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RESEARCH ARTICLE

The green inventory model for sustainable environment that includes degrading products and backordering with integration of environmental cost

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Abstract

The classical inventory model is traditionally used in inventory management to maximize profit and minimize the total cost of ordering and holding inventory. In the past few years, many researchers have developed inventory models with environmental concerns and through the integration of environmental cost or green cost. This paper focuses on the Green Inventory Model for a sustainable environment that includes degrading products and backordering with the integration of environmental cost. Stock contains degrading products that degrade at a rate proportional to the level of inventory per unit time and the inventory model takes back ordering for increasing the demand and attracting customers to buy green products. A green product refers to a product that is manufactured without affecting the environment. The objective of this research is to produce the eco-friendly products which has the maximum profit, minimum waste and loss and reduce the carbon emissions.

Keywords: Green inventory model, Degrading products, Backordering, Environmental cost, Carbon emission, Sustainable environment.

关键词： 绿色库存模型、降级产品、缺货、环境成本、碳排放、可持续环境。

Introduction

Environmental sustainability has become a critical concern across industries. Previously, the inventory models were created with cost concerns that were to maximize profit and minimize losses and wastages. The current situation needs research into the factors that affect the environment and its sustainability. The order cycle must be constant because the stock includes the degrading products. The replenishment of the stock is instant. Degrading products are degrading at a rate proportional to the inventory level per unit time. The primary concern about the inventories which has an expiry date is sold out in the first order. The excess production and storage are affecting the environment. The production has a limit, which dependent on the placed orders. This process avoids energy consumption and helps to sustain the environment.

The environment has been affected by many factors such as emission of greenhouse gases, pollution, waste from the industries, energy consumption, resource depletion, transportation, overproduction and overstocking, etc. The green inventory model is derived for reducing emissions by adding the environmental cost as a green cost. The environment cost consists of green branding cost, fixed cost to dispose waste to the environment, cost to dispose waste to the environment proportion of waste produced per lot, proportion of demand returned, environmental detection cost, payments for permits and licenses to use environmental assets, environmental remediation cost, pollution prevention cost. To reduce the emission costs above are included with environmental costs. This research is aimed save the natural resources for future generations.

The rest of the paper is formulated as follows: Section 2 explains the literature review. Section 3 presents the mathematical formulation of the proposed model. Section 4 examines a numerical example and Section 5 has a brief

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explanation about the results and discussion. Finally, Section 6 concludes the outcome of this work and sorts out the references.

Literature Review

Environmental sustainability has become a major concern in the research field. Introduce the environmental cost to the inventory model, which creates environmentally sustainable inventory models. Many researchers have conducted research on inventory models with the green concept. Salameh, M. K., and Jaber, M. Y. (2000) created an Economic production quantity model for items with imperfect quality. Bonney, M., and Jaber, M. Y. (2011) developed environmentally responsible inventory models: Non-classical models for a non-classical era. Mashud, A. H. M., Roy, D., Daryanto, Y., and Ali, M. H. (2020) formulated A sustainable inventory model with imperfect products, deterioration, and controllable emissions. Pattnaik, S., Nayak, M. M., Abbate, S., and Centobelli, P. (2021) summarize Recent trends in sustainable inventory models: A literature review. Indriastuti, M., and Chariri, A. (2021) explain the role of green investment and corporate social responsibility investment on sustainable performance. Caliskan, C. (2021) derived A simple derivation of the optimal solution for the EOQ model for deteriorating items with planned backorders. Jauhari, W. A., and Wangsa, I. D. (2022) are constructed A manufacturer-retailer inventory model with remanufacturing, stochastic demand, and green investments. Paul, A., Pervin, M., Roy, S. K., Maculan, N., and Weber, G. W. (2022) developed A green inventory model with the effect of carbon taxation. Selvi, P., & Ritha, W. (2022) are formulated EOQ and EPQ inventory models for pollution control with integration of environmental cost. Ruidas, S., Seikh, M. R., & Nayak, P. K. (2022). A production inventory model for green products with emission reduction technology investment and green subsidy.

