



RESEARCH ARTICLE

Economic Order Quantity under Perishability: Analytical and Iterative Approaches to Cost Minimization

Nalini. S¹, Ritha. W^{1*}, Sasitharan Nagapan²

Abstract

Perishability is a critical factor in inventory control that necessitates adjustments to traditional models like the Economic Order Quantity (EOQ). This paper explores the EOQ problem under perishable conditions and employs numerical and analytical methods to derive optimal ordering policies. The study is structured around three cases: (i) calculating EOQ with a fixed percentage of perishable goods, (ii) calculating EOQ with time-based perishability, and (iii) determining the optimal ordering time interval with time-based perishability. To solve the transcendental equations that arise, particularly in cases (ii) and (iii) due to the introduction of time-sensitive decay, the paper utilizes both a numerical iterative method and the Newton-Raphson method.

The numerical method iteratively refines the EOQ or ordering interval until convergence, while the Newton-Raphson method employs derivatives of the cost function to find the optimal solution. The comparison of the two solution methods reveals that while both converge to similar EOQ values, the Newton-Raphson method generally provides a more precise and cost-effective solution, especially in cases with time-dependent perishability.

Keywords: Perishability, EOQ, Optimum Quantity, Optimum Cycle Time, Numerical method, Newton-Raphson Method, Total cost.

Introduction

The Economic Order Quantity (EOQ) model is a cornerstone of inventory management, aiming to minimize the total costs associated with ordering and holding inventory. The classical EOQ model, however, rests on assumptions that limit its applicability to many real-world situations. One such limiting assumption is that inventory items can be stored indefinitely without deterioration. In numerous

industries, including food, pharmaceuticals, and chemicals, this assumption is invalid as goods have a finite shelf life and are subject to perishability.

The challenge of incorporating perishability into inventory models has attracted researchers for several decades. Early inventory models largely focused on non-perishable items, with the emphasis on balancing ordering costs and holding costs. However, as the importance of managing perishable goods became more apparent, researchers began to extend the EOQ framework to account for the time-dependent nature of inventory value.

The development of inventory models for perishable goods has evolved through several stages. Initial efforts involved adapting the EOQ formula to account for a constant rate of perishability. These models provided a foundation for understanding the basic trade-offs between ordering frequency and spoilage. Later research introduced more complex perishability patterns, such as time-varying decay rates, and considered factors like preservation techniques and salvage value.

Over time, various mathematical techniques have been employed to solve perishable inventory problems. Analytical solutions were derived for simplified scenarios, providing insights into optimal ordering policies. However, many realistic perishability models result in complex equations

¹Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirapalli 620002, India.

²Faculty of Civil Engineering and Built Environment, Universiti Tun Hussein Onn, Malaysia, Johor, MALAYSIA

***Corresponding Author:** Ritha. W, Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirapalli 620002, India., E-Mail: ritha_prakash@yahoo.co.in

How to cite this article: Nalini, S., Ritha, W., Nagapan, S. (2025). Economic Order Quantity under Perishability: Analytical and Iterative Approaches to Cost Minimization. *The Scientific Temper*, 16(9):4704-4714.

Doi: 10.58414/SCIENTIFICTEMPER.2025.16.9.01

Source of support: Nil

Conflict of interest: None.

that cannot be solved analytically. As a result, numerical methods and computer simulations have become essential tools for determining optimal inventory strategies in these situations. These computational approaches allow for the analysis of more intricate factors, such as stochastic demand, lead time variability, and sophisticated perishability functions.

Literature Review

A foundational review of perishable inventory models, categorized by product lifetime and demand behavior, laid the theoretical groundwork for future advancements in the field (Nahmias, 1982). The inclusion of inflation effects and payment delays in inventory models marked a significant development toward realistic financial modeling for deteriorating items (Liao *et al.*, 2000).

A comparative analysis of age-based and stock-level control policies provided critical insight into control mechanisms suitable for perishable inventory systems (Tekin *et al.*, 2001). Multi-location storage complexity was addressed through a two-warehouse inventory model that incorporated FIFO dispatch policies and deterioration dynamics (Lee, 2006).

Comprehensive reviews on deteriorating inventory systems highlighted trends and gaps in the literature since the early 2000s, setting the stage for integrated and sustainable approaches (Bakker *et al.*, 2012). In grocery perishables, the optimization of ordering and pricing strategies was explored through models that reflect the operational challenges of freshness and demand uncertainty (Li *et al.*, 2012).

The application of RFID technology enabled more responsive inventory decisions by incorporating freshness and shelf-space constraints into perishable goods management (Piramuthu & Zhou, 2013). A large-scale literature analysis identified thematic patterns and suggested directions for incorporating technology and sustainability into future research (Chaudhary *et al.*, 2018).

A time decay-linked logistic approach was developed to more closely align perishability with inventory control and logistics decisions (Ali *et al.*, 2013). Optimization strategies across the supply chain were examined from an integrated perspective, emphasizing efficiency in production, storage, and distribution of perishable items (Mirabelli & Solina, 2022).

Environmental concerns were incorporated into perishable inventory models through the inclusion of carbon-sensitive demand and fuzzy logic, promoting green supply chain practices (Rani *et al.*, 2019). The healthcare sector's need for inventory systems tailored to pharmaceutical perishables was addressed through models featuring price sensitivity, variable deterioration, and partial backlogging (Rastogi & Singh, 2019).

Realistic constraints related to product expiration and

customer backordering were captured in models featuring fixed lifetimes and backlogging allowances (Olsson & Tydesjö, 2010). A hospital inventory system for platelets was optimized using regular and expedited replenishment policies to maintain supply of perishable medical items (Zhou *et al.*, 2011).

The integration of trade credit with price-sensitive demand was explored in models that addressed retailer financing and consumer responsiveness (Rameswari & Uthayakumar, 2018). The joint optimization of selling price and credit period for sellers of perishable items was addressed in an early pricing-inventory model that considered demand elasticity (Abad & Jaggi, 2003)

Materials And Methods

The model focusses on finding out the optimum quantity Q^* and the optimum Time T^* under different cases of perishability. Two methods have also been used under cases-(ii) and (iii).

