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RESEARCH ARTICLE

Optimization of an Advanced Integrated Inventory Model Considering Shortages and Deterioration across Varying Demand Functions

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Abstract

To determine and emphasize the importance of Internet of Things (IoT)-enabled investment in an inventory model confronted with shortages, storage costs, and deterioration of goods, this study focuses on maximizing maximum stock level while minimizing overall inventory-related expenditures. Conventional inventory models frequently ignore the effect of digital evaluation on sustaining inventory levels and preventing deterioration, resulting in inefficient decision-making. An enhanced inventory model is offered, which uses internet of things (IoT) technology to track inventory factors in real time, hence lowering degradation, shortages and holding costs. To account for the influence of demand fluctuation, three distinct demand structures are investigated: (i) linear price and stock-dependent demand, (ii) a price function with a negative power of a constant, and (iii) an exponential function of price. These demand structures explain several competitive scenarios in which demand is influenced by costs and availability of inventory. To assess the efficacy of the developed IoT-based model, a comparative investigation is carried out under these three demand situations. Secondary data from Abu Hashan Md Mashud's research are used to support the numerical analysis. Results shows that the maximum inventory level per cycle for the Cases I, II and III are 188.584482, 402.584988, 303.434275 and the total costs for the Cases I, II and III are $1108.00326, $786.214411, $1373.11204 respectively. Amongst the three demand variations, the demand model that involves raising the price to a negative power of a constant outperforms the others, resulting in the highest optimum stock levels. The numerical research's findings reveal that IoT integration not only improves operational effectiveness, but also leads to a substantial rise in maximum stock level every cycle. The research's key innovation resides in its integration of IoT technology with inventory models in a variety of demand situations, an approach that has yet to be completely explored in the existing literature. The findings indicate that IoT-based inventory models are exceptionally successful at controlling stock, reducing degradation, and enhancing profitability, particularly when demand follows nonlinear patterns such as the negative power form.

Keywords: Inventory model, Demand patterns, Shortages, Deterioration, Inventory level, Internet of Things.

Introduction

Inventory management is essential for ensuring supply chain efficacy, adaptation, and sustainability in the present rapidly changing and highly competitive marketplace. Conventional inventory strategies that balance purchase and storage expenses can often be insufficient to address issues such as shortages, product deterioration, and shifting demand patterns. A evaluation was performed on inventory models optimized for controlling decaying products (Freddy Perez *et al*., 2020). With the fast development of technological advances, globalization, and unanticipated client behavior, organizations have to develop new inventory strategies intended for optimizing stock levels, reducing cost ineffectiveness, and increasing overall profitability. One of the most essential components of practical inventory models is decay, which is the gradual decline in product value or utility. This is especially crucial in industries that manage perishable products, high-value commodities with deteriorating quality, and time-critical merchandise. A survey was performed on solutions for inventory management for deteriorating products (Mahdi Karim, 2025). An EOQ model was developed that integrates carbon emissions and rising prices for degrading products

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of defective quality due to the learning effect (Osama Abdulaziz Alamri, *et al*., 2022). Neglecting deterioration often leads to undervalued costs, inaccurate demand forecasts, and wasted resource allocation. Several research studies have examined inventory models in various demand circumstances, such as shortages, price changes, and carbon emission constraints. A model for the best allocation in preservation technology was developed under fluctuating demand, combining trade credit and shortage situations (Mrudul Y. Jani *et al*., 2021). An EOQ-driven inventory framework was created that incorporates nonlinear costs of holding dependent on inventory level, nonlinear need impacted by stock availability, and trade credit concerns (Leopoldo Eduardo Cardenas-Barron *et al*., 2020). An EOQ-based degrading inventory structure has been built, accounting for various demand patterns and entirely backlogged shortages (Abu Hashan Md Mashud, 2020). However, little emphasis has been placed on using newly developed technologies such as the Internet of Things (IoT) to maximize stock levels while cutting prices in varied demand patterns. A literature assessment was undertaken on automatic inventory management systems that integrate IoT to improve stock maximization and save carrying costs (Friday Ugbebor *et al*., 2024).

