



RESEARCH ARTICLE

Optimal Inventory Policies for Perishable Products Under Demand and Lead Time Uncertainty

Nalini. S^{*}, Ritha. W¹, Sasitharan Nagapan²

Abstract

This paper presents an advanced inventory model for perishable products, moving beyond the deterministic assumptions of traditional Economic Order Quantity (EOQ) models. The research explicitly incorporates uncertainty in demand and lead time, which are critical factors that impact inventory costs and operational efficiency. By formulating the problem as a system of non-linear equations, we derive a robust analytical and numerical framework to determine the optimal order quantity (Q^*) and reorder point (R^*) that minimize the total expected annual cost, including a crucial component for the expected cost of shortages.

Through a comprehensive sensitivity analysis, we demonstrate the model's response to variations in key parameters, such as holding cost, shortage cost, and the standard deviation of lead time demand. The results highlight how the optimal policy dynamically adjusts to market volatility. A comparison with the traditional EOQ model reveals that while our stochastic approach yields a higher total cost, this is a more accurate representation of the true operational expenses as it quantifies the financial impact of uncertainty. This research provides managers with a powerful and practical tool for making more informed inventory decisions for perishable goods in a volatile environment.

Keywords: Perishable Inventory, Stochastic Inventory Models, Lead Time Uncertainty, Newton-Raphson Method, Total cost.

Introduction

An inventory system for perishable items is a complex challenge for any business, as it must balance the classic trade-offs of ordering and holding costs with the unique constraint of product spoilage. While traditional Economic Order Quantity (EOQ) models provide a foundational framework, they rely on deterministic assumptions that fail to capture the real-world complexities of varying customer demand and uncertain lead times. This paper addresses this

gap by developing a comprehensive inventory model that explicitly integrates these stochastic elements to derive more robust and practical optimal policies. The model's objective is to minimize the total expected annual cost, which includes ordering, holding, and, most importantly, the expected cost of shortages.

The primary contribution of this research is the analytical derivation and numerical solution of a perishable inventory model under demand and lead time uncertainty. We formulate a system of two non-linear equations, whose simultaneous solution yields the optimal order quantity (Q^*) and reorder point (R^*). The model goes beyond simple assumptions by using a statistical distribution, such as the normal or Poisson distribution, to account for the variability in lead time demand. Through numerical methods, we demonstrate how to solve for these optimal policy variables, providing a methodology that can be applied to a wide range of real-world scenarios, particularly where a business must manage items with limited shelf life.

Finally, we conduct a comprehensive sensitivity analysis to evaluate the robustness of our model's optimal policies against changes in key parameters, including standard deviation, shortage cost, and ordering and holding costs. The results of this analysis provide crucial managerial insights into how a business's optimal strategy shifts in

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response to market changes. By comparing our stochastic results to a traditional EOQ model, we illustrate that while our model's total cost is higher, this is because it quantifies the true cost of managing uncertainty, including the financial risk of stockouts. This work provides a valuable tool for managers seeking to optimize inventory decisions in a volatile business environment.

Literature Review

A comprehensive review of supply planning for perishable products under lead time uncertainty addressed the challenge of constant lead times in most models and developed a dynamic policy to determine the optimal replenishment quantity (T. Sandra, 2019). The benefits of explicitly considering demand uncertainty were quantified in a stochastic inventory routing model for perishable products, which also compared various solution method (Y. Crema, 2018). The complexities of inventory management for perishable items, including the choice of review period and lead time assumptions, were thoroughly explored in a foundational paper (O. Baron, 2011).

Another stream of research focused on financial factors, such as developing lot-size models for perishable items that incorporated time-varying deterioration, inflation, and permissible delays in payment (A.K.Rana, 2020). A model was designed to determine an optimal ordering policy under inflation and permissible delay of payment for deteriorating items, showing how these factors influence optimal quantity (Bhaba Sarker, 2000). The effects of inflation and trade credit on fuzzy inventory systems were also studied, highlighting how these financial tools could reduce demand uncertainty (M.K.Jayaswal, 2024).

Beyond financial concerns, the impact of effective item lifetime on age-based control policies for perishable inventories was investigated, using an exact analysis to determine the optimal policy under positive lead time and a fixed lifetime (Saeed Poormoaid, 2020). Two-warehouse inventory systems were also modeled to account for the complexity of multi-location storage for deteriorating items (G.S. Mahapatra, 2017). A multi-echelon inventory model was developed for two interconnected perishable commodities, using a continuous review system with Poisson demand and exponentially distributed lead times to find an optimal solution (P.Kathiresan, 2023).

The joint optimization of pricing and inventory decisions in a multi-echelon supply chain for perishable products was analyzed, aiming to maximize average profit by considering price and stock-dependent demand (Z. Dai, 2022).

