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STUDY OF BARDEEN COOPER STATE (BCS) TO BOSE EINSTEIN CONDENSATION (BEC) CROSSOVER

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ABSTRACT

This study presents a simple idea and a conceptual understanding of the crossover from the Bardeen Cooper (BCS) state of weakly-correlated pairs of fermions to the Bose{Einstein condensation (BEC) of diatomic molecules in the atomic Fermi gas. In the context of ultra cold Fermi gases, a BCS-BEC crossover means that by tuning the interaction strength, one goes from a BCS state to a BEC state without encountering a phase transition. BEC state is a Bose-Einstein condensate of two-atom molecules (bound fermions), while the BCS state is made up of pair of atoms. The difference between the pairs and the molecules is that the molecules are localized in the real space, whereas the BCS pairs are made of two particles with opposite momenta in the momentum space.

INTRODUCTION

There we have discussed the theoretical development of BCS-BEC crossover physics. Superconductivity was discovered when the resistance of Hg goes to zero below a critical temperature and its resistivity becomes zero. The super-fluid phase of liquid ^{4}He was revealed, when the viscosity of the liquid ^{4}He goes below 2.17K. Actually they observed mesophase of super-fluid ^{3}He in which two phase ^{3}He-A phase and ^{3}He-B phase were very popular .In 1986, high temperature superconductors were discovered in Cooper oxide materials. The physical properties of this system vary widely, they are all linked by their behavior. These effects depend upon the system in electrically charged (superconductors) or neutral (super-fluid). The phenomenon of super-fluid were in the context of Bose- Einstein Condensation (BEC) which is of an ensemble of bosonic particle BEC is a consequence of the quantum statistics of bosons, which are particles with integral spin . ^{4}He is boson because it is made up of even number of ½ integer spin fermions electron, proton and neutron, fermion. BEC was overshadowed by the amazing success in 1957 of Bardeen Cooper Schieffer (BCS) theory of superconductivity (Bardeen et al, 1957). Cooper found that pair of fermions in the presence of filled Fermi sea, form a bound pair with an arbitrarily small interaction. The BCS theory solved this problem in many pair in Fermi sea.

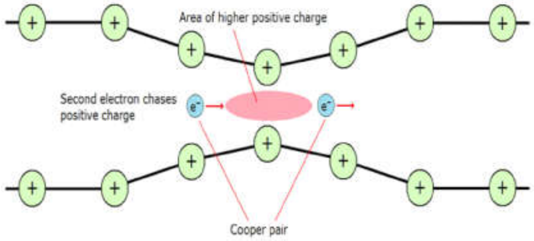
The BCS theory described conventional superconductors for which the attraction between fermions is much less than Fermi energy. The BCS theory ground state was able to describe super-fluid ^{3}He and many aspects of high critical temperature superconductors. The composite bosons were alkali atom such as ^{87}Rb has been cooled as a gas down to 10^{-9} K temperature. At these temperature, the thermal de Broglie wavelength of particles becomes on order of the inter-particle spacing in the gas and BEC is formed. M. R. Andrews et al (1997) observed condensates properties of coherent matter wave, M.R. Mathews and C. Raman et al (1999) observed the super-fluid nature of a condensate. In this way both the BEC and super-fluid properties could be clearly seen in one system and understood extremely well theoretically.

Recently a technique has been discovered in condense matter which is around to study highly condense matter. Experimentally verified BEC is much stronger interaction than alkali gases. A quantum phase transition to the high-correlated Mott insulator(MI) state was observe though studies of quantum gases in optical potential. These bosonic systems required that this theory goes beyond mean field interaction.

BSC theory was originally applied in the limit where interaction energy is extremely small to the Fermi energy. For this purpose the chemical potential can be fixed at Fermi energy and his calculations becomes easy. Nozieres (1985) and Randeria et al (1990) also used the gap equation and gave the structure of the crossover theory of physics. In the BCS limit, pairing and phase transition to a super-fluid state occurs at this temperature. In the BEC limit this is not the case because the constituent fermions are very tightly bound pairs and can form for above critical temperature. It is nature to expect that there would be a cross between these two behavior which is BCS-BEC crossover.

BCS theory and Superconductivity: A theory of superconductivity formulated by John Bardeen, Leon Cooper, and Robert Schrieer, explaining the phenomenon in which a current of electron pairs flows without resistance in certain materials at low temperatures.

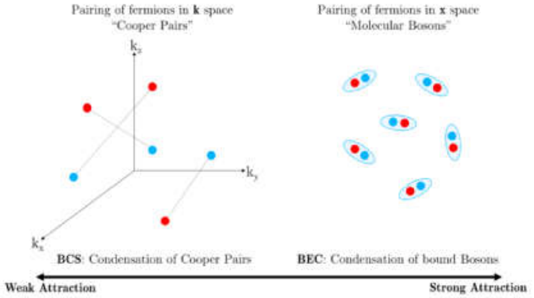
According to theory, superconductivity arises when a single negatively charged electron slightly distorts the lattice of atoms in the superconductor, drawing toward it a small excess of positive charge creating an electron-phonon pair. This excess, afterwards, attracts a second electron (electron-phonon interaction). It is this weak and indirect attraction that binds the electrons together, into a Cooper pair in momentum space as shown in the figure below.



The Cooper pairs within the superconductor carry the current without any resistance, but why? Mathematically, because the Cooper pair is more stable than a single electron within the lattice, it experiences less resistance. Physically, the Cooper pair is more resistant to vibrations within the lattice as the attraction to its partner will keep it more stable. Hence, Cooper pairs move through the lattice relatively unaffected by thermal vibrations (phonons) below the critical temperature.