Description of the Model

Notation and assumptions

Notation

CASE-(i)

Model formulation

Calculate Q^* (with perishability of 2.5% of goods)

This case deals with the EOQ problem, wherein, a certain percentage of goods becomes perishable.

D Annual demand

A Ordering cost

H Holding cost

P Purchase cost

X % of goods perish (useless and no salvage value)

Numerical examples are provided under each case to have more clarity and continuity

EOQ Model

$$\text{Adjusted Demand } D' = \frac{D}{1 - X}$$

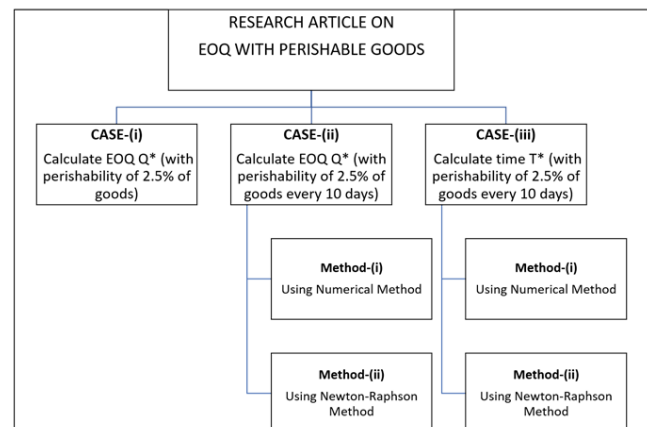


Figure 1: Details of Cases and Methods

$$EOQ Q^* = \sqrt{\frac{2AD'}{H}}$$

Numerical Example

D=200 units (annual demand)

A=10 Rs/order (ordering cost)

H=2 Rs/unit/year (holding cost)

P=5 Rs/unit (purchase cost)

X=2.5% of item perishes (useless and no salvage value)

Step 1: Adjust Demand for Perishable Goods

Since 2.5% of the goods perish and cannot be used, the actual quantity that needs to be ordered to meet demand is more than 200 units.

So, effective quantity needed =

$$\text{Adjusted Demand} = \frac{D}{1-X} = \frac{200}{1-0.025} = \frac{200}{0.975} = D' = 205.13 \text{ units}$$

Step 2: EOQ Formula

$$EOQ = \sqrt{\frac{2AD'}{H}} = \sqrt{\frac{2 \times 10 \times 205.13}{2}} = 45.29 \text{ units}$$

Step 3: Total Cost

$$\text{Ordering cost} = \frac{D'}{Q} * A = \frac{205.13}{45.29} * 10 = \text{Rs } 45.29$$

$$\text{Holding cost} = \frac{Q}{2} * H = \frac{45.29}{2} * 2 = \text{Rs } 45.29$$

$$\text{Purchase cost} = D' * P = 205.13 * 5 = \text{Rs } 1025.64$$

$$\text{Total cost} = \text{Rs } 1116.22$$

Summary

- Adjusted Demand: ≈ 205.13 units/year
- EOQ: ≈ 45.29 units
- Total Cost: $\approx \text{Rs } 1116.22/\text{year}$

Sensitivity Analysis is carried out by varying the Ordering cost, Holding Cost and Perished Goods % and their effect on the Total Cost is calculated.

Sensitivity Analysis Table

The details of EOQ and Total Cost for each scenario is tabulated.

Base values

Values: D=200, P=5, A=10, H=2, X=2.5% and D'=210.53

Case-1: Varying only the Ordering Cost

Missing text

Case-2: Varying only the Holding Cost

Missing text

Case-3 Varying only the Perishability %

Missing text

Observations

- EOQ increases with higher order cost and perished percentage, but decreases with higher holding cost.
- Total cost increases when any of the parameters go up

Table 1: Varying Ordering Cost

Unit Ord Cost ₹	EOQ	Ord Cost ₹	Total Cost ₹
5	32.03	64.05	1121.72
10	45.29	45.29	1116.22
15	55.47	36.98	1118.09
20	64.05	32.03	1121.72

Table 2: Varying Holding Cost

Unit Holding Cost ₹	EOQ	Hold Cost ₹	Total Cost ₹
1	64.05	32.03	1089.69
2	45.29	45.29	1116.22
3	36.98	55.47	1136.58
4	32.03	64.05	1153.74

Table 3: Varying Perished Goods %

Perished Goods X %	Adj Demand	EOQ	Total Cost ₹
0.025	205.13	45.29	1116.22
0.05	210.53	45.88	1144.40
0.075	216.22	46.50	1174.08
0.1	222.22	47.14	1205.39

— particularly due to the increase in adjusted demand with higher perishability.

- The system is most sensitive to perishability (X) when it comes to total cost, as it affects both ordering and purchasing volumes.

CASE-(ii)

Calculate Q^* (with perishability of 2.5% of goods every 10 days)

This adds a time-based perishability element to the EOQ model, which makes it more dynamic and realistic for some industries (e.g., food, pharma).

2.5% of inventory perishes every 10 days, instead of just 2.5% of annual demand.

This changes the perishability from a one-time loss to a continuous decay over time, which accumulates based on how long inventory is held.

Model Formulation

EOQ model with % of perishability every x day

D Annual demand

A Ordering cost

H Holding cost

P Purchase cost

X % of goods perish in every 10 days

Step 1: Define EOQ Cycle Time

If $EOQ = Q$, then:

$$\text{Cycle time } t = \frac{Q}{D} * 365$$

Step 2: Model Perishability

Let's define:

$r=0.025$ (2.5% perish every 10 days)

Perish rate per day: $k = \frac{0.025}{10} = 0.0025 = 0.25\%$ per day.