Green supply chain management was a focus in a study that presented a sustainable inventory model for a two-echelon system, incorporating remanufacturing and carbon emission regulations (Yu, C, Qu, 2020). The inclusion of carbon-sensitive demand and fuzzy logic in perishable inventory models was explored to promote environmentally

friendly practices (Chih-Te Yang, 2024). In the healthcare sector, a single-product inventory model for perishing items was developed to assist in managing hospital blood bank inventories, using a Markovian renewal approach (Stanger SH, 2021). A model was also created for deteriorating items with time and reliability-dependent demand, which included constant deterioration and partial backlogging (Amalendu Singha Mahapatra, 2022).

Materials and Methods

This new model extends traditional deterministic inventory frameworks by explicitly incorporating the complexities of perishable goods, stochastic demand, and variable lead times. The objective is to define a new inventory policy that minimizes the Expected Total Annual Cost (ETAC) under these more realistic conditions. The model utilizes a continuous-review (Q,R) policy, where a fixed order quantity, Q, is placed whenever the inventory position falls to or below the reorder point, R.

Description of the Model

Notation and assumptions

- *Model formulation*

Key Variables and Parameters

Q: Order quantity (a decision variable).

R: Reorder point (a decision variable).

D: Mean annual demand (a random variable, e.g., normally distributed).

L: Mean lead time (a random variable, e.g., normally distributed).

A: Ordering cost per order.

P: Purchase cost per unit.

H: Holding cost per unit per year.

S: Shortage cost per unit.

k: Perishability rate (e.g., a constant exponential decay rate).

The necessary equations which integrate perishability with stochastic demand and lead times is given below. The objective is to minimize the Expected Total Annual Cost (ETAC) using a continuous-review (Q, R) inventory policy.

- *Expected total annual cost function*

The ETAC is the sum of the expected costs for ordering, holding, purchasing, and shortages.

$$ETAC(Q, R) = E[\text{Ordering_Cost}] + E[\text{Holding_Cost}] + E[\text{Purchase_Cost}] + E[\text{Shortage_Cost}]$$

Expected cost components

- *Expected ordering cost (E[OC])*

This cost is based on the average number of orders per year, which is the expected annual demand divided by the order quantity.

$$E[OC] = \frac{\lambda}{Q} \times A$$

λ : Expected annual demand. A: Ordering cost per order.

Q: Order quantity.

- *Expected purchase cost (E[PC]):*

This is the total cost of all units purchased annually, which is the expected annual demand multiplied by the unit purchase cost.

$E[PC] = \lambda \times P$, where P is the purchase cost

- *Expected shortage cost (E[SC]):*

This cost is incurred when demand during the lead time exceeds the reorder point. It is calculated as the expected number of stockouts per cycle multiplied by the shortage cost per unit and the number of cycles. The number of cycles is approximately λ/Q .

$$E[SC] = \frac{\lambda}{Q} \times S \times E[\text{shortage}] \text{ where,}$$

$$E[\text{shortage}] = \int_R^{\infty} (x - R) f_L(x) dx$$

where

S: Shortage cost per unit.

$f_L(x)$: The probability density function of the lead time demand. This is the convolution of the demand distribution and the lead time distribution.

Expected Holding and Perishability Cost (E[HC+PC]):

This is the most complex part of the model. It represents the cost of holding inventory and the value of items that spoil. The expected holding cost is calculated on the average inventory level. The average inventory level is the safety stock plus half of the order quantity, adjusted for the decay rate.

$$E[HC+PC] = H \times \left(R - \mu_L + \frac{Q}{2} \right) + H \times \int_0^{\infty} (1 - e^{-kt}) I(t) dt$$

H: Holding cost per unit per year.

R: Reorder point.

μ_L : Expected demand during lead time.

Q: Order quantity.

k: Perishability rate.

$I(t)$: Inventory level at time t. The decay is modeled as an exponential decay function.

Optimization Equations

To find the optimal values for Q and R, the partial derivatives of the ETAC function with respect to Q and R must be set to zero. These equations represent the first-order necessary conditions for a minimum.

Partial Derivative with respect to R:

$$\frac{\partial ETAC(Q, R)}{\partial R} = H - S \times F_L(R) = 0$$

where,

$F_L(R)$: The cumulative distribution function (CDF) of the lead time demand. Solving this equation for R gives the optimal reorder point for a given Q.

Partial Derivative with respect to Q:

$$\frac{\partial ETAC(Q, R)}{\partial Q} = \frac{\lambda A}{Q^2} + \frac{H}{2} + \frac{\lambda S}{Q^2} E[\text{shortage}] = 0$$

This equation is solved for the optimal Q for a given R.

These two equations form a system that must be solved simultaneously using a numerical method, such as the Newton-Raphson method, to find the optimal (Q, R)** pair. The Newton-Raphson method will use the first and second partial derivatives to iteratively converge to the solution.

