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# AN EVALUATION OF SUPER-FLUID DENSITY $\left(\frac{\rho_s}{\rho}\right)$ AS A FUNCTION OF $\left(\frac{T}{T_c}\right)$ FOR BCS-BEC CROSSOVER REGIME

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## ABSTRACT

In a Fermi gas with a Fashbach resonance, one can tune the strength of the paring interaction by adjusting the threshold energy of Feshbach resonance. The super-fluid density $\rho_s$ is always equal to the total carrier density $\rho$ at $T = 0$ which it vanishes at the super-fluid phase transition $T_c$. These properties are satisfied in both Fermi and Bose super-fluid. There is a crucial difference between $\rho_s$ in a Fermi superfluid and that in a Bose super-fluid. In a mean field BCS theory originates from the thermal dissociation of cooper pairs.

There it was observed that that if one increases the strength of the pairing interaction BCS type normal fluid density dominated by quasi-particle excitation change into BES type normal fluid density dominated by Bogoliubov collective excitations. As super-fluid density $\rho_s$ plays an important role in two fluid hydrodynamics these evaluated results would be useful in the study of dynamical properties in the BCS-BEC crossover region at finite temperature.

## INTRODUCTION

In this study, we have evaluated the super-fluid density$(\rho_s/\rho)$, quasi particle contribution $(\rho_n^F/\rho)$ and fluctuation particle contribution $(\rho_n^B/\rho)$ as a function of $(T/T_c)$ for BCS limit $\left(\frac{1}{K_F a_s} = -2.07\right)$, pseudo gap limit$\left(\frac{1}{K_F a_s} = 0.0\right)$ and BEC limit $\left(\frac{1}{K_F a_s} = 2.0\right)$, where $K_F$ is Fermi wave vector and $a_s$ is the s-wave scattering length.

As we know that in a Fermi gas with a Feshbach resonance, one can tune the strength of the pairing interaction by adjusting the threshold energy of Feshbach resonance (Timmerman et al, 2001). The BCS –BEC crossover has been realized by using the unique property (Bartenstein et al, 2004). Here, if one increases the strength of the paring interaction the character of super-fluidity continuously changes from weak coupling BCS type to strong coupling BEC type of tightly bound cooper pairs (Randeria, 1995; Noziers and Schmitt, 1985). In the super-fluid phase, the super-fluid density $\rho_s$ is the most fundamental quantities. The value of $\rho_s$ is always equal to the total carrier density $\rho$ at temperature $T=0$ while it vanishes at the super-fluid phase transition $T_c$. These properties are satisfied in both Fermi and Bose super -fluid, irrespective of the strength of the pairing interactions. There is a crucial difference between $\rho_s$ in a Fermi super-fluid and that in a Bose super-fluid. In a mean field BCS theory $\rho_s$ $(T>0)$ originates from the thermal dissociation of cooper pairs. The resulting normal fluid density $\rho_n = \rho - \rho_s$ is determined by quasi particle excitations. On the other hand $\rho_n$ in the Bose super-fluid is dominated by Bogoliubov collective excitation. Therefore, it is very interesting problem to see as to how $\rho_s$ in a Fermi superfluid changes into $\rho_s$ in a Bose super-fluid in BCS-BEC crossover.

In this chapter, we used the theoretical formalism of Y. Ohashi (2002) and Y. Ohashi and Griffin (2003), Maxwell et al (2014), Chui S and Rizvi (2014), KG Zloschchasliey (2014). There we have theoretically evaluated the super-fluid density in the BCS-BEC crossover. Y. Ohashi and A. Griffin have taken an uniform super-fluid Fermi gas at finite temperature and extended the strong coupling Gaussian fluctuation theory for transition temperature $T_c$ developed by Nozieres and Schmitt-Rink (Bartenstein et al, 2004; Noziers and Schmitt, 1985) to super-fluid phase below transition temperature $T_c$. Self consistently determined energy gap $\Delta$ and chemical potential $\mu$. We have used their formalism to evaluate super-fluid density $(\rho_s/\rho)$ and quasi particle contribution $(\rho_n^F/\rho)$ fluctuation quasi particle contribution $(\rho_n^B/\rho)$ for BCS-BEC crossover.

**Mathematical formulae used in the evaluation of superfluid density, quasi particle contribution and fluctuation quasi particle contribution.**

The BCS Hamiltonian in Nambau representation is

$$H = \frac{\Delta^2}{U}\sum_p \xi_p + \sum_p \psi_p^+[\xi_p\tau_3 - \Delta\tau_1]\psi_p$$
$$-\frac{U}{4}\sum_p [\rho_{1,q}\rho_{1,-q} + \rho_{2,q}\rho_{2,-q}] \quad (1)$$

One assumes two atomic hyperfine states described by pseudo-spin $\sigma = \uparrow,\downarrow$, $\psi_p^+ = \left(C_{p\uparrow}^+, C_{-p\downarrow}\right)$ is a Nambau field operator, $C_{p\sigma}^+$ is the creation operator of a Fermi atom and $\tau_j$ are the Pauli matrices $(j=1,2,3)$ which act on the particle –hole space. $\xi_p = \left(\frac{p^2}{2m} - \mu\right)$ is the atomic kinetic energy measured from the chemical potential $\mu$, $U$ is the tunable pairing interaction associated with Feshbach resonance. $\rho_{1q}$ and $\rho_{2q}$ are the amplitude fluctuation and phase fluctuation of the order parameter $\Delta$.