Assuming exponential decay, the fraction remaining after t days is:

Remaining fraction $= e^{-kt}$

So, lost fraction $= 1 - e^{-kt}$

Thus, perished units per cycle $= Q * (1 - e^{-kt})$

These are wasted, so effective demand per cycle becomes

Useful units per order $= Q * e^{-kt}$

To meet annual demand D , we must order

$$\text{Adjusted annual demand} = \frac{D}{e^{-kt}}$$

Step 3: Plug into EOQ Formula

EOQ with perishability becomes:

$$EOQ = \sqrt{\frac{2AD}{H * e^{kt}}}$$

But notice: EOQ now appears on both sides, since $t = \frac{Q}{D} * 365$ and k depend on t .

This becomes a transcendental equation (can't solve algebraically)

Solving Transcendental equation in EOQ through various methods

Unlike algebraic equations, transcendental equations generally do not have closed-form analytical solutions that can be expressed in terms of elementary functions. Therefore, numerical methods are typically employed to find approximate solutions. Here are some common approaches:

Graphical Method:

Rearrange the equation to the form $f(Q) = 0$. Plot the function $y = f(Q)$.

The roots of the equation (the values of Q for which $f(Q)=0$) are the points where the graph intersects the Q -axis. This method provides a visual understanding of the solutions and can give rough estimates.

Numerical Methods

These are iterative techniques that start with an initial guess and progressively refine it to get closer to the actual solution.

Bisection Method

This method requires an initial interval $[a, b]$ where $f(a)$ and $f(b)$ have opposite signs, guaranteeing at least one root in the interval. The interval is repeatedly halved, always containing a root, until the desired accuracy is achieved.

Newton-Raphson Method

This is a more efficient method that uses the derivative of

the function, $f'(Q)$, to iteratively find the root. The formula for the next approximation is:

$$Q_{n+1} = Q_n - \frac{f(Q_n)}{f'(Q_n)}$$

The convergence of this method depends on the initial guess and the behaviour of the function and its derivative.

Secant Method

This is similar to the Newton-Raphson method but approximates the derivative using the function values at two previous points:

$$Q_{n+1} = Q_n - f(Q_n) * \frac{Q_n - Q_{n-1}}{f(Q_n) - f(Q_{n-1})}$$

Fixed-Point Iteration

Rearrange the equation to the form $Q = g(Q)$. Start with an initial guess Q_0 and iterate using $Q_{n+1} = g(Q_n)$. If the iteration converges, the limit is a solution to the equation.

Root-Finding Algorithms in Software

Many mathematical software packages (e.g., Python with libraries like SciPy, MATLAB, Mathematica) have built-in functions for finding roots of non-linear equations, including transcendental ones. These algorithms often implement sophisticated numerical methods.

In our case, the transcendental equation will be solved by the following two methods

- Solving through Numerical Method
- Solving through Newton-Raphson Method

Case-(ii) Method-(i)

Solving transcendental equation through numerical method

This employs an iterative numerical method to find the Economic Order Quantity (EOQ) under potentially complex conditions. It starts with an initial EOQ and calculates the cycle time, an exponential decay factor, and an adjusted demand. A new EOQ (EOQ') is then computed, and the difference between successive EOQ values is tracked. The process repeats until this difference becomes minimal, indicating convergence to a near-optimal EOQ. The corresponding total cost at this point represents the estimated minimum. This approach is useful when standard EOQ assumptions of constant demand and costs do not hold.

Numerical Example

Assumptions: The following attributes have constant values
 $D = 200$; $A = 10$; $H = 2$; $k = 0.0025$

Analysis

Impact of Perishability

As the initial EOQ increases, the cycle time also increases.

Table 4: Case-(ii) Method-(i) Numerical Method results

EOQ	Cycle time $t = (EOQ/D*365)$	e^{-kt}	Adj.Demand = (D/e^{-kt})	EOQ'	EOQ' DIFF	Total Cost
45.00	82.13	0.81	246	49.56	49.5	1327
49.56	90.44	0.80	251	50.07	0.5	1354
50.07	91.38	0.80	251	50.13	0.06	1357
50.13	91.49	0.80	251	50.14	0.01	1357
50.14	91.51	0.80	251	50.14	0.00	1357
50.14	91.51	0.80	251	50.14	0.00	1357
50.14	91.51	0.80	251	50.14	0.00	1357
50.14	91.51	0.80	251	50.14	0.00	1357
50.14	91.51	0.80	251	50.14	0.00	1357

With a longer cycle time, the e^{-kt} factor decreases (from 0.81 to 0.80 in the observed range), indicating a higher proportion of inventory is expected to perish. To compensate for this perishability, the adjusted demand (Adj. Demand) increases. Consequently, the recalculated EOQ (EOQ') also tends to increase to meet the higher adjusted demand.

Convergence of EOQ'

Notice that as the iterations progress, the difference between successive EOQ' values (EOQ' DIFF) decreases. This suggests that the model is iteratively converging towards a more stable, perishability-adjusted optimal order quantity.

Total Cost Optimization

The total cost initially increases as the EOQ and EOQ' rise. This is likely due to higher holding costs associated with larger order quantities and the cost of perished items. However, the total cost then starts to decrease and eventually seems to stabilize around ₹1357. This indicates that there's an optimal balance where the increased order quantity minimizes the combined costs of ordering, holding, and perishability.

Relationship between Cycle Time and Perishability

The table clearly shows a direct relationship between the chosen EOQ and the cycle time. Larger EOQs lead to longer cycle times, which in turn result in greater losses due to perishability (lower e^{-kt}).

The Iterative Adjustment

The process of moving from EOQ to EOQ' demonstrates an iterative approach to finding the optimal order quantity when dealing with perishable goods. The initial EOQ is adjusted based on the estimated perishability during its cycle, leading to a revised order quantity. This process repeats until a stable EOQ' is reached.

Findings

The suggested Economic Order Quantity (EOQ') is approximately 50 units, because of:

Minimized total cost

The table shows that the total cost stabilizes at ₹1357 when the EOQ' reaches around 50.14. This suggests that ordering in this quantity balances the costs of ordering, holding inventory, and losses due to perishability in the most efficient way.