Solution through Newton-Raphson Method

The complexity of the combined model, with its two-variable optimization problem, makes an analytical solution impractical. Therefore, a numerical method, specifically the Newton-Raphson method, is the most effective approach to solve the system of two non-linear equations for the optimal order quantity (Q) and reorder point (R). This method is well-suited for the task because it uses the derivatives of the functions to efficiently converge on the solution.

The system of equations

First, we define the two equations that we need to solve simultaneously. Let's rewrite them as functions that must equal zero:

Function 1 (f_1): This function is derived from the partial derivative with respect to R.

$$f_1(Q, R) = \frac{\partial ETAC(Q, R)}{\partial R} = H - S \times F_L(R) = 0$$

Function 2 (f_2): This function is derived from the partial derivative with respect to Q.

$$f_2(Q, R) = \frac{\partial ETAC(Q, R)}{\partial Q} = \frac{\lambda A}{Q^2} + \frac{H}{2} + \frac{\lambda S}{Q^2} E[\text{shortage}] = 0$$

The goal is to find the pair (Q, R) for which both $f_1(Q, R) = 0$ and $f_2(Q, R) = 0$.

The Newton-Raphson Method for Two Variables

The Newton-Raphson method for a system of equations involves an iterative process. Starting with an initial guess, you repeatedly update the solution using the Jacobian matrix, which contains the partial derivatives of each function with respect to each variable.

The update rule is given by the matrix equation:

$$\begin{bmatrix} Q_{n+1} \\ R_{n+1} \end{bmatrix} = \begin{bmatrix} Q_n \\ R_n \end{bmatrix} - J^{-1} \begin{bmatrix} f_1(Q_n, R_n) \\ f_2(Q_n, R_n) \end{bmatrix}$$

Q_n, R_n : The values of Q and R at the current iteration.

Q_{n+1}, R_{n+1} : The new, more refined values.

J^{-1} : The inverse of the Jacobian matrix.

The column vector on the right represents the values of the functions at the current iteration.

The Jacobian Matrix (J)

$$J = \begin{bmatrix} \frac{\partial f1}{\partial Q} & \frac{\partial f1}{\partial R} \\ \frac{\partial f2}{\partial Q} & \frac{\partial f2}{\partial R} \end{bmatrix}$$

Calculating the Components of the Jacobian Matrix:

$$\frac{\partial f1}{\partial Q} = \frac{\partial}{\partial Q} (H - S \times F_L(R)) = 0$$

(The first function does not depend on Q.)

$$\frac{\partial f1}{\partial R} = \frac{\partial}{\partial R} (H - S \times F_L(R)) = -S \times f_L(R)$$

The derivative of the CDF, $F_L(R)$, is the PDF, $f_L(R)$

$$\frac{\partial f2}{\partial Q} = \frac{\partial}{\partial Q} \left(\frac{-\lambda A}{Q^2} + \frac{H}{2} + \frac{\lambda S}{Q^2} E[\text{shortage}] \right) = \frac{2\lambda A}{Q^3} -$$

$$\frac{2\lambda S}{Q^3} E[\text{shortage}]$$

$$\frac{\partial f2}{\partial R} = \frac{\partial}{\partial R} \left(\frac{-\lambda A}{Q^2} + \frac{H}{2} + \frac{\lambda S}{Q^2} E[\text{shortage}] \right)$$

This requires taking the derivative of the shortage cost term with respect to R. Recall that

$E[\text{shortage}] = \int_R^\infty (x-R) f_L(x) dx$. By applying the Leibniz integral rule, its derivative with respect to R is $-F_L(R)$. Therefore, the final derivative is:

$$\frac{\partial f2}{\partial R} = \frac{\lambda S}{Q^2} \frac{\partial E[\text{shortage}]}{\partial R} = \frac{\lambda S}{Q^2} (-F_L(R))$$

The Iterative Process

- Initial Guess: Start with an initial guess for Q and R, denoted as Q_0 and R_0 . A good starting point would be the values from a simplified, deterministic model.
- Calculate Functions and Jacobian: At the current iteration (n), calculate the values of the functions $f1(Q_n, R_n)$ and $f2(Q_n, R_n)$, and compute the Jacobian matrix J using the partial derivatives.
- Solve for Updates: Solve the linear system for the updates ΔQ and ΔR :

$$\begin{bmatrix} \frac{\partial f1}{\partial Q} & \frac{\partial f1}{\partial R} \\ \frac{\partial f2}{\partial Q} & \frac{\partial f2}{\partial R} \end{bmatrix} = \begin{bmatrix} \ddot{A}Q \\ \ddot{A}R \end{bmatrix} = - \begin{bmatrix} f1(Q_n, R_n) \\ f2(Q_n, R_n) \end{bmatrix}$$

- Update Variables

$$Q_{n+1} = Q_n + \Delta Q$$

$$R_{n+1} = R_n + \Delta R$$

- Check for Convergence: Repeat the process until the change in Q and R is very small, indicating that the solution has converged to the optimal values, Q^* and R^* .