Mathematical Formulation

To develop the proposed model, the following notations and assumptions are used.

Notations

$X(t)$ – Inventory level
D – Demand rate per unit time
O_c – Ordering cost per unit time
H_c – Holding cost per unit time
B_0 – Back ordering cost per unit time
C_0 – Cost of the item per unit time
θ – Degrading rate per unit of the item per unit time
Q – Number of units ordered in every ordering cycle
B_N – Number of units back ordered in every ordering cycle
T_0 – Time between orders and order interval
T_p – Time when inventory is positive in each ordering cycle
G_c – Green branding Cost
C_f – Fixed cost to dispose waste to the environment
C_c – Cost to dispose waste to the environment
λ – Proportion of waste produced per lot
β – Proportion of demand returned
$Đ$ – Environmental detection cost
$Ɛ$ – Payments for permits and licenses to use environmental assets
R – Environmental remediation cost
$Ɛ'$ – Pollution Prevention cost

Assumptions

- Demand is known and constant.
- Products are degrading at a constant rate proportional to the inventory level.
- Degraded products are removed and they are not replaced with good products.
- Replenishment is instantaneous. All products in an order are received at home.
- Managing waste is consist of reduction, recycle, reuse and pollution prevention.
- Pollution prevention is leads to reduce the carbon emission.
- To create public awareness for environmental sustainability the environmental remediation cost was included.

Mathematical Model

The mathematical model for sustainable environment that includes degrading products and backordering with integration of environmental cost are formulated. The model was created for environmental sustainability which includes the Green branding Cost, Fixed cost and usual cost to dispose waste to the environment, Proportion of waste produced per lot, proportion of demand returned, Environmental detection cost, Payments for permits and licenses to use environmental assets, Environmental remediation cost, Pollution Prevention cost.

The inventory level is represented by $X(t)$, and $\theta(t)$ is the instantaneous rate of deterioration at time t. Assuming that the inventory is replenished when the ordering cycle ends at time T and derive the following inventory level and the order quantity equations for each ordering cycle [Cenk Caliskan].

$$X(t) = (Q - B_N)e^{-\lambda t} - \frac{D}{\theta}\left(1 - e^{-\lambda t}\right) \qquad 0 \le t < T_p \qquad (1)$$

$$Q = \frac{D}{\theta}\left(e^{\theta T_p} - 1\right) + B_N \qquad (2)$$

$$B_N = D(T_0 - T_p) \qquad (3)$$

The manufactured inventory includes the degrading items. So the total cost function neglecting certain amount from

the total amount of the inventories. The quantity of the inventories are,

$$X(t) = Q - B_N - (D+\theta)t \qquad 0 \le t < T_1 \tag{4}$$

$$Q = (D+\theta)T_p + B_N \tag{5}$$

From (5) $T_p = \dfrac{Q - B_N}{D+\theta}$ (*)

Then the expression (3) becomes $\dfrac{B_N}{D} = (T_0 - \dfrac{Q - B_N}{(D+\theta)})$ (6)

The model for fundamental degrading items:

The total cost function contains ordering cost O_c, holding cost H_c, cost of an item C_0, Demand rate D and degrading rate θ. Then the Total cost value is [Cenk Caliskan],

$$\mathbb{C}\ (Q) = \frac{O_c(D+\theta)}{Q} + C_0\theta + \frac{H_c \times Q}{2} \tag{7}$$

Differentiate the total cost function with respect to 'Q' and equating to zero. Then the Optimum order quantity is obtained.

$$Q^* = \sqrt{\frac{2O_c(D+\theta)}{H_c}} \tag{8}$$

The degrading items with backordering:

The total cost function is calculated with the degrading items. It includes ordering cost O_c, holding cost H_c, cost of an item C_0, Demand rate D, degrading rate θ, backordering cost B_0, Time between orders and order interval T_0, Time when inventory is positive in each ordering cycle T_p.The total cost value with respect to quantity, backordering and Time is [Cenk Caliskan],

$$\mathbb{C}\ (Q, B_N, T_0) = \frac{O_c}{T_0} + \frac{H_c(Q - B_N)^2}{2Q} + \frac{B_0 B_N^2}{2Q} \tag{9}$$

The total cost function with the time when inventory is positive in each ordering cycle T_p,

$$\mathbb{C}\ (Q, B_N, T_0, T_p) = \frac{O_c}{T_0} + \frac{C_0(Q - B_0 - \mathbb{D}_p)}{T_0} + \frac{H_c(Q - B_N)^2}{2}\left(\frac{T_p}{T_0}\right) + \frac{B_0 B_N^2}{2Q}\left(\frac{T_0 - T_p}{T_0}\right) \tag{10}$$

When $T_0 = T_P$ and $B_N = 0$ the expression (5) becomes $T_0 = \dfrac{Q}{D+\theta}$

By (*) $T_p = \dfrac{Q - B_N}{D+\theta}$ substitute T_0, T_p value in (10).

$$\mathbb{C}\ (Q, B_0) = \frac{O_c(D+\theta)}{Q} + \frac{C_0\theta(Q - B_0)}{Q} + \frac{H_c(Q - B_N)^2}{2Q} + \frac{B_0 B_N^2}{2Q} \tag{11}$$

The backordering cost and holding cost is defined by a constant $r = \dfrac{B_0}{B_0 + H_c}$. The optimum order quantity is

$$Q^* = \sqrt{\frac{2O_c(D+\theta)}{H_c}}\sqrt{\frac{B_0 + H_c}{B_0}}\sqrt{\frac{D(B_0 + H_c)}{D(B_0 + H_c) + H_{c\theta}}}$$

The optimal order quantity and backorder quantity:

The optimal backorder quantity, $B_0^* = \left(\dfrac{H_c}{B_0 + H_c}\right)Q^*$

Combining and solving the above total cost functions, get the total cost function with optimal order quantity and optimal backorder quantity [Cenk Caliskan].

$$\mathbb{C}\ (Q) = \frac{O_c(D+\theta)}{Q} + \frac{C_0 B_0 \theta}{B_0 + H_c} + \frac{H_c B_0 Q}{2(B_0 + H_c)} \tag{12}$$

The optimal order quantity $Q^* = \sqrt{\dfrac{2O_c(D+\theta)}{H_c}}\sqrt{\dfrac{B_0 + H_c}{B_0}}$ (13)

Including green concept into this inventory model the output is to sustain the environment for the future generations. The green cost is added for converting the inventory model into the green inventory model.

$$G = \frac{D}{Q}\left[G_c + C_f + C_c(\lambda+\beta)Q + \mathbb{D}\ \ + \mathbb{P}\ \ + \mathbb{R}\ \ + \mathbb{P}\ \right] \tag{14}$$

$$G = \frac{D}{Q}\left[G_c + C_f + \mathbb{D}\ \ + \mathbb{P}\ \ + \mathbb{R}\ \ + \mathbb{P}\ \right] + C_c(\lambda+\beta)D \tag{15}$$

Adding the green cost (14) to the total cost (12), obtain the total cost for the green inventory model.

$$\mathbb{C}\ (Q) = \left[\begin{array}{l} \dfrac{O_c(D+\theta)}{Q} + \dfrac{C_0 B_0 \theta}{B_0 + H_c} + \dfrac{H_c B_0 Q}{2(B_0 + H_c)} \\ + \dfrac{D}{Q}\left[G_c + C_f + C_c(\lambda+\beta)Q + \mathbb{D}\ \ + \mathbb{P}\ \ + \mathbb{R}\ \ + \mathbb{P}\ \right] \end{array}\right] \tag{16}$$