Accounting for perishability

The EOQ' calculation explicitly considers the impact of e^{-kt} on the demand. By ordering a larger quantity (EOQ' compared to the initial EOQs), you ensure that you can still meet the actual demand after some units perish during the longer cycle time.

Stable order quantity

The EOQ' DIFF values become very small (approaching zero) as the iterations progress towards 50.14. This indicates that this adjusted order quantity is relatively stable and likely close to the true optimal value considering the perishability factor.

Therefore, to minimize total inventory costs while accounting for the perishability of the product, the business should aim to order approximately 50 units per order cycle. This will lead to a cycle time of around 92 days based on the demand and the adjusted order quantity.

Case-(ii) Method-(ii)

Solving the Case-(ii) problem with Newton-Raphson method

Steps for using Newton-Raphson method

Formulate the total cost function

Develop the total annual cost equation for the specific inventory control model, taking into account the relaxed assumptions that lead to the transcendental equation. This cost function will typically include ordering costs, holding costs, and potentially shortage costs, purchase costs, etc.

Find the first derivative

To minimize the total cost, differentiate the total cost

function with respect to the order quantity Q and set the derivative equal to zero:

$$\frac{dTC(Q)}{dQ} = 0$$

This step will yield the transcendental equation that needs to be solved for the optimal order quantity.

Solve the Transcendental Equation

Use one of the methods described above (graphical or numerical) to find the value(s) of Q that satisfy the equation $\frac{dTC(Q)}{dQ} = 0$

Verify Optimality (Second Derivative Test)

To ensure that the solution corresponds to a minimum cost, you can use the second derivative test. Calculate the second derivative $\frac{d^2TC(Q)}{dQ^2} = 0$ and evaluate it at the critical point Q^* . If $\frac{d^2TC(Q)}{dQ^2} > 0$, then Q^* corresponds to a local minimum.

Consider Constraints and Practical Implications

Ensure that the obtained optimal order quantity is feasible and makes practical sense within the context of the inventory problem. For example, the order quantity must be non-negative.

Model Formulation

D Annual Demand
A Ordering Cost
H Holding Cost
K Perishing rate

$$EOQ = \sqrt{\frac{2AD}{H * e^{kt}}}$$

$$\text{Where } t = \frac{Q}{D} * 365$$

$$EOQ = Q$$

$$Q = \sqrt{\frac{2 * A * D}{e^{(-k * (Q/D) * 365)}}}$$

$$f(Q) = Q - \sqrt{\frac{2 * A * D}{H * e^{(-k * (Q/D) * 365)}}} \quad (1)$$

$$f'(Q) = 1 - \frac{d}{d(Q)} \left[\sqrt{\frac{2 * A * D}{H * e^{(-k * (Q/D) * 365)}}} \right]$$

$$= 1/2 * \left(2AD / H * e^{(-k * (Q/D) * 365)} \right)^{(-1/2)} * (2AD / H) * e^{(-k * (Q/D) * 365)} * (-k * 365 / D)$$

$$= \frac{1}{2} * \left(\frac{2 * A * D}{H * e^{(-k * (Q/D) * 365)}} \right)^{(-1/2)} * \left(\frac{2 * A * D}{H} \right) *$$

$$e^{(-k * (Q/D) * 365)} * \left(\frac{-k * 365}{D} \right)$$

$$f'(Q) = 1 + \left(\frac{365 * k}{2 * D} \right) * \sqrt{\frac{2AD * e^{(-k * (Q/D) * 365)}}{H}} \quad (2)$$

The Newton-Raphson method uses the iterative formula:

$$Q_{\text{next}} = Q_{\text{current}} - \frac{f(Q_{\text{current}})}{f'(Q_{\text{current}})}$$

For each iteration, the values of Q_{current} , t_0 , t , $f(Q)$, $f'(Q)$, Q_{next} and $Q_{\text{next_diff}}$ are calculated. The initial value of Q_{current} for the first iteration is derived from the standard EOQ formula. The value of Q_{current} in the second iteration is the same as the value of Q_{next} in the previous (first) iteration. In this way, a total of 10 iterations have been processed and the results are displayed below.

Numerical Example

Assumptions: The following attributes have constant values
D = 200; A = 10; H = 2; k = 0.0025

Observations from the Iterations

Convergence

The iteration appears to be converging towards a solution. The value of $f(Q)$ is progressively approaching zero.

Stabilizing Q_{next}

The value of Q_{next} starts with a jump from the initial guess (49.12) to 49.95. However, in subsequent iterations, the change in Q_{next} (indicated by $Q_{\text{next_diff}}$) rapidly decreases, suggesting the method is settling on a specific value.

Cycle Time t

The cycle time t increases as Q_{current} increases and then stabilizes as Q_{current} converges. This makes sense because a larger order quantity will naturally lead to a longer cycle time before a reorder is needed.

Function Value $f(Q)$

The value of $f(Q)$ starts at -4.80 and steadily moves towards zero. By iteration 6, it reaches 0.00, indicating that the condition $f(Q) = 0$ (our target for the root) is met within the precision of the calculations.

Derivative $f'(Q)$

The derivative $f'(Q)$ remains relatively stable around 1.09 throughout the iterations. This suggests that the function

Table 5: Case-(ii) Method-(ii) Newton Raphson Method results

Q_{current}	t_0	$f(Q)$	$f'(Q)$	Q_{next}	$Q_{\text{next_dif}}$	Tot. Cost
44.72	81.62	-4.80	1.09	49.12	0.00	1092.06
49.12	89.64	-0.90	1.09	49.95	0.83	1092.44
49.95	91.16	-0.17	1.09	50.10	0.16	1092.60
50.10	91.44	-0.03	1.09	50.13	0.03	1092.63
50.13	91.49	-0.01	1.09	50.14	0.01	1092.63
50.14	91.50	0.00	1.09	50.14	0.00	1092.63
50.14	91.51	0.00	1.09	50.14	0.00	1092.63
50.14	91.51	0.00	1.09	50.14	0.00	1092.63
50.14	91.51	0.00	1.09	50.14	0.00	1092.63
50.14	91.51	0.00	1.09	50.14	0.00	1092.63

is well-behaved around the root, which contributes to the convergence of the Newton-Raphson method.