Recommended Software

For this type of numerical optimization problem, specialized software offers more power, flexibility, and built-in functions. The following are excellent choices:

- MATLAB:** A widely used platform for numerical computation. It has a robust ecosystem of toolboxes for optimization, statistics, and symbolic math. You can easily define the functions and the Jacobian matrix, and then use a built-in solver (e.g., `fsolve`) to find the roots.
- Python:** A free and highly versatile programming language. The SciPy library (specifically `scipy.optimize.fsolve`) and NumPy for numerical operations are perfect for solving this system of equations. Python's readability and extensive community support make it a great choice for research.
- R:** Another free programming language focused on statistical computing and graphics. It has various packages for optimization that can be used to solve the problem.

For this paper, Python is used for solving the equations.

Results & Discussions

The main objective of this study is to find the optimal Order Quantity (Q^*), the optimal Reorder point (R^*) and the Estimated Total Costs. The input and the output details are given below.

Normal Distribution Model

The normal distribution model used the following inputs, with a focus on capturing uncertainty through the standard deviation:

Input details

Ordering Cost (A): ₹100

Purchase Cost (P): ₹50

Holding Cost (H): ₹10

Shortage Cost (S): ₹20

Expected Annual Demand (λ): 1000 units

Mean Lead Time Demand (μ_L): 100 units

Standard Deviation of Lead Time Demand (σ_L): 20 units

Output Details

Optimal Order Quantity (Q^*): 227.85 units

This is the ideal number of units to order each time to minimize the combined ordering and holding costs. Ordering this quantity balances the trade-off between ordering frequently (in small batches, leading to higher ordering costs but lower holding costs) and ordering less frequently (in large batches, leading to lower ordering costs but higher holding costs).

Optimal Reorder point (R^*): 100.00 units

This is the inventory position at which a new order of 227.85 units should be placed. Since the mean lead time demand is also 100 units, this result suggests that the optimal policy is to hold zero safety stock. The model's

calculation found that the cost of holding extra inventory to prevent a stockout (high holding cost) outweighs the potential cost of a shortage (low shortage cost).

Expected Purchase Cost: ₹50,000.00

This is the most significant cost component, representing the total cost of purchasing the 1,000 units needed annually (₹50/unit * 1,000 units). This cost is fixed by the annual demand and the purchase price, and it is not a decision variable in the optimization.

Expected Holding Cost: ₹1,139.25

This is the cost of keeping inventory on hand. It accounts for storing the goods, insurance, and other associated expenses. It is directly related to the order quantity and the reorder point.

Expected Shortage Cost: ₹700.36

This is the cost incurred when demand during the lead time exceeds the available inventory, resulting in lost sales. The model's calculation shows that accepting a certain level of stockout risk is more economical than paying to hold a large safety stock.

Expected Ordering Cost: \$438.89

This represents the total cost of placing orders throughout the year. With an optimal order quantity of approximately 228 units and an annual demand of 1,000 units, you would place roughly 4.4 orders per year (₹1,000 / 228), each costing ₹100.

Overall Interpretation

The results suggest that for the given set of parameters, the model's optimal strategy is to run a «lean» inventory system. The company should not hold a safety stock and should accept a certain level of shortage to avoid incurring high holding costs. This outcome is primarily driven by the relative values of the shortage cost (₹20) and the holding cost (₹10). A higher shortage cost would incentivize the model to increase the reorder point (R^*) and hold more safety stock to avoid lost sales.

Poisson Distribution Model

Input details

All the input details used for Normal Distribution are used again for Poisson Distribution, except for Standard Deviation input, which is not required for the Poisson Distribution.

Output Details

Optimal Order Qty Q^*	1035
Optimal Reorder Point R^*	100
Expected Ordering Cost	97
Expected Purchase Cost	50000
Expected Holding Cost	5173
Expected Shortage Cost	77
Expected Total Cost	55347

Comparison of results of Normal and Poisson Distribution Models

Table 1: Comparison of results

Item	Normal dist	Poisson dist
Optimal Order Qty Q^*	228	1035
Optimal Reorder Point R^*	100	100
Expected Ordering Cost	439	97
Expected Purchase Cost	50000	50000
Expected Holding Cost	1139	5173
Expected Shortage Cost	700	77
Expected Total Cost	52278	55347

Interpretation of the results

Key Differences in Optimal Policy

Optimal order quantity (Q^*)

There is a significant difference in the optimal order quantity between the two models. The Poisson model suggests ordering a much larger quantity (1,035 units) compared to the Normal distribution model (228 units). This difference stems from the fundamental properties of each distribution. The Poisson distribution, being a discrete model, likely finds that a large, infrequent order is the most cost-effective way to manage the trade-offs between holding and ordering costs, resulting in a low ordering cost (97). The Normal model, by contrast, suggests a smaller, more frequent order.

