= 10 ZAR/kg; α= 51 kg; β= 5; λ= 0.12/week. *x*=0.9, *a* = 0.04.

Considering trade credit under two cases

Case (i) M = 0.15 years, Let =0.05 and =0.08 using this parameter the optimal order quantity is y^* = 114 and total profit per unit time is TPU_{sc} = 2419 ZAR/week.

Case (ii) M = 0.3 years, Let =0.05 and =0.08 using this parameter the optimal order quantity is y^* = 114 and total profit per unit time is TPU_{sc} = 2789 ZAR/week.

Opting for an integrated policy is favourable to all supply chain members. When the proposed model is using the trade credit financing. An increased trade credit period can lead to higher order quantities as it will lead to maximize the overall profit for the supply chain system. By utilizing trade credit financing, companies improving their working capital and reduce the pressure on cash flow. And also in this study witnessing that to increase the credit period it would increase the profit TPU_{sc} = 2789 ZAR/week of the supply chain. This research work combined the study of four echelons inventory model for growing items under trade credit financing.

Conclusion

This proposed inventory model concentrates four echelons inventory model under trade credit financing. Using this technique, the seller can retain their retailer for their products. The main objective is to maximize the overall profit of the supply chain from the numerical it's clear that as the permissible period (M) increases the total profit per unit time is also increases.

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