Analysis of Convergence

The Q_{next_diff} column clearly shows the convergence:

- The difference is initially 0.83.
- It then drops significantly and finally 0.00 from iterations 6 onwards

This rapid decrease in the difference between successive estimates of Q indicates that the method is efficiently finding the root of the equation.

Optimum EOQ Value

Based on the convergence observed in the table, the optimum EOQ value appears to be approximately 50.14 say 50 units. This is the value at which $f(Q)$ becomes equal to zero, satisfying the condition for solving the transcendental equation.

Reasons for the value being the likely optimum

Root of the equation

The Newton-Raphson method aims to find the root of $f(Q) = 0$, which corresponds to the EOQ that balances the costs in the inventory model with perishability.

Convergence

The stable convergence of the iterations towards 50.14 suggests that this value is a consistent solution found by the numerical method.

Small $f(Q)$

At the final iterations, the value of $f(Q)$ is essentially zero, indicating that the estimated Q satisfies the EOQ equation with a high degree of accuracy.

Total cost

The analysis of total cost indicates that the total cost remains constant within a narrow band.

Comparison Report: Case-(ii) Method (i) vs Method (ii)

Case-(ii) Method (i): Numerical Method

Method (i) involves an iterative adjustment approach where EOQ is refined based on adjusted demand and decay rates. The process converges after several iterations to the following:

- Final EOQ: 50.14 units
- Cycle Time: 91.51 days
- Total Cost: ₹1357

Case-(ii) Method (ii): Newton-Raphson Approach

Method (ii) applies the Newton-Raphson method, using the derivative of the cost function to iteratively reach the EOQ that minimizes total cost. It converges within six iterations:

- Final EOQ: 50.14 units
- Cycle Time: 91.51 days

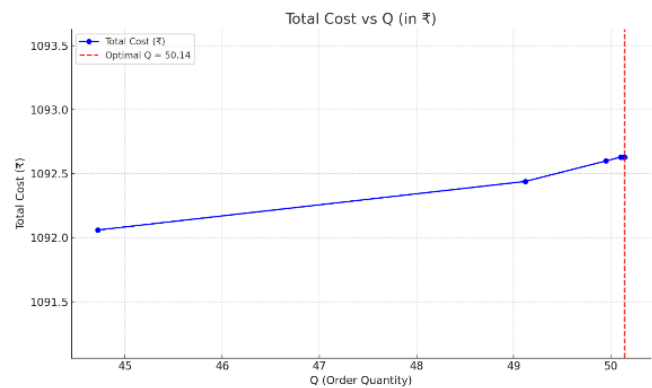


Figure 2: Total Cost (vs) Q^*

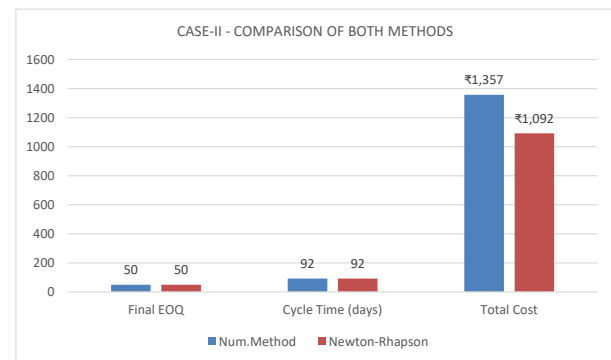


Figure 3: Case-(ii) Comparison of both methods

- Total Cost: ₹1092.63

Conclusion

While both methods converged to the same EOQ value (50.14 units) and cycle time (91.51 days), the Newton-Raphson method achieved a significantly lower total cost (₹1092.63 vs ₹1357). This demonstrates the Newton-Raphson method's strength in precision and efficiency when optimizing continuous cost functions. Method (ii) is thus preferred in applications requiring cost minimization under perishability constraints.

CASE-(iii)

Calculate time T^* (with perishability of 2.5% of goods every 10 days)

Instead of ordering whenever inventory hits a threshold (as in EOQ), you order every fixed number of days (T) — say, every 30 days, 60 days, etc. You don't wait for stock to run out. But here's the twist: when perishability is involved, time becomes the enemy. Because the longer you hold stock, the more you lose to spoilage.

Case-(iii) Method-(i) Find T^*

Solving Transcendental equation through numerical method
Let:

D Demand

T Ordering interval in days (say, 30 days)
 K Perishing rate per day
 A Order Cost
 H Holding Cost
 P Purchase price

Model formulation

Number of orders per year $N = \frac{365}{T}$
 Order Quantity per Cycle Q without perishability $= D * \frac{T}{365}$
 Inventory Decay Over Time accounting for continuous decay during the average holding time.
 Effective average inventory $\approx \frac{Q}{2} * e^{-kt/2}$
 Total annual cost (with fixed interval T)
 Ordering cost $= \frac{365}{T} * A$
 Holding cost $= \frac{Q}{2} * e^{-kt/2} * H * \frac{365}{T}$
 Purchase cost due to perishability $= \frac{P * D}{e^{-kt/2}}$

$$\text{Total cost due to perishability} = \frac{365}{T} * A + \frac{Q}{2} * e^{-kt/2} * H * \frac{365}{T} + \frac{P * D}{e^{-kt/2}}$$

Numerical Example

D = 200, A = 10, H = 2, k = 0.0025

Ordering every 60 days seems optimal in this case. With perishability, smaller, more frequent orders tend to reduce waste and holding costs. The total cost is also minimum for this cycle time.