The generalized density operator is written as

$$\rho_{j,q} = \sum_p \psi^+_{p+\frac{q}{2}} \tau_j \psi_{p-\frac{q}{2}} \quad (j=1,2) \quad (2)$$

where $\rho_{jq}$ is the generalized density operator, $\rho_{1q}$ and $\rho_{2q}$ describe the amplitude and phase fluctuations of the order parameter $\Delta$ respectively.

In equation (1) the interaction is described by the sum of the interaction between amplitude fluctuations $(\rho_{1,q}\rho_{1,-q})$ and the phase alterations $(\rho_{2,q}\rho_{2,-q})$. In the Noziers Schmitt-Rink theory (1985), transition temperature $T_c$ is described by the Thou less criterion in the $t$-matrix approximation. The resulting equation for $T_c$ has the same form as the mean –field BCS gap equation with $\Delta \to 0$. However in contrast to the weak coupling BCS theory (where $\mu = E_F$), $\mu$ remarkably deviates from $E_F$ in the BCS-BEC crossover regime due to strong pairing fluctuations. The NSR theory includes the strong coupling effect by solving the equation of state within the Gaussian fluctuation approximation (Randeria, 1995) in terms of pairing fluctuation.

One extends the NSR theory to the

superfluid phase below $T_c$. To calculate $\Delta$, one uses the BCS gap equation

$$1=\frac{-4\pi a_s}{m}\sum_p\left\{\frac{\tanh(\beta E_{F/2})}{2E_p}-\frac{1}{2\xi_p}\right\} \quad (3)$$

where $E_p=\sqrt{(\varepsilon_p-\mu)^2+\Delta^2}$ is the single excitation spectrum. In equation (3) one eliminates the well known ultraviolet divergence by employing a two-body scattering length $a_s$

$$\frac{4\pi a_s}{m}=\frac{-U}{1-\sum_p^{p_c}\frac{1}{2\varepsilon_p}} \quad (4)$$

Where is two-body scattering length.

**Calculation of chemical potential $\mu$ :**

One consider the thermodynamic potential $\Omega$

Density is given as

$$\rho=\frac{-\partial\Omega}{\partial\mu} \quad (5)$$

Fluctuation contribution to $\Omega(=\partial\Omega)$ is calculated from relevant Feynman diagrams. Now summing up these diagrams, One obtains (Ohasi and Griffin, 2002) total densities

$$\rho=\rho_F^0-\frac{1}{2\beta}\sum_{q,v_n}\frac{\partial}{\partial\mu}\ln\det\left[1-\frac{4\pi a_s}{m}\left\{\Xi(q,iv_n)+\frac{1}{2\varepsilon_p}\right\}\right] \quad (6a)$$

$$\rho_F^0=\sum_p\left(-\frac{\xi_p}{E_p}\right)\tanh\left(\frac{\beta E_p}{2}\right) \quad (6b)$$

$\rho_F^0$ is the number of Fermi atoms in the meanfield approximation . In equation (6a) the second term describes the fluctuation contribution. $\Xi(q,i\omega_m)$ is the matrix correlation function. $\Pi_{ij}$ is the generalized density correlation function. $v_n$ is the boson Matsubara frequency. $\omega_m$ is the fermion- Matsubara frequency. Superfluid density in the BCS-BEC crossover is determined as

$$\rho_s=\rho-\rho_n \quad (7)$$

$\rho$ is the total carrier density given by

$$\rho=\sum_p 1+\frac{1}{\beta}\sum_{p,\omega_m}Tr[\tau_3 G(p,i\omega_m)] \quad (8)$$

$$G(p,i\omega_m)=G_0(p,i\omega_m)+G_0(p,i\omega_m)\sum_s(p,i\omega_m)G_0(p,i\omega_m) \quad (9a)$$

where

$$G(p,i\omega_m)^{-1}=i\omega_m-\delta\varepsilon_p\tau_3+\Delta\tau_1 \quad (9b)$$

$G_0(p,i\omega_m)$ is the matrix single particle thermal Green' function. $\Sigma$ is the self –energy which involves corrections to go $G_0$. $\rho_n$ is the well known BCS normal fluid density $\rho_n$ is calculated both for boson and fermion.