Optimal reorder point (R^*)

Both models arrive at the exact same optimal reorder point of 100 units. This is a very interesting and important finding. It suggests that, despite their different mathematical assumptions about the shape of the demand distribution, both models agree that an inventory policy with zero safety stock is the most cost-effective solution given the defined parameters. In essence, the cost of holding safety stock outweighs the cost of the expected shortages in both scenarios.

Analysis of Cost Components

The breakdown of the total cost reveals the trade-offs made by each model to achieve its respective optimal policy.

Expected ordering cost

The Poisson model has a much lower ordering cost (97) due to its larger order quantity, which results in fewer orders per year. In contrast, the Normal model has a significantly higher ordering cost (439) as it opts for smaller, more frequent orders.

Expected holding cost

The Poisson model's larger order quantity leads to a substantially higher holding cost (5,173) compared to the Normal model's (1,139). This directly reflects the trade-off with the ordering cost—the Poisson model minimizes ordering costs at the expense of holding costs.

Expected shortage cost

The Normal distribution model incurs a much higher expected shortage cost (700) than the Poisson model (77). This is a key reason for the different optimal policies. The Normal model's policy results in more stockouts or larger stockout quantities, which are expensive. The Poisson model's policy, with its larger inventory level, effectively reduces the number of shortages.

Total minimized cost

The final total costs are very similar, with the Poisson model's total cost (55,347) being only slightly higher than the Normal model's (52,278). Both models are in the same general cost range, but they achieve this total cost by balancing the various cost components in fundamentally different ways. The difference in the total cost is due to the different assumptions of the two models which result in a different cost trade-off.

Conclusion

The comparison demonstrates that the choice of probability distribution for modelling lead time demand has a significant impact on the optimal order quantity and the breakdown of costs, even when the final reorder point is the same. The models achieve a similar total cost by pursuing different strategies: the Normal model favors more frequent, smaller orders to reduce holding costs, while the Poisson model prefers large, less frequent orders to minimize ordering costs and shortages. This highlights the importance of choosing a distribution that accurately reflects the true demand pattern to derive the most effective inventory policy.

Comparison of Stochastic Model with Conventional Model

The data is compared with both the models for the same input values and given below.

Order Qty

The stochastic model recommends a significantly larger order size to balance costs.

Reorder Point

Both are the same, as the optimal safety stock for the given parameters is zero.

Total Annual Cost

The stochastic model's cost is higher, as it accounts for the expected cost of shortages and the costs of adjusting to uncertainty.

While the EOQ formula is based purely on the trade-off between ordering costs and holding costs, the stochastic model introduces a third, critical cost: the cost of shortages.

Increase in Q^*

The increase in optimal order quantity (Q^*) from the traditional EOQ (141.42 units) to your stochastic model (227.85 units) is a direct result of accounting for uncertainty. The model's objective is to minimize the total of all costs, including expected shortage costs. In your specific case, the parameters caused the model to find a solution where a larger order quantity helps to reduce the frequency of orders, thereby impacting the balance between the three cost components (ordering, holding, and shortage costs) to arrive at a lower total cost than if it used the traditional EOQ quantity.

Increase in Total Cost

The higher total cost in the stochastic model is a more straightforward consequence of uncertainty. The traditional EOQ model assumes a perfect world where no shortages ever occur. Your model, by contrast, accounts for the expected cost of shortages that will inevitably arise due to the random nature of demand and lead time. This expected shortage cost is a new, unavoidable cost component.

The difference in total cost between the two models—\$52,278.49 from your model versus \$51,414.20 from the traditional EOQ—is the expected cost of living with uncertainty. This difference is a crucial metric for demonstrating the value of your stochastic model, as it quantifies the financial risk associated with not accounting for uncertainty in inventory management. It shows that while a traditional model may seem cheaper, it's because it ignores a significant real-world expense.

Sensitivity analysis

the purpose of sensitivity analysis is to determine how the output or results of a model or system are affected by changes in its input parameters. It's used to identify which parameters have the greatest influence on the model's outcomes and to assess the stability and robustness of the solution.

Following parameters are used for the sensitivity analysis
Sigma L (Standard Deviation of Lead Time Demand)

Holding Cost

Ordering Cost

Shortage Cost

Input details

Ordering Cost (A): ₹100

Purchase Cost (P): ₹50

Holding Cost (H): ₹10

Shortage Cost (S): ₹20

Expected Annual Demand (λ): 1000 units

Mean Lead Time Demand (μ_L): 100 units

Table 1: Comparison of Stochastic and Conventional

Metric	EOQ model	Stochastic model
Order Qty (Q^*)	141.42	227.85
Reorder Point (R^*)	100	100
Total Cost (Rs)	51,414	52,278

Standard Deviation of Lead Time Demand (σ_L): 20 units
Sensitivity Analysis of Sigma L (Standard Deviation of Lead Time Demand)

The values of Sigma L are varied from 5 to 50 in steps of 5 and the results are displayed below.