Case-(iii) Method-(ii) Find T*

Solving transcendental equation through newton-raphson method

To develop the required equations for using Newton-Raphson to find the optimal value of T (the ordering interval in days) that minimizes the Total Cost (TC), we first need to construct the full expression for TC(T), and then derive the first and second derivatives of TC(T), since Newton-Raphson uses the iteration

$$T_{n+1} = T_n - \frac{TC'(T_n)}{TC''(T_n)}$$

Model Formulation

Given Demand D = 200 units / year, Ordering cost A = 10, Holding cost H = 2 per unit per year, Purchase cost P = 5 per unit, Decay rate k = 0.005, Ordering interval T in days.

Converting T from days to years $T_{\text{years}} = \frac{T}{365}$
 Number of orders per year $N = \frac{365}{T}$

Order qty per cycle Q = D * $\frac{T}{365}$
 Effective average inventory with perishability $= \frac{Q}{2} * e^{-kt} = \frac{D * T}{2 * 365} * e^{-kt}$

Ordering cost $C_o = \frac{365}{T} * A$

Holding cost $C_h = (\frac{D * T}{2 * 365} * e^{-kt}) * H$

Table 6: Case-(iii) Method-(i) Numerical Method

T	N	Q	$e^{-kt/2}$	D'	TOT.COST
120	3.04	65.75	0.861	232.37	1364.39
110	3.32	60.27	0.872	229.48	1354.89
100	3.65	54.79	0.882	226.63	1346.15
90	4.06	49.32	0.894	223.81	1338.35
80	4.56	43.84	0.905	221.03	1331.76
70	5.21	38.36	0.916	218.29	1326.83
60	6.08	32.88	0.928	215.58	1324.27
50	7.30	27.40	0.939	212.90	1325.38
40	9.13	21.92	0.951	210.25	1332.77
30	12.17	16.44	0.963	207.64	1352.52
20	18.25	10.96	0.975	205.06	1402.88

$$\text{Total cost } T(C) = \frac{365}{T} * A + \frac{D * H}{2 * 365} * T * e^{-kT}$$

First Derivative TC'(T)

$$TC'(T) = \frac{d}{dT} \left(\frac{365}{T} * A + \frac{D * H}{2 * 365} * T * e^{-kT} \right)$$

Differentiating term by term

$$\frac{d}{dT} \left(\frac{365A}{T} \right) = - \frac{365A}{T^2}$$

$$\frac{d}{dT} \left(\frac{D * H}{2 * 365} * T * e^{-kT} \right) = \frac{D * H}{2 * 365} * (e^{-kT} - k * T * e^{-kT})$$

$$TC'(T) = - \frac{365A}{T^2} + \frac{D * H}{2 * 365} * (e^{-kT} - k * T * e^{-kT})$$

Second Derivative TC''(T)

Differentiate TC'(T)

$$TC'(T) = \frac{d}{dT} \left(- \frac{365A}{T^2} + \frac{D * H}{2 * 365} * (e^{-kT} - k * T * e^{-kT}) \right)$$

Differentiating term by term

$$\frac{d}{dT} \left(- \frac{d}{dT} \left(\frac{D * H}{2 * 365} * (e^{-kT} - k * T * e^{-kT}) \right) \right)$$

$$\frac{d}{dT} (e^{-kT} - k * T * e^{-kT}) = -k * e^{-kT} - k * (e^{-kT} - k * T * e^{-kT})$$

$$= -2 * k * e^{-kT} + k^2 * T * e^{-kT}$$

$$TC''(T) = \frac{730A}{T^3} + \frac{D * H}{2 * 365} * (-2 * k * e^{-kT} + k^2 * T * e^{-kT})$$

$$\text{Let } C = \frac{D * H}{2 * 365}$$

Then

$$TC(T) = \frac{365}{A} + C * T * e^{-Kt}$$

$$TC'(T) = -\frac{365 * A}{T^2} + C * (e^{-Kt} - K * T * e^{-Kt})$$

$$TC''(T) = \frac{730 * A}{T^3} + C * (-2 * K * e^{-Kt} + K^2 * T * e^{-Kt})$$

$$\text{Newton-Raphson formula } T_{n+1} = T_n - \frac{TC'(T_n)}{TC''(T_n)}$$

Total cost with purchase cost

$$TC(T) = \frac{365A}{T} + \frac{D * H}{2 * 365} * T * e^{-Kt} + P * D$$

Observations

The Newton-Raphson method converges rapidly: the change becomes negligible (<0.01) by the 8th iteration.

Final optimal value of T≈110

Both the first and second derivatives (TC' and TC'') approach 0 at the optimum, which confirms we have found a local minimum.

Comparison of Numerical Method and Newton-Raphson for Optimal Ordering Interval

Numerical method

The Numerical method evaluated total cost over a range of T values (from 15 to 120 days). It found the minimum total cost at T = 60 days:

- Optimal T (from table): 60 days
- Total Cost: ₹1324.27

Newton-Raphson Method

The Newton-Raphson method iteratively finds the minimum of the total cost function by using its first and second derivatives. It converged in 7–8 iterations and identified the optimal ordering interval as:

- Optimal T: 109.97 days
- Minimum Total Cost: ₹1078.96

The Newton-Raphson method provides a more precise and cost-effective solution compared to the Numerical method in this scenario. Its ability to use derivative

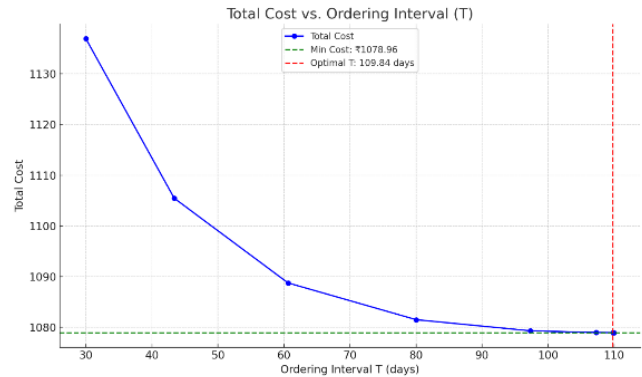


Figure 4: Total cost (vs) T Ordering interval

information enables it to locate the true global minimum, while the Numerical method, limited by the granularity of sampled T values, may identify only a local minimum. Cycle Time value is higher and Cost is lower in the case of Newton-Raphson method.