IoT technology enables continuous tracking, analysis of trends, and automated inventory modifications. IoT adoption reduces holding and degrading costs by monitoring product qualities, warehouse conditions, and demand changes. A literature review was conducted on the effect of the Internet of Things (IoT) on the management of inventory (Yasaman Mashayekhy *et al*., 2022). The Internet of Things (IoT) was investigated in the retail industry as a means of connecting supply and demand (Felipe Caro *et al*., 2019). Despite its transformative potential, the role of IoT in maximizing the greatest quantity of inventory every cycle under diverse demand models has yet to be completely examined. An examination has been given on smart inventory management solutions that use the Internet of Things (Souvik Paul, *et al*., 2019). IoT provides small-scale stock tracking, allowing for real-time management of shortage costs, storage expenses, and degradation rates. Additionally, incorporating IoT into inventory structures allows stakeholders to maintain larger levels of inventory while lowering the risk of overproduction or waste, ensuring an equilibrium between availability and effectiveness. Incorporating IoT into inventory models enhances demand response. Real-time data collection provides valuable insights into client purchasing behavior, enabling businesses to adjust pricing strategies, replenishment plans, and stockholding policies in reaction to market developments. This is especially important in complex demand circumstances, such as price-based or inventory-dependent demand structures, where traditional models usually fail to represent dynamic variations. Businesses that combine choices regarding inventory with internet-of-things based intelligence can decrease operational risks while enhancing profitability. However, there is a major gap in the literature for examining IoT-driven inventory management models using different demand models. Prior research has mostly concentrated on single demand patterns or traditional technical solutions, ignoring the comparable impact of several demand-side features when paired with IoT.

To bridge this gap and demonstrate the unpredictable nature of market-driven consumer demand, the current work develops an IoT-enabled inventory model under three distinct demand structures: (i) Demand fluctuating linearly with price and stock accessibility; (ii) Demand is represented as a negative power function of price; and (iii) Demand is expressed as an exponentially increasing function of price. These demand forms represent a wide range of client responses to price and product availability in real-world circumstances. Under these demand situations, the proposed technique evaluates how continuous digital monitoring could optimize inventory levels while lowering deterioration, holding, and shortage-related costs by incorporating IoT technology into inventory. The results of this study show that, among the three demand types, the negative power function of price surpasses the linear and exponential demand models, resulting in higher maximum stock levels and more lucrative outcomes. This underscores the significance of demand patterns in assessing the impact of IoT-facilitated models, and it provides a new insight by combining variation in demand analysis with IoT use in inventory models. By carefully integrating IoT technologies into inventory frameworks with shortages and deterioration, this study contributes to the bridge between traditional inventory models and digitally improved supply chain management. The findings offer important recommendations to decision-makers and supervisors for using Internet of Things (IoT)-based solutions to optimize stock levels, reduce inefficiencies, and ensure long-term viability in unpredictable markets.

Methodology

A unified inventory model is developed that employs internet of things (IoT) technology to monitor inventory conditions in real time, reducing shortage costs, holding charges, and deterioration while raising optimal stock levels. To evaluate the model's efficiency under various market conditions, three demand patterns are considered: (i) demand varies linearly with price and availability of stocks (ii) demand expressed as a negative power function of price and (iii) demand defined by an exponential function of price. This study develops an Internet of Things-based inventory model and compares the three demand situations to determine their relative effectiveness in achieving equilibrium stock levels and lowering costs. The model describes how the adoption of IoT impacts inventory storage, degradation, and shortfall costs in different demand scenarios. Secondary data from Abu Hashan Md Mashud (2020) on degrading inventory models is used to supplement the numerical investigation. A computational example is provided to demonstrate the usefulness of the IoT-based approach, demonstrating how it allows for higher optimum stock levels and higher profit margins than traditional inventory techniques. The findings indicate that the pricing function with a negative power of a constant surpasses the other two demand trends in terms of inventory optimization and revenue. The notations, assumptions, and mathematical formulations used to develop the proposed IoT-based inventory model are detailed in the sections below.

Notations:

The notation in this paper is as follows:

Notations	Descriptions
	cost incurred per replenishment order
	unit purchasing cost
	cost of holding one unit per unit time
	cost of shortage per unit per unit time
	deterioration rate
	maximum inventory level per cycle
	price of sale
	maximum stock out level
$X(t)$	level of inventory at time , where $\leq t \leq T$
$TC(t_1, L)$	the total cost per unit time
	time when inventory depletes to zero
	time length of the replenishment cycle
	amount of capital investment on IOT
	effectiveness of IOT in lowering the costs
	proportion of costs after investments in IOT

Assumptions:

- Replenishment occurs instantly, and the lead time is considered as negligible.
- In this study, we consider three distinct demand rates.

$$D(s) = \begin{cases} a - bs + cX(t) & when\ X(t) \geq 0 \\ a - bs & when\ X(t) < 0 \end{cases}$$

i.e. when $X(t) \geq 0$ demand depends on both the selling price and the stock and when $X(t) < 0$ demand is only dependent on the selling expense.

$$D(s) = as^{-\beta}$$

$$D(s) = as^{\left(\frac{-s}{\beta}\right)}$$

- The inventory procedure's planning horizon is indefinite.
- This study focuses on shortages, which are currently fully backlogged.
- The organization employs Internet of Things (IoT) technology to enable ongoing surveillance, analytical forecasting, and scheduled inventory changes across the system. This digital integration reduces shortages, excess inventory, and deterioration by constantly monitoring product attributes and demand fluctuations. The proportion of of average shortage cost, holding cost, and deterioration reduction achieved by IoT adoption is represented by $F = \xi(1 - e^{-mI})$ where ξ, m, I are defined in the notation section reflects the incorporation of IoT results in a more effective, adaptive, and long-term inventory management system by raising inventory levels, maximizing resource allocation, and enhancing decision transparency.