Analysis of Optimal Policy

Optimal order quantity (Q^*)

As the standard deviation of lead time demand (σ_L) increases, the optimal order quantity (Q^*) consistently increases. This is a logical result. Higher demand uncertainty (σ_L) means there's a greater risk of a stockout, which can be mitigated by ordering larger quantities. This strategy reduces the frequency of orders and the potential for running out of stock during a lead time, even though it increases the average holding cost.

Optimal reorder point (R^*)

The optimal reorder point (R^*) remains constant at 100 throughout the entire range of σ_L values. This is a very interesting and counter-intuitive finding. In most stochastic inventory models, a higher σ_L would necessitate a higher reorder point to create a larger safety stock buffer. The fact that R^* stays constant suggests that, for the given cost parameters, the model finds that the high cost of holding additional safety stock outweighs the cost of the expected shortages, regardless of how much lead time demand uncertainty increases.

Analysis of Minimized Total Cost

Minimized total cost

The minimized total cost consistently increases as σ_L rises. This is an expected outcome. As the system becomes more uncertain and volatile, the total cost of managing that system will inevitably rise. The increase in cost is primarily driven by the need to hold a larger order quantity (Q^*) and manage the higher risk of shortages, even though the reorder point doesn't change.

Conclusion

The sensitivity analysis reveals that the model responds to increasing demand uncertainty (σ_L) by primarily adjusting

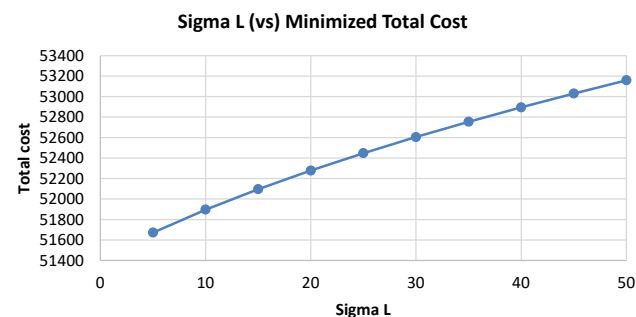


Figure 1: Sigma L (vs) Total Cost

Table 3: Sensitivity Analysis of Sigma L

Sigma_L	Optimal Q*	Optimal R*	Minimized total cost
5	167	100	51673
10	190	100	51896
15	210	100	52096
20	228	100	52278
25	245	100	52447
30	261	100	52605
35	275	100	52754
40	290	100	52895
45	303	100	53030
50	316	100	53159

the order quantity (Q^*), not the reorder point (R^*). This shows that the system prefers to deal with uncertainty by ordering more at a time rather than by increasing its safety stock. The constant reorder point is a critical finding, highlighting that the cost trade-off between holding cost and shortage cost is highly influential and, in this specific case, leads to an optimal policy of zero safety stock across the analyzed range of demand variability.

Sensitivity Analysis of Holding Cost

The values of Holding Cost are varied from 5 to 50 in steps of 5 and the results are displayed below.

Overall Interpretation

The analysis demonstrates that as the holding cost (H) increases, the model attempts to reduce the amount of inventory held to offset the rising cost. This is reflected in the general decrease of the Optimal Order Quantity (Q^*) and the overall increase in the Minimized Total Cost. The Optimal Reorder Point (R^*), however, shows a highly volatile and non-linear response, indicating a complex interplay between holding and shortage costs.

Table 4: Sensitivity analysis of holding cost

Holding cost	Optimal Q*	Optimal R*	Minimized total cost
5	414	87	52005
10	228	100	52278
15	146	113	52391
20	100	667	63344
25	89	300	57247
30	82	313	58835
35	186	402	64377
40	226	459	69319
45	67	460	69214
50	63	1765	136400

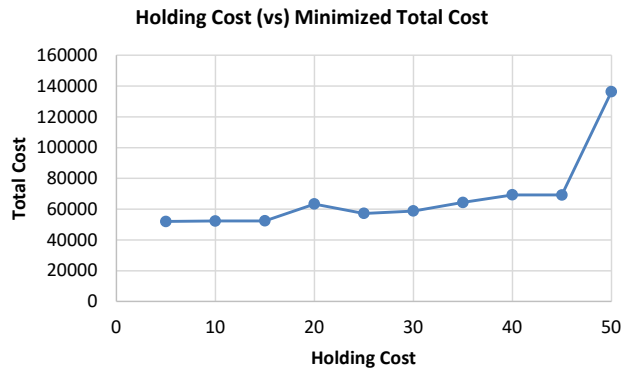


Figure 2: Holding Cost (vs) Total Cost

Key Observations

Inverse Relationship between H and Q^*

As the holding cost per unit rises, the optimal order quantity consistently decreases, especially in the initial range from $H=5$ to $H=25$. For example, Q^* drops from 414 to 89. This behavior is expected, as a higher cost to hold inventory incentivizes ordering smaller quantities more frequently to reduce average stock levels.