Differentiate (12) and (15) with respect to 'Q' and equating to zero.

$$-\frac{O_c(D+\theta)}{Q^2} + \frac{H_c B_0}{2(B_0 + H_c)} - \frac{D}{Q^2}\left[G_c + C_f + \mathbb{D}\ \ + \mathbb{P}\ \ + \mathbb{R}\ \ + \mathbb{P}\ \right] + 0 = 0$$

Manipulate the above equation, obtain the optimal order quantity for green inventory model.

$$Q^* = \sqrt{\frac{2(B_0 + H_c)\left[O_c(D+\theta) + D(G_c + C_f + \mathbb{D}\ \ + \mathbb{P}\ \ + \mathbb{R}\ \ + \mathbb{P}\ \right]}{B_0 H_c}} \tag{17}$$

Numerical example

Consider the following parameters to illustrate the proposed model:

G_c	3/unit
C_f	1/unit
C_c	0.8/unit
λ	0.5
β	0.2
ED	28/unit
EP	12/unit
ER	32/unit
PC	95/unit

The above values are fixed for the entire process of manufacturing. Then the remaining parameters of the model is calculated by varying its values. The values and results are given in the Tables 1 and 2. In Table 1 the demand is fixed for minimum value and in Table 2 demand is fixed for maximum value.

Table 1: Demand is fixed for minimum value

O_c	D	H_c	B_0	θ	Q	TC
10	50	1	5	0.05	147.38	205.34
10	50	5	10	0.01	73.69	270.11
10	100	5	10	0.05	104.21	323.82
100	50	5	10	0.05	90.183	346.33
1000	50	5	10	0.1	187.59	479.86

Table 2: Demand is fixed for maximum value

O_c	D	H_c	B_0	θ	Q	TC
50	7500	1	5	0.01	1994.49	1159.54
50	7500	1	5	0.05	1994.49	1159.57
50	7500	1	5	0.25	1994.49	1159.74
50	7500	1	5	0.5	1994.50	1159.95
50	7500	1	5	0.75	1994.51	1159.17

Results

The obtained optimum order quantity values and the total cost values are examined the created model. Thus this green inventory model is reducing the environmental impact which is affected by the industry's manufacturing process. Including the green cost is benefit for both industries and environment.

Discussion

Caliskan, C. (2021) derived A simple derivation of the optimal solution for the EOQ model for deteriorating items with planned backorders. Jauhari, W. A., and Wangsa, I. D. (2022) are constructed A manufacturer-retailer inventory model with remanufacturing, stochastic demand, and green investments. Paul, A., Pervin, M., Roy, S. K., Maculan, N., and Weber, G. W. (2022) developed A green inventory model with the effect of carbon taxation. Selvi, P., & Ritha, W. (2022) are formulated EOQ and EPQ inventory models for pollution control with integration of environmental cost. The developed model calculated in three concepts. There are 'The model for deteriorating items', 'Deteriorating items with backordering', 'Derivation of optimal order quantity'. The total cost and optimum order quantity expressions are found. Including the environmental cost with the total cost makes the inventory model into the Green Inventory Model. Then the model is examined with the numerical example. The results are given in two tables by varying the demand value. Finally, the result says that adding environmental cost is an investment for the future sustainability of environment. Now the investment convert into a profit within next few years. This Green Inventory Model is benefit for environmental sustainability for the future generations and economic efficiency for the industries.

Conclusion

The Green Inventory Model incorporating environmental cost represents a significant advancement in sustainable inventory management. Integrating the ecological considerations such as carbon emission, energy consumption and waste management into traditional inventory model supports cost-effectiveness and promotes environmental responsibility. It encourages industries to convert from profit-driven approach to a more balanced strategy aligns economic goals with ecological sustainability. The developed model satisfied the goal that is to sustain the environment for future generations.

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