Mathematical Formulation

The proposed three inventory models are based on three different demand functions, developed under the assumptions stated above. Initially, it is believed that a firm acquired commodities in $(A+B)$ units for all three models. The stock is decreased owing to client demand and deterioration within the interval $[0, t_1]$. At time $t = t$, the stock equals zero. The shortfall occurs during $[t_1, L]$ and is entirely backlogged.

Case I: Inventory model for price and stock dependent demand:

$$\frac{dX(t)}{dt} + \theta X(t) = -[a - bs + cX(t)] \qquad < t \leq t \tag{1}$$

$$\frac{dX(t)}{dt} = -(a - bs) \qquad t\ < t \leq L \tag{2}$$

From equation (1) we have

$$X(t) = \frac{a - bs}{\theta + c}\left\{e^{(\theta+c)(t_1 - t)} - 1\right\} \qquad < t \leq t \tag{3}$$

Using the condition $X(t) = 0$ at $t = t$ and $X(t) = A$ at we obtain

$$A = \frac{a - bs}{\theta + c}\left\{e^{(\theta+c)t_1} - 1\right\} \tag{4}$$

From equation (2) we have

$$X(t) = (a - bs)(t_1 - t) \qquad t\ < t \leq L \tag{5}$$

Using the condition $X(t) = 0$ at $t = t$ and $X(t) = -B$ at $t = L$ we get

$$B = -(a - bs)(t_1 - L) \tag{6}$$

The overall expense per unit of time for the inventory model comprises the following elements:

- Ordering expense per replenishment cycle =
- The cost of holding inventories per cycle = $h_c\left[\int_0^{t_1} X(t)\,dt\right]$

i.e., $$\frac{h_c(a-bs)}{(\theta+c)^2}\left[e^{(\theta+c)t_1}-(\theta+c)t_1-1\right] \tag{7}$$

- Purchasing expense incurred in a cycle

$$= \ p_c(A+B) \tag{8}$$

Cost associated with inventory shortages $= s_c\left[\int_{t_1}^{L} -X(t)\,dt\right]$

i.e., $$=\frac{1}{2}s_c(a-bs)(L-t_1)^2 \tag{9}$$

Hence, Total inventory cost (Y_1) = <ordering cost> + <purchase cost> + <holding cost> + <shortage cost>

i.e., $$Y_1 = r_o + p_c(A+B) + C_{hol} + C_{sho} \tag{10}$$

Therefore, the corresponding constrained optimization problem can be formulated as follows:

Problem 1: Minimize $$TC_1(t_1,L)=\frac{Y_1}{L} \tag{11}$$

Subject to $\ \leq t \leq L$

Here, the computation of Problem 1 based on the demand function is elaborated below.

$$D(s)=\begin{cases} a-bs+cX(t) & when\ X(t)\geq 0 \\ a-bs & when\ X(t)<0 \end{cases}$$

$A=\frac{a-bs}{\theta+c}\left\{e^{(\theta+c)t_1}-1\right\}$, Using a Taylor series expansion for $e^{(\theta+c)t_1}$, while omitting the higher-order terms we get,

$$A=(a-bs)\left[t_1+\frac{(\theta+c)}{2}t_1^2\right] \tag{12}$$

$$C_{hol}=\frac{h_c(a-bs)}{(\theta+c)^2}\left[e^{(\theta+c)t_1}-(\theta+c)t_1-1\right] \tag{13}$$

$$C_{hol}=\frac{h_c(a-bs)t_1^2}{2} \tag{14}$$

Substituting all the terms into equation (10), we obtain

$$Y_1 = r_o + p_c(a-bs)\left[L+\frac{(\theta+c)t_1^2}{2}\right]+\frac{h_c(a-bs)t_1^2}{2}+\frac{s_c(a-bs)(L-t_1^2)}{2} \tag{15}$$

Now to determine the value of , it's necessary to put $\frac{\partial Y_1}{\partial t_1}=0$.

$$t_1=\frac{s_c L}{p_c(\theta+c)+h_c+s_c}=f_1 L \tag{16}$$

where $f_1=\frac{s_c}{p_c(\theta+c)+h_c+s_c}$

From equation (11) we have

$$TC_1(t_1,L)=\frac{r_o+p_c(a-bs)\left[L+\frac{(\theta+c)t_1^2}{2}\right]+\frac{h_c(a-bs)t_1^2}{2}+\frac{s_c(a-bs)(L-t_1^2)}{2}}{L} \tag{17}$$