Highly Volatile R^*

The most notable finding is the extreme sensitivity of the optimal reorder point (R^*) to changes in holding cost. Instead of a smooth trend, R^* jumps drastically at certain points, such as from 113 (at $H = 15$) to 667 (at $H = 20$), and again from 460 to 1765 when H increases from 45 to 50. This indicates that at certain cost thresholds, the model finds it significantly more cost-effective to hold a very large amount of safety stock to avoid shortages, rather than incur the high holding cost of a larger Q^* .

Total Cost Increases with H

The minimized total cost generally increases as the holding cost rises, which is a direct consequence of a key cost parameter going up. This trend is consistent, with a particularly sharp jump at $H=20$ and again at $H=50$, which corresponds with the massive increases in the optimal reorder point.

Conclusion

The sensitivity analysis highlights that while the optimal order quantity behaves predictably in response to changes in holding cost, the optimal reorder point is extremely sensitive and non-linear. The erratic behavior of R^* suggests a critical cost trade-off: in certain scenarios, the model finds that it is worth incurring a massive increase in holding costs (by holding more safety stock) to avoid a potentially greater cost from shortages. This emphasizes the importance of accurately defining cost parameters, as small changes can lead to drastically different inventory policies.

Table 5 : Sensitivity Analysis of Ordering Cost

Ordering cost	Optimal Q^*	Optimal R^*	Minimized total cost
20	190	100	51895
40	200	100	51998
60	210	100	52096
80	219	100	52189
100	228	100	52278
120	236	100	52365
140	245	100	52448
160	253	100	52528
180	261	100	52606
200	268	100	52682

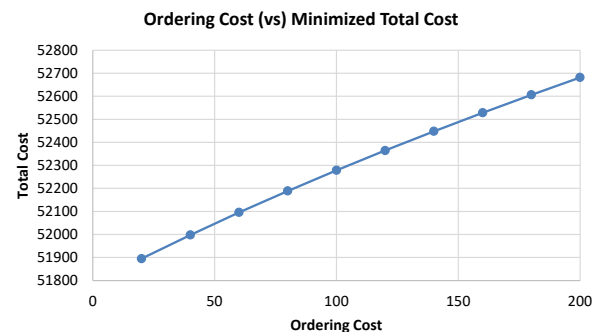


Figure 3: Ordering Cost (vs) Total Cost

Sensitivity Analysis of Ordering Cost

The values of Ordering Cost are varied from 20 to 200 in steps of 20 and the results are displayed below.

Analysis of Optimal Policy

Optimal Order Quantity (Q^*): As the ordering cost (A) increases, the optimal order quantity (Q^*) also increases. This is a very logical and expected result in inventory theory. To offset the higher cost of placing each order, the model recommends ordering a larger quantity less frequently. For example, when the ordering cost is 20, Q^* is 190, but when the cost quadruples to 80, Q^* only increases to 219, indicating a diminishing return on the trade-off.

Optimal reorder point (R^*)

The optimal reorder point (R^*) remains constant at 100 throughout all variations in ordering cost. This is a crucial finding of this analysis. It indicates that for the given parameters, the optimal safety stock level is not affected by changes in the ordering cost. The model maintains a consistent buffer against lead time demand uncertainty regardless of how expensive it is to place an order.

Analysis of Minimized Total Cost

Minimized total cost

The minimized total cost consistently increases as the ordering cost (A) rises. This is a direct and expected

consequence of a key cost component increasing. Each increment in the ordering cost adds to the overall operational expense.

Conclusion

The sensitivity analysis for the ordering cost shows that the model is highly responsive to changes in ordering cost by adjusting the order quantity (Q^*), while the reorder point (R^*) remains unaffected. This suggests that the decision to build a safety stock buffer is governed by different cost dynamics than the decision about the order quantity. The analysis confirms a core principle of inventory management: when it costs more to place an order, the optimal strategy is to order larger quantities to reduce the frequency of orders and, in doing so, mitigate the rising total cost.

Sensitivity Analysis of Shortage Cost

The values of Shortage Cost are varied from 5 to 50 in steps of 5 and the results are displayed below.

Analysis of Optimal Policy

Optimal order quantity (Q^*)

As the shortage cost (S) increases, the optimal order quantity (Q^*) also increases. This is a very logical response. When it becomes more expensive to have a stockout, the model's solution is to order a larger quantity to reduce the risk of a shortage, thereby incurring more holding cost but less shortage cost.