CASE-(ii) – Effect of variation of Perishability and Demand on the Total Cost.

Use case scenarios have been worked out by varying Perishability and Demand and their effect on Total Cost has been worked out, under both the methods.

Variation of Perishability

Given Demand = 200, Ordering Cost = 10, Holding Cost = 2, Purchase Cost = 5

Perishability = Varied from 0.0015 to 0.0095. The effect on total cost is given Figure 6.

As the perishability is increased, the total cost under Numerical Method is much more than that of Newton-Raphson method. In the Newton-Raphson method, the variation in perishability practically does not have any significant impact on the total cost.

Variation of Demand

Given Perishability = 0.0025 Ordering Cost = 10, Holding Cost = 2, Purchase Cost = 5

Demand = Varied from 200 to 1200 units. The effect on total cost is given Figure 7.

Table 7: Case-(iii) Method-(ii) Newton Raphson

T_n	$TC'(T_n)$	$TC''(T_n)$	T_{n+1}	$T_{n+1} - T_n$
30	-3.59	0.27	43.38	13.38
43.38	-1.50	0.09	60.62	17.24
60.62	-0.59	0.03	80.03	19.41
80.03	-0.21	0.01	97.29	17.27
97.29	-0.06	0.01	107.30	10.01
107.30	-0.01	0.00	109.84	2.54
109.84	0.00	0.00	109.97	0.13
109.97	0.00	0.00	109.97	0.00
109.97	0.00	0.00	109.97	0.00

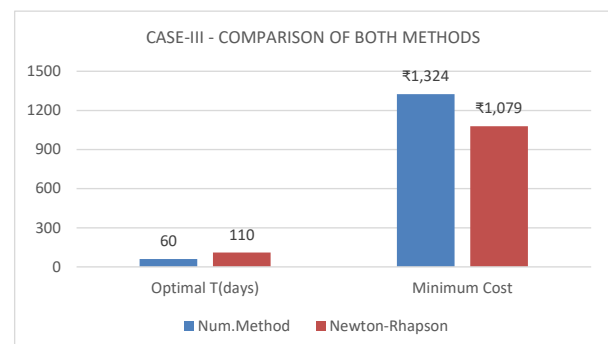


Figure 5: Case-(iii) Comparison of both methods

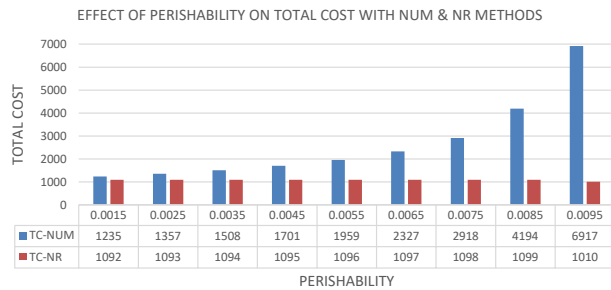


Figure 6: Effect of variation of Perishability on Cost

The Cost under Newton-Raphson method is lower than that of Numerical method.

Discussions

The enhanced EOQ models presented in this study effectively address perishability in inventory systems, offering practical relevance for industries like food and pharmaceuticals. Case-(i), which considers a flat 2.5% loss of demand, provided a baseline with a total cost of ₹1116.22 and demonstrated that perishability significantly influences total cost more than ordering or holding costs. Case-(ii) introduced time-based perishability, yielding more dynamic insights. Both the numerical and Newton-Raphson methods converged to an EOQ of 50.14 units, but the Newton-Raphson method achieved a lower total cost (₹1092.63), proving more efficient and precise in handling complex cost functions.

Case-(iii) shifted the focus from quantity to ordering interval (T), with the Newton-Raphson method again outperforming the numerical method by identifying a more cost-effective interval (T = 109.97 days, ₹1078.96 cost). Across all cases, models that explicitly incorporate perishability and apply advanced optimization methods offer superior inventory decisions. The study highlights the importance of selecting appropriate methods for minimizing total cost and suggests that derivative-based approaches are particularly effective in real-world, non-linear scenarios.

Potential Applications

Perishable food supply chains

Industries dealing with fresh produce, dairy, seafood, and meat face acute perishability issues. Applying the time-sensitive EOQ models (Case-ii and Case-iii) enables

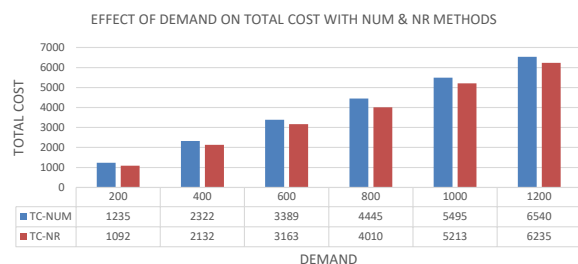


Figure 7: Effect of variation of Demand on Cost

wholesalers, retailers, and cold storage operators to optimize order sizes and replenishment frequency, thereby reducing spoilage, preserving quality, and minimizing total cost.

Retail with seasonal or expiring inventory

Fashion apparel, electronics, and promotional items often have limited shelf-lives due to seasonal trends or rapid technological changes. By incorporating decay rates into inventory planning, retailers can better align procurement cycles with demand patterns, minimizing end-of-season losses and clearance-related markdowns.

Healthcare logistics

Hospitals, clinics, and pharmaceutical distributors manage inventories of temperature- or time-sensitive products such as vaccines, insulin, blood products, and medications. The Newton-Raphson method, with its precision in minimizing costs under continuous decay, is particularly effective in this context, ensuring timely replenishment and minimizing expired stock.

Floriculture and nursery products

Cut flowers, ornamental plants, and nursery saplings are highly perishable commodities. Employing EOQ models that consider exponential decay enables growers and exporters to synchronize harvest, storage, and shipment schedules, preserving product value and reducing return rates.