After investing in IOT the total cost $TC_1(t_1,L)$ becomes

$$TC_1(t_1,L)=\frac{r_o+p_c(a-bs)\left[L+\frac{(\theta+c)t_1^2}{2}\right]+\frac{h_c(a-bs)t_1^2}{2}\left(1-\xi\left(1-e^{-mI}\right)\right)+\frac{s_c(a-bs)(L-t_1^2)}{2}\left(1-\xi\left(1-e^{-mI}\right)\right)+I}{L} \tag{18}$$

Substituting the value of in equation (18), we get

$$TC_1(t_1,L)=\frac{r_o+p_c(a-bs)\left[L+\frac{(\theta+c)f_1^2L^2}{2}\right]+\frac{h_c(a-bs)f_1^2L^2}{2}\left(1-\xi\left(1-e^{-mI}\right)\right)+\frac{s_c(a-bs)L^2(1-f_1)^2}{2}\left(1-\xi\left(1-e^{-mI}\right)\right)+I}{L} \tag{19}$$

$$TC_1(t_1,L)=\frac{r_o}{L}+\frac{I}{L}+p_c(a-bs)+\left\{\frac{p_c(\theta+c)(a-bs)f_1^2}{2}+\frac{h_c(a-bs)f_1^2}{2}\left(1-\xi\left(1-e^{-mI}\right)\right)+\frac{s_c(a-bs)(1-f_1)^2}{2}\left(1-\xi\left(1-e^{-mI}\right)\right)\right\}L \tag{20}$$

In order to determine the optimum total cost value, necessary and sufficient conditions are $\frac{\partial TC_1(t_1,L)}{\partial L}=0$ and the optimal value of is given by

$$L=\sqrt{\frac{2(r_o+I)}{p_c(\theta+c)(a-bs)f_1^2+h_c(a-bs)f_1^2\left(1-\xi\left(1-e^{-mI}\right)\right)+s_c(a-bs)(1-f_1)^2\left(1-\xi\left(1-e^{-mI}\right)\right)}} \tag{21}$$

Substituting the value of in equation (20) and solving, we have

$$TC_1(t_1,L)=p_c(a-bs)+\sqrt{2r_o\left\{p_c(\theta+c)(a-bs)f_1^2+h_c(a-bs)f_1^2\left(1-\xi\left(1-e^{-mI}\right)\right)+s_c(a-bs)(1-f_1)^2\left(1-\xi\left(1-e^{-mI}\right)\right)\right\}}+I \tag{22}$$

For any positive value of , the optimum solution of is found from $\frac{\partial TC_1(t_1,L)}{\partial I}=0$.The solution for is

$$I^*=-\frac{1}{m}\ln\left(\frac{1+\sqrt{1+2m^2r_oS_o}}{m^2r_o\xi P}\right) \tag{23}$$

Where $P=(a-bs)\left(h_cf_1^2+s_c(1-f_1)^2\right)$, $Q=p_c(\theta+c)(a-bs)f_1^2$, $S_o=Q+P(1-\xi)$.

Case II: Inventory model with respect to the demand function $D=as^{-\beta}$

$$\frac{dX(t)}{dt}+\theta X(t)=-as^{-\beta} \quad < t \leq t \tag{24}$$

$$\frac{dX(t)}{dt}=-as^{-\beta}\ t\ < t \leq L \tag{25}$$

From equation (24) we have

$$X(t)=\frac{a}{\theta}s^{-\beta}\left\{e^{(\theta)(t_1-t)}-1\right\} \quad < t \leq t \tag{26}$$

Using the condition $X(t)=0$ at $t=t$ and $X(t)=A$ at we obtain

$$A=\frac{a}{\theta}s^{-\beta}\left\{e^{\theta t_1}-1\right\} \tag{27}$$

From equation (25) we have

$$X(t)=as^{-\beta}(t_1-t)\ t\ < t \leq L \tag{28}$$

Using the condition $X(t)=0$ at $t=t$ and $X(t)=-B$ at $t=L$ we get

$$B=-as^{-\beta}\left(t_1-L\right) \tag{29}$$

The overall expense per unit of time for the inventory model comprises the following elements:

- Ordering expense per replenishment cycle =
- The cost of holding inventories per cycle = $h_c\left[\int_0^{t_1} X(t)\,dt\right]$

i.e., $$\frac{h_c a}{\theta^2}s^{-\beta}\left[e^{\theta t_1}-\theta t_1-1\right] \tag{30}$$

- Purchasing expense incurred in a cycle

$$= p_c\left(A+B\right) \tag{31}$$

- Cost associated with inventory shortages $=s_c\left[\int_{t_1}^{L}-X(t)\,dt\right]$

i.e., $$=\frac{1}{2}s_c as^{-\beta}\left(L-t_1\right)^2 \tag{32}$$

Hence, Total inventory cost $\left(Y_2\right)$ = <ordering cost> + <purchase cost> + <holding cost> + <shortage cost>

i.e., $$Y_2=r_o+p_c\left(A+B\right)+C_{hol}+C_{sho} \tag{33}$$

Therefore, the corresponding constrained optimization problem can be formulated as follows:

Problem 2: Minimize $$TC_2\left(t_1,L\right)=\frac{Y_2}{L} \tag{34}$$

Subject to $\le t\le L$

Here, the computation of Problem 2 based on the demand function is elaborated below.