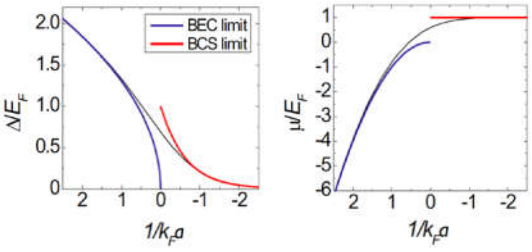
The existence of a single fermion pair, however, is not sufficient to describe the macroscopic behavior of superconductors. It is necessary to invent a collective and correlated state in which many electron (fermion) pairs acting together to produce a zero-resistance state. For that, BCS theory proposes a many particle wave function corresponding to largely overlapping fermion pairs with zero center of mass momentum, zero angular momentum (s-wave), and zero total spin (singlet) (DeMelo, 2008). Moreover, one of the most fundamental features of the BCS state is the existence of correlations between fermion pairs, which lead to an order parameter 1=kF as, for the superconducting state. This order parameter is found to be directly related to the energy gap Eg in the elementary excitation spectrum. Because all relevant fermions participating in the ground state of an s-wave superconductor are paired, creating a single fermion excitation requires breaking a Cooper pair with a energy cost. Thus the contribution of elementary excitations to the specific heat and other thermodynamic properties shows exponential behavior _ exp (ÀÛÜEg=T) at low temperature (Regal, 2006). BCS theory successfully applied to many experimental results and explained the physics behind the conventional superconductors until the discovery of cuprates.

From BCS to BEC super-fluids: Even though the BCS theory became successfully and widely applicable to many phenomena, it is basically a weak attraction theory. A generalization of the BCS theory has been developed to include the strong attraction regime in which fermion pairs become tightly bound diatomic Bose molecules and undergo Bose{Einstein condensation. BEC state is on the the strong attraction side of the phase space and is formed by the condensation of bound fermions in real space known as molecular bosons.



The simplest conceptual picture of the BCS to BEC super-fluidity evolution for s-wave (l = 0) pairing can be constructed for low fermion densities and short-ranged interactions that are much smaller than the average separation between fermions (DeMelo, 2008). In the BCS regime, the attraction is weak, the pairs are much larger than their average separation, and they overlap substantially in the momentum space. In the BEC regime, the attraction is strong, the pairs are much smaller than their average separation, and they overlap very little in the real space. However the clear picture of evolution between these two regimes is understood at zero temperature, in 1980, when Leggett proposed a simple description in real space of paired fermions with opposite spins. Leggett considered a zero-ranged attractive potential (contact interaction) between fermions and showed that when the attraction is weak, a BCS super-fluid appeared, and when the attraction is strong, a BEC super-fluid emerged. And this result is generalized in 1993 by Melo, Randeria, and Engelbrecht by using a zero-ranged attraction potential characterized by the experimentally measurable length scale as, (s wave scattering length). Also, since the natural momentum scale is Fermi wave-vector kF , the strength of the attractive interactions can be characterized by the dimensionless scattering parameter 1=kF as this attraction strength parameter changes sign from BCS to BEC side and becomes negative for attractive pairs and positive for repulsive. That is why, in the BCS side pairs substantially overlap for negative as unlike the positive and repulsive as of the BEC side.

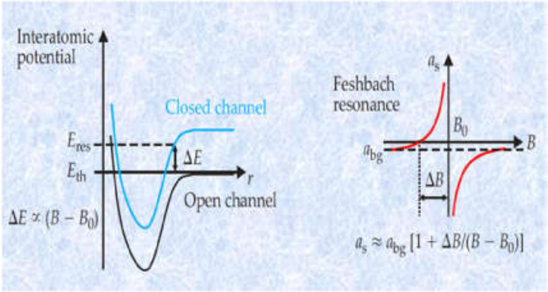
The phase diagram illustrates two important physical concepts of the BCS-to-BEC crossover. First, the normal, non-condensed state for weak attractions is a Fermi liquid that evolves smoothly into a molecular Bose liquid without a phase transition. These two regimes are separated by a pair formation (or molecular dissociation)



temperature T pair characterized by chemical equilibrium between bound fermion pairs and unbound fermions. Second, T pair and the critical temperature Tc are more or less the same in the BCS limit which means that pairs form and condense at the same temperature.

It is the qualitative change in the elementary excitation spectrum at μ = 0 not the emergence of a bound state at the unitarity limit, where as diverges that separates the BCS region from the BEC region. But how tune the interaction strength to cross the line of μ = 0 and realize the crossover?

Feshbach resonances: Feshbach resonances allow to tune the s-wave interaction by changing the scattering length by applying a magnetic field and are the essential tool to control the interaction between atoms in ultra-cold quantum gases (Cheng et al, 2010). In the context of scattering processes in many-body systems, the Feshbach resonance occurs when the energy of a bound state of an interatomic potential is equal to the kinetic energy of a colliding pair of atoms.



The underlying requirement, shown schematically on the left, is that at zero magnetic field, the interatomic potentials of two atoms in their ground state (the open channel) and in an excited state (the closed channel) be not too different in energy. The resonance, characterized by a divergence in the scattering length as, occurs when the energy difference _E between a bound state with energy Eres in the closed channel and the asymptotic threshold energy Eth of scattering states in the open channel is brought to zero by an applied external magnetic field B0. In short, a Feshbach resonance occurs when the bound molecular state in the closed channel energetically approaches the scattering state in the open channel. Then even weak coupling can lead to strong mixing between the two channels (Cheng et al, 2010). By this way the short range interaction strength which depends on the s-wave scattering length as can be tuned and the BCS-to-BEC crossover can be realized.

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