Chemical and laboratory reagents

Reagents and certain chemical compounds degrade under environmental stress or over time. Laboratories and industrial users can apply the time-based decay models to determine safe inventory levels that minimize obsolescence while ensuring operational readiness.

Meal kit and subscription services

Companies offering ready-to-cook meals, fresh produce boxes, or curated monthly kits face logistical challenges related to perishability. Implementing optimized EOQ models helps such businesses manage lead times, reduce inventory waste, and enhance customer satisfaction through reliable, fresh deliveries.

E-Commerce with fast fashion and trend-driven goods

Online platforms dealing in fast fashion, limited-edition merchandise, or viral products face perishability not of material but of consumer interest. The EOQ framework with decay parameters can be adapted to model declining demand or relevance, guiding sellers to prevent overstock and maximize revenue within narrow sales windows.

Emergency stockpiling and disaster relief

Organizations stockpiling critical items like food kits, water, and medical supplies for disaster response must manage inventories that degrade or expire. Perishable EOQ models allow planners to optimize replenishment cycles, ensuring

preparedness while minimizing waste due to shelf-life expiry.

Conclusion

The study demonstrates that incorporating perishability into EOQ models leads to more realistic and cost-effective inventory strategies. Among the approaches, Case-(ii)—which models continuous time-based decay—offers the most accurate optimization, with the Newton-Raphson method delivering the lowest total cost. These findings help inventory planners make better decisions by aligning order quantities and intervals with the rate of perishability, ultimately minimizing waste and enhancing operational efficiency.

Acknowledgements

The authors would like to thank Universiti Tun Hussein Onn Malaysia, Holy Cross College India, and BP Renalcare Sdn. Bhd. for the financial support through Industry Grant Vot M085.

References

- Abad, P.L., & Jaggi, C.K. (2003). A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. *International Journal of Production Economics*, 83, 115–122. [https://doi.org/10.1016/S0925-5273\(02\)00142-1](https://doi.org/10.1016/S0925-5273(02)00142-1)
- Ali, S.S., Madaan, J., Chan, F.T.S., & Kannan, S. (2013). Inventory management of perishable products: A time decay linked logistic approach. *International Journal of Production Research*, 51(13), 3864–3879. <https://doi.org/10.1080/00207543.2012.752587>
- Bakker, M., Riezebos, J., & Teunter, R. (2012). Review of inventory systems with deterioration since 2001. *European Journal of Operational Research*, 221(2), 275–284. <https://doi.org/10.1016/j.ejor.2012.03.004>
- Chaudhary, V., Kulshrestha, R., & Routroy, S. (2018). State-of-the-art literature review on inventory models for perishable products. *Journal of Advances in Management Research*, 15(3), 306–346. <https://doi.org/10.1108/JAMR-09-2017-0091>
- Lee, C.C. (2006). Two-warehouse inventory model with deterioration under FIFO dispatching policy. *European Journal of Operational Research*, 174(2), 861–873. <https://doi.org/10.1016/j.ejor.2005.03.027>
- Li, Y., Cheang, B., & Lim, A. (2012). Grocery perishables management. *Production and Operations Management*, 21(3), 504–517. <https://doi.org/10.1111/j.1937-5956.2011.01288.x>
- Liao, H.C., Tsai, C.H., & Su, C.T. (2000). An inventory model for deteriorating items under inflation when delay in payment is permissible. *International Journal of Production Economics*, 63(2), 207–214. [https://doi.org/10.1016/S0925-5273\(99\)00015-8](https://doi.org/10.1016/S0925-5273(99)00015-8)
- Mirabelli, G., & Solina, V. (2022). Optimization strategies for the integrated management of perishable supply chains: A literature review. *Journal of Industrial Engineering and Management*, 15(1), 1–24. <https://doi.org/10.3926/jiem.3603>
- Nahmias, S. (1982). Perishable inventory theory: A review. *Operations Research*, 30(4), 680–708. DOI:10.1287/opre.30.4.680
- Olsson, F., & Tydesjö, P. (2010). Inventory problems with perishable items: Fixed lifetimes and backlogging. *European Journal of Operational Research*, 202(1), 131–137. <https://doi.org/10.1016/j.ejor.2009.05.010>
- Piramuthu, S., & Zhou, W. (2013). RFID and perishable inventory management with shelf-space and freshness-dependent demand. *International Journal of Production Economics*, 144(2), 635–640. <https://doi.org/10.1016/j.ijpe.2013.04.035>
- Rameswari, M., & Uthayakumar, R. (2018). An integrated inventory model for deteriorating items with price-dependent demand under two-level trade credit policy. *International Journal of Systems Science: Operations & Logistics*, 5(3), 253–267. <https://doi.org/10.1080/23302674.2017.1292432>
- Rani, S., Ali, R., & Agarwal, A. (2019). Fuzzy inventory model for deteriorating items in a green supply chain with carbon concerned demand. *Opsearch*, 56(1), 91–122. <https://doi.org/10.1007/s12597-019-00361-8>
- Rastogi, M., & Singh, S.R. (2019). An inventory system for varying deteriorating pharmaceutical items with price-sensitive demand and variable holding cost under partial backlogging in healthcare industries. *Sādhanā*, 44(4), 95. <https://doi.org/10.1007/s12046-019-1075-3>
- Tekin, E., Gürler, Ü., & Berk, E. (2001). Age-based vs. stock-level control policies for a perishable inventory system. *European Journal of Operational Research*, 134(2), 309–329. [https://doi.org/10.1016/S0377-2217\(00\)00250-2](https://doi.org/10.1016/S0377-2217(00)00250-2)
- Zhou, D., Leung, L.C., & Pierskalla, W.P. (2011). Inventory management of platelets in hospitals: Optimal inventory policy for perishable products with regular and optional expedited replenishments. *Manufacturing & Service Operations Management*, 13(4), 420–438. <https://doi.org/10.1287/msom.1110.0334>