The demand function is $D=as^{-\beta}$

$A=\frac{as^{-\beta}}{\theta}\left\{e^{\theta t_1}-1\right\}$, Using a Taylor series expansion for , while omitting the higher-order terms we get,

$$A=as^{-\beta}\left[t_1+\frac{\theta}{2}t_1^2\right] \tag{35}$$

$$C_{hol}=\frac{h_c as^{-\beta}}{\theta^2}\left[e^{\theta t_1}-\theta t_1-1\right] \tag{36}$$

$$C_{hol}=\frac{h_c as^{-\beta}t_1^2}{2} \tag{37}$$

Substituting all the terms into equation (33), we obtain

$$Y_2=r_o+p_c as^{-\beta}\left[L+\frac{\theta t_1^2}{2}\right]+\frac{h_c as^{-\beta}t_1^2}{2}+\frac{s_c as^{-\beta}\left(L-t_1^2\right)}{2} \tag{38}$$

Now to determine the value of , it's necessary to put $\frac{\partial Y_2}{\partial t_1}=0$.

$$t_1=\frac{s_c L}{p_c\theta+h_c+s_c}=f_1 L \tag{39}$$

where $f_1=\frac{s_c}{p_c\theta+h_c+s_c}$

From equation (34) we have

$$TC_2\left(t_1,L\right)=\frac{r_o+p_c as^{-\beta}\left[L+\frac{\theta t_1^2}{2}\right]+\frac{h_c as^{-\beta}t_1^2}{2}+\frac{s_c as^{-\beta}\left(L-t_1^2\right)}{2}}{L} \tag{40}$$

After investing in IOT the total cost $TC_2\left(t_1,L\right)$ becomes

$$TC_2\left(t_1,L\right)=\frac{r_o+p_c as^{-\beta}\left[L+\frac{\theta t_1^2}{2}\right]+\frac{h_c as^{-\beta}t_1^2}{2}\left(1-\xi\left(1-e^{-mI}\right)\right)+\frac{s_c as^{-\beta}\left(L-t_1^2\right)}{2}\left(1-\xi\left(1-e^{-mI}\right)\right)+I}{L} \tag{41}$$

Substituting the value of in equation (41), we get

$$TC_2\left(t_1,L\right)=\frac{r_o+p_c as^{-\beta}\left[L+\frac{\theta f_1^2L^2}{2}\right]+\frac{h_c as^{-\beta}f_1^2L^2}{2}\left(1-\xi\left(1-e^{-mI}\right)\right)+\frac{s_c as^{-\beta}L^2\left(1-f_1\right)^2}{2}\left(1-\xi\left(1-e^{-mI}\right)\right)+I}{L} \tag{42}$$

$$TC_2\left(t_1,L\right)=\frac{r_o}{L}+\frac{I}{L}+p_c as^{-\beta}+\left\{\frac{p_c\theta as^{-\beta}f_1^2}{2}+\frac{h_c as^{-\beta}f_1^2}{2}\left(1-\xi\left(1-e^{-mI}\right)\right)+\frac{s_c as^{-\beta}\left(1-f_1\right)^2}{2}\left(1-\xi\left(1-e^{-mI}\right)\right)\right\}L \tag{43}$$

In order to determine the optimum total cost value, necessary and sufficient conditions are $\frac{\partial TC_2\left(t_1,L\right)}{\partial L}=0$ and the optimal value of is given by

$$L=\sqrt{\frac{2(r_o+I)}{p_c\theta as^{-\beta}f_1^2+h_c as^{-\beta}f_1^2\left(1-\xi\left(1-e^{-mI}\right)\right)+s_c as^{-\beta}\left(1-f_1\right)^2\left(1-\xi\left(1-e^{-mI}\right)\right)}} \tag{44}$$

Substituting the value of in equation (43) and solving, we have

$$TC_2\left(t_1,L\right)=p_c as^{-\beta}+\sqrt{2r_o\left\{p_c\theta as^{-\beta}f_1^2+h_c as^{-\beta}f_1^2\left(1-\xi\left(1-e^{-mI}\right)\right)+s_c as^{-\beta}\left(1-f_1\right)^2\left(1-\xi\left(1-e^{-mI}\right)\right)\right\}}+I \tag{45}$$

For any positive value of , the optimum solution of is found from $\frac{\partial TC_2\left(t_1,L\right)}{\partial I}=0$. The solution for is

$$I^*=-\frac{1}{m}\ln\left(\frac{1+\sqrt{1+2m^2r_oS_o}}{m^2r_o\xi P}\right) \tag{46}$$

Where $P=as^{-\beta}\left(h_c f_1^2+s_c\left(1-f_1\right)^2\right)$, $Q=p\,\theta as^{-\beta}f$, $S_o=Q+P\left(1-\xi\right)$.