Table 6: Sensitivity analysis of shortage cost

Shortage cost	Optimal Q^*	Optimal R^*	Minimized total cost
20	190	100	51895
40	200	100	51998
60	210	100	52096
80	219	100	52189
100	228	100	52278
120	236	100	52365
140	245	100	52448
160	253	100	52528
180	261	100	52606
200	268	100	52682

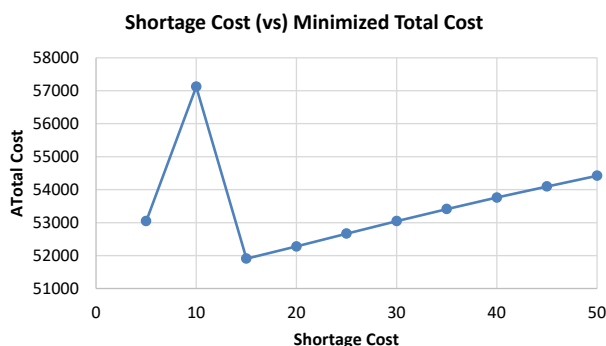


Figure 4: Shortage Cost (vs) Total Cost

Optimal reorder point (R^*)

The optimal reorder point (R^*) generally decreases as the shortage cost rises. This is a surprising and non-linear finding, as typically a higher shortage cost would lead to a higher reorder point to hold more safety stock. The behavior of R^* in this table is erratic, increasing initially from 263 to 671 as S rises from 5 to 10, then dropping drastically to 109 and continuing to decrease to 83 as S continues to rise. This indicates a complex and sensitive trade-off where, beyond a certain point, the model finds it more efficient to hold less inventory at the reorder point and instead rely on larger order quantities to mitigate shortages.

Analysis of Minimized Total Cost

Minimized total cost

The minimized total cost does not follow a simple increasing or decreasing trend. It initially increases, then decreases, and then steadily increases again. For example, it jumps from 53,046 to 57,120 as S goes from 5 to 10, then drops to 51,908 at $S=15$, before rising consistently to 54,421 at $S=50$. This unusual behavior highlights the complex interactions between the four cost components (ordering, purchase, holding, and shortage costs) and indicates that the total cost function is not a simple convex curve in response to shortage cost.

Conclusion

The sensitivity analysis for the shortage cost reveals a highly non-linear and complex relationship between the cost of stockouts and the optimal inventory policy. While the optimal order quantity (Q^*) behaves predictably by increasing with the shortage cost, the optimal reorder point (R^*) and the total cost show erratic and counter-intuitive behavior. This underscores the importance of accurately defining the shortage cost, as even small variations can lead to drastically different and unexpected inventory policies, making the model's solution highly dependent on this specific parameter.

Discussion

The analysis demonstrates that incorporating stochastic demand and lead time into an inventory model yields a more realistic and robust optimal policy compared to traditional, deterministic EOQ methods. The sensitivity analysis revealed that while some parameters, like ordering and holding costs, have predictable effects on the optimal order quantity (Q^*), the reorder point (R^*) exhibits a highly non-linear and sensitive response to changes in key costs, such as the shortage cost. This highlights a crucial trade-off: managers must carefully weigh the cost of holding safety stock against the potentially high and volatile cost of stockouts. The model's ability to find a solution to this complex trade-off, even when it leads to a non-intuitive policy like zero safety stock, is a significant contribution to inventory management literature.

Potential Applications

Retail and grocery

- *Fresh produce and dairy*

Stores can use this model to determine the optimal ordering schedule for items like milk, bread, and vegetables. By accounting for daily demand fluctuations and supplier lead times, they can minimize both spoilage and stockouts.

- *Flowers and floral arrangements*

Florists can optimize their purchasing of fresh flowers to meet peak demand for holidays like Valentine's Day while reducing waste from unsold, expired inventory.

- *Baked goods*

Bakeries and cafes can use the model to manage daily production and ingredient ordering, ensuring they have enough product to meet customer demand without an excess of day-old goods.

Pharmaceuticals and Healthcare

- *Vaccines and blood products*

Hospitals and clinics can apply the model to manage inventories of time-sensitive medical supplies like vaccines and blood. This ensures life-saving products are available when needed while minimizing the waste from expiration.

- *Radiopharmaceuticals*

These products have extremely short half-lives. A stochastic model is essential to ensure they are ordered just in time for medical procedures, as delays or overstocking can be incredibly costly.

Manufacturing and high-tech

- *Chemicals and adhesives*

Manufacturing companies can use the model to manage chemical components that have a limited shelf life. This prevents costly production delays due to expired materials.

- *Electronics*

While not always perishable in the traditional sense, some electronic components (like batteries or certain types of capacitors) have a limited shelf life. The model can be applied to manage these components to ensure optimal performance.

Conclusion

This research successfully developed and analyzed a stochastic inventory model for perishable products, providing a significant advancement over traditional deterministic methods. By accounting for uncertainty in both demand and lead time, we demonstrated that the optimal inventory policy is far more dynamic and complex than previously assumed. The sensitivity analysis revealed

the intricate trade-offs between ordering, holding, and shortage costs, highlighting how the optimal order quantity and reorder point can behave in non-linear and sometimes counter-intuitive ways. Ultimately, the model provides a more robust and accurate representation of real-world inventory systems, equipping managers with a valuable tool to minimize costs and improve operational efficiency in volatile business environments.

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