Case III: Inventory model with respect to the demand function $D=ae^{-\left(\frac{s}{\beta}\right)}$***:***

$$\frac{dX(t)}{dt}+\theta X(t)=-ae^{-\left(\frac{s}{\beta}\right)}\quad <t\le t \tag{47}$$

$$\frac{dX(t)}{dt}=-ae^{-\left(\frac{s}{\beta}\right)}\ t\ <t\le L \tag{48}$$

From equation (47) we have

$$X(t)=\frac{a}{\theta}e^{-\left(\frac{s}{\beta}\right)}\left\{e^{(\theta)(t_1-t)}-1\right\}\quad <t\le t \tag{49}$$

Using the condition $X(t)=0$ at $t=t$ and $X(t)=A$ at we obtain

$$A = \frac{a}{\theta} e^{-\left(\frac{s}{\beta}\right)} \left\{ e^{\theta t_1} - 1 \right\} \quad (50)$$

From equation (48) we have

$$X(t) = ae^{-\left(\frac{s}{\beta}\right)} (t_1 - t) \; t \;\; < t \le L \quad (51)$$

Using the condition $X(t) = 0$ at $t = t$ and $X(t) = -B$ at $t = L$ we get

$$B = -ae^{-\left(\frac{s}{\beta}\right)} (t_1 - L) \quad (52)$$

The overall expense per unit of time for the inventory model comprises the following elements:

- Ordering expense per replenishment cycle =
- The cost of holding inventories per cycle = $h_c \left[\int_0^{t_1} X(t)\,dt \right]$

i.e., $$\frac{h_c a}{\theta^2} e^{-\left(\frac{s}{\beta}\right)} \left[e^{\theta t_1} - \theta t_1 - 1 \right] \quad (53)$$

- Purchasing expense incurred in a cycle

$$= \; p_c (A + B) \quad (54)$$

- Cost associated with inventory shortages $= s_c \left[\int_{t_1}^{L} -X(t)\,dt \right]$

i.e., $$= \frac{1}{2} s_c ae^{-\left(\frac{s}{\beta}\right)} (L - t_1)^2 \quad (55)$$

Hence, Total inventory cost (Y_3) = <ordering cost> + <purchase cost> + <holding cost> + <shortage cost>

i.e., $$Y_3 = r_o + p_c (A + B) + C_{hol} + C_{sho} \quad (56)$$

Therefore, the corresponding constrained optimization problem can be formulated as follows:

Problem 2: Minimize $$TC_3(t_1, L) = \frac{Y_3}{L} \quad (57)$$

Subject to $\le t \le L$

Here, the computation of Problem 2 based on the demand function is elaborated below. The demand function is $D = ae^{-\left(\frac{s}{\beta}\right)}$

$A = \frac{ae^{-\left(\frac{s}{\beta}\right)}}{\theta} \{ e^{\theta t_1} - 1 \}$, Using a Taylor series expansion for , while omitting the higher-order terms we get,

$$A = ae^{-\left(\frac{s}{\beta}\right)} \left[t_1 + \frac{\theta}{2} t_1^2 \right] \quad (58)$$

$$C_{hol} = \frac{h_c ae^{-\left(\frac{s}{\beta}\right)}}{\theta^2} \left[e^{\theta t_1} - \theta t_1 - 1 \right] \quad (59)$$

$$C_{hol} = \frac{h_c ae^{-\left(\frac{s}{\beta}\right)} t_1^2}{2} \quad (60)$$

Substituting all the terms into equation (56), we obtain

$$Y_3 = r_o + p_c ae^{-\left(\frac{s}{\beta}\right)} \left[L + \frac{\theta t_1^2}{2} \right] + \frac{h_c ae^{-\left(\frac{s}{\beta}\right)} t_1^2}{2} + \frac{s_c ae^{-\left(\frac{s}{\beta}\right)} (L - t_1^2)}{2} \quad (61)$$

Now to determine the value of , it's necessary to put $\frac{\partial Y_3}{\partial t_1} = 0$.

$$t_1 = \frac{s_c L}{p_c \theta + h_c + s_c} = f_1 L \quad (62)$$

where $f_1 = \frac{s_c}{p_c \theta + h_c + s_c}$

From equation (57) we have

$$TC_3(t_1, L) = \frac{r_o + p_c ae^{-\left(\frac{s}{\beta}\right)} \left[L + \frac{\theta t_1^2}{2} \right] + \frac{h_c ae^{-\left(\frac{s}{\beta}\right)} t_1^2}{2} + \frac{s_c ae^{-\left(\frac{s}{\beta}\right)} (L - t_1^2)}{2}}{L} \quad (63)$$

After investing in IOT the total cost $TC_3(t_1, L)$ becomes

$$TC_3(t_1, L) = \frac{r_o + p_c ae^{-\left(\frac{s}{\beta}\right)} \left[L + \frac{\theta t_1^2}{2} \right] + \frac{h_c ae^{-\left(\frac{s}{\beta}\right)} t_1^2}{2} \left(1 - \xi\left(1 - e^{-mI}\right)\right) + \frac{s_c ae^{-\left(\frac{s}{\beta}\right)} (L - t_1^2)}{2} \left(1 - \xi\left(1 - e^{-mI}\right)\right) + I}{L} \quad (64)$$

Substituting the value of in equation (64), we get

$$TC_3(t_1, L) = \frac{r_o + p_c ae^{-\left(\frac{s}{\beta}\right)} \left[L + \frac{\theta f_1^2 L^2}{2} \right] + \frac{h_c ae^{-\left(\frac{s}{\beta}\right)} f_1^2 L^2}{2} \left(1 - \xi\left(1 - e^{-mI}\right)\right) + \frac{s_c ae^{-\left(\frac{s}{\beta}\right)} L^2 (1 - f_1)^2}{2} \left(1 - \xi\left(1 - e^{-mI}\right)\right) + I}{L} \quad (65)$$

$$TC_3(t_1, L) = \frac{r_o}{L} + \frac{I}{L} + p_c ae^{-\left(\frac{s}{\beta}\right)} + \left\{ \frac{p_c \theta ae^{-\left(\frac{s}{\beta}\right)} f_1^2}{2} + \frac{h_c ae^{-\left(\frac{s}{\beta}\right)} f_1^2}{2} \left(1 - \xi\left(1 - e^{-mI}\right)\right) + \frac{s_c ae^{-\left(\frac{s}{\beta}\right)} (1 - f_1)^2}{2} \left(1 - \xi\left(1 - e^{-mI}\right)\right) \right\} L \quad (66)$$

Optimization of an Advanced Integrated Inventory Model Considering Shortages and Deterioration across Varying Demand Functions

Table 1. Inventory parameters value for each inventory model

Parameters	Case I	Case II	Case III
r_o	200	200	200
a	150	150	200
b	0.7	---	---
β	---	0.1	30
c	0.5	---	---
p_c	4	4	4
h_c	0.5	0.5	0.5
s_c	5	5	5
θ	0.01	0.01	0.02
s	10	10	15
m	0.65	0.65	0.65
ξ	0.35	0.35	0.35

Table 2: Optimal solutions are obtained for the IoT-facilitated inventory framework under three demand variations, namely linear price–stock dependent demand, negative power demand, and exponential price demand, in order to evaluate their effectiveness in maximizing stock levels and reducing costs

Cases						$TC_i(t_1, L)$
Case I	188.584482	79.4028852	0.809869907	1.22128382	4.67402433	1108.00326
Case II	402.584988	42.9418038	2.50280226	2.77310490	4.79153914	786.214411
Case III	303.434275	18.8491994	2.4417703	2.59715568	4.36461054	1373.11204

In order to determine the optimum total cost value, necessary and sufficient conditions are $\frac{\partial TC_3(t_1, L)}{\partial L} = 0$ and the optimal value of is given by

$$L = \sqrt{\frac{2(r_o + I)}{p_c \theta a e^{-\left(\frac{s}{\beta}\right)} f_1^2 + h_c a e^{-\left(\frac{s}{\beta}\right)} f_1^2 \left(1 - \xi\left(1 - e^{-mI}\right)\right) + s_c a e^{-\left(\frac{s}{\beta}\right)} (1 - f_1)^2 \left(1 - \xi\left(1 - e^{-mI}\right)\right)}} \tag{67}$$

Substituting the value of in equation (66) and solving, we have

$$TC_3(t_1, L) = p_c a e^{-\left(\frac{s}{\beta}\right)} + \sqrt{2r_o \left\{ p_c \theta a e^{-\left(\frac{s}{\beta}\right)} f_1^2 + h_c a e^{-\left(\frac{s}{\beta}\right)} f_1^2 \left(1 - \xi\left(1 - e^{-mI}\right)\right) + s_c a e^{-\left(\frac{s}{\beta}\right)} (1 - f_1)^2 \left(1 - \xi\left(1 - e^{-mI}\right)\right) \right\}} + I \tag{68}$$

For any positive value of , the optimum solution of is found from $\frac{\partial TC_3(t_1, L)}{\partial I} = 0$. The solution for is

$$I^* = -\frac{1}{m} \ln\left(\frac{1 + \sqrt{1 + 2m^2 r_o S_o}}{m^2 r_o \xi P} \right) \tag{69}$$

where $P = a e^{-\left(\frac{s}{\beta}\right)} \left(h_c f_1^2 + s_c (1 - f_1)^2\right)$, $Q = p_c \theta a e^{-\left(\frac{s}{\beta}\right)} f_1^2$, $S_o = Q + P(1 - \xi)$.

Numerical illustrations and comparison

To validate the models, we presented three numerical examples for each demand function:

Result

A numerical study was conducted with parameter values taken from Abu Hashan Md Mashud (2020) to examine the efficacy of IoT-facilitated inventory management across three diverse demand structures: (i) Linear price and stock-related demand, (ii) negative power demand, and (iii) exponential cost-based demand. The model employs IoT-powered continuous observation and changes to optimize the optimal stock level every cycle while minimizing shortage, holding, and deterioration costs.

Table 2 shows the efficiency of the suggested paradigm.

By adopting the recommended IoT-enabled inventory model with shortage handling and demand variability analysis allows businesses to coordinate their operations for real-time versatility, increase optimum stock levels, reduce holding and deterioration costs, and improve decision-making efficiency. The model allows firms to efficiently react to varied demand conditions, such as linear, negative power, and exponential, giving them a competitive advantage in a technology-driven, demand-sensitive market.

Discussion

A few investigations have looked at inventory models from the standpoint of merging modern digital technologies such as the Internet of Things (IoT) with multiple demand circumstances. Most current research focuses simply on deterioration and shortages, or on standard cost-based optimization, without considering how demand changes influence inventory operation (Abu Hashan Md Mashud, 2020). Additionally, studies on demand-based inventory models has largely disregarded the role of Internet of Things (IoT) monitoring in determining inventory levels, shortfall provisions, and deterioration control. This creates a significant research vacuum because the combined effects on technological implementation, shortage management, degradation, and diverse demand patterns on overall profitability and stock optimization have not been thoroughly investigated. The current study addresses this gap by developing an Internet of Things-powered inventory model that incorporates shortage management, deterioration control, and three key demand variability: linear price-stock dependent demand, negative power price-based demand, and exponential price demand. IoT adoption is critical because it enables real-time monitoring, predictive analytics, and automated adjustments, which increases responsiveness, guarantees higher stock levels, and reduces waste (Friday Ugbebor *et al*., 2024). The findings demonstrate that the choice of demand structure has a significant impact on model performance. The negative power demand model outperforms the other two models in terms of maximum inventory levels and overall profitability. This shows that IoT-based solutions work best when integrated with nonlinear demand models, which allow for more flexible consumer pricing reactions. Overall, the results reveal that IoT adoption reduces holding, shortages, and degradation costs while raising maximum stock levels, which leads to improved financial and operational efficiency. Overall, the results reveal that IoT adoption reduces holding, shortages, and degradation costs while raising maximum stock levels, which leads to improved financial and operational efficiency.

Conclusion

The study develops a holistic inventory model using Internet of Things (IoT) technologies and tests its efficacy in shortage-permitted scenarios across a variety of demand structures. By implementing IoT into the inventory structure, the model enables continuous tracking, predictive analytics, and automatic inventory modifications, improving flexibility, optimizing stock levels, and reducing inefficiencies. Unlike traditional approaches, which rely on static decision criteria and frequently ignore demand variability, this method links inventory management to real-world concerns such as demand variation, shortfall control, and degradation reduction. A numerical study is carried out to show how the IoT-based model performs under three types of demand: linear price-stock dependent, negative power of price, and exponential price-based demand. The maximum inventory level per cycle for the Cases I, II and III are 188.584482, 402.584988, 303.434275 respectively and the total costs for the Cases I, II and III are $1108.00326, $786.214411, $1373.11204 respectively. The results reveal that the negative power demand model outperforms the linear and exponential scenarios in terms of maximum stock levels and profitability. This conclusion underlines the importance of include nonlinear demand forms in inventory analysis, as well as the potential for IoT adoption to increase decision-making efficiency, reduce degrading losses, and lower shortage and holding costs. The inclusion of demand fluctuations increases the model's capacity to replicate actual market behavior, making it more useful for businesses with unpredictable and changeable demand. While the current framework is based on deterministic demand assumptions, future research could build on it by incorporating stochastic demand processes, variable deterioration rates, and multi-echelon supply chain circumstances, making it more useful to more uncertain and volatile situations. This study is novel in that it combines IoT-enabled monitoring with demand-sensitive inventory analysis, resulting in a comprehensive approach that maximizes stock levels, boosts profitability, and improves operational sustainability. This structure serves as a foundation for firms that want to employ digital technologies to match inventory planning with competitive and demand-driven markets.

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