$$\rho_n^F=\frac{-2}{\beta}\sum_{p,\omega_m}p_z\frac{\partial}{\partial Q}Tr[G_{so}\{G(p,i\omega_m)\}]Q\to 0 \quad (10)$$

where $G_0$ is replaced by $G_{so}$. Super current state is described (Ohashi and Takada, 1997) by order parameter.

$$\Delta(z)=\Delta\exp(iQz)$$

Superfluid velocity $V_s=\frac{Q}{2m}$

$G_s$ is the matrix single particle Green's function in the super current state.

$$(G_{so})^{-1}=\left(i\omega_m-\frac{Q_{pz}}{2m}-\varepsilon_p\tau_3+\Delta\tau_1\right) \quad (11)$$

$$\rho_n^F=\frac{-2}{3m}\sum_p p^2\frac{\partial f(E_p)}{2E_p} \quad (12)$$

where $f(E)$ is the Fermi-Dirac distribution function. Boson normal density (fluctuation correction) is given by

$$\rho_n^B=\frac{-2}{\beta},\sum_{p,\omega_m}p_z\frac{\partial}{\partial Q}Tr\left[G_{s0}(p,i\omega_m)\sum_s(p,i\omega_m)G_{so}(p,i\omega_m)\right]Q\to 0 \quad (13)$$

In the weak coupling BCS regime $(K_F a_s)^{-1}\ll 1$, pairing fluctuation are weak and one finds that $\rho_n\approx\rho_n^F$ or $\rho_s=\rho-\rho_n^F$. In this regime equation (12) shows that $\rho_n$ is dominated by the quasi-particle excitation with excitation gap $\Delta$. In the BCS-BEC crossover regime, the chemical potential deviates from the Fermi energy $E_F$ and becomes negative in the strong coupling BEC regime (Noziers, 1985; Bartastan et al, 2004). One can calculate the chemical potential $\mu$ in the BEC limit where $(K_F a_s)^{-1}\gg 1$. Using equation (3) chemical potential is calculated as

$$\mu=\frac{-1}{2ma_s^2} \quad (14)$$

In BEC regime the chemical potential $\mu$ works as a large expectation gap therefore quasi-particle excitation as well as $\rho_n^F$ are suppressed. This shows that Cooper pair do not dissociate in the Fermi atoms due to large binding energy.

From equation (13) one can calculate the fluctuation contribution $\rho_n^B$. This is the dominant term in the strong coupling regime BEC. This is the dominant term in the strong coupling regime BEC. From equation (13) one obtains $\rho_n^B$ as

$$\rho_n^B = \frac{-2}{3M}\sum_q q^2 \frac{\partial n_B\left(E_q^B\right)}{\partial E_q^B} \tag{15}$$

where $n_B(E)$ is the Bose distribution function. $M = 2m$ is the molecular mass

$$E_q^B = \left[\frac{q}{2m}\left(\frac{q^2}{2m} + 2v_B\phi^2\right)\right]^{1/2} \tag{16}$$

Equation (16) the Bogoliubov phonon spectrum in dilute molecular Bose gas with a repulsive interaction $v_B = \frac{4\pi(2a_s)}{M}$ and the BCS order parameter $\phi = \left(\frac{a_s}{8\pi m\Delta}\right)^{1/2}$. In the BEC regime the normal fluid density is dominated by Bogoliubov collective excitations in a molecular Bose super-fluid.

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## DISCUSSION OF RESULTS

In this study the method of evaluation of ratio of super-fluid density $\left(\frac{\rho_s}{\rho}\right)$ as a function of $\left(\frac{T}{T_c}\right)$ for BCS limit have estimated, Pseudo gap limit $(K_F a_s)^{-1} = 0$ and BEC limit $(K_F a_s)^{-1} = 2$. Our theoretical evaluated results show that $\left(\frac{\rho_s}{\rho}\right)$ is larger in BEC limit and smaller in BCS limit as a function to $\left(\frac{T}{T_c}\right)$ in which $\left(\frac{\rho_s}{\rho}\right)$ declines with $\left(\frac{T}{T_c}\right)$ for all cases. The evaluated results are shown in table-$T_1$. We have presented the method of evaluation of quasi particle contribution $\left(\frac{\rho_n^F}{\rho}\right)$ as a function of $\left(\frac{T}{T_c}\right)$ for all three limits. Our evaluated results are shown in table- 4 $T_2$. There theoretical evaluated results shows that $\left(\frac{\rho_n^F}{\rho}\right)$ are larger for BCS limit and smaller in Pseudo gap limit. We have shown the theoretical evaluated results of quasi particle fluctuation contribution $\left(\frac{\rho_n^B}{\rho}\right)$ as a function of $\left(\frac{T}{T_c}\right)$ in the above three limits. The evaluated results are shown in table-$T_3$. Our theoretical results show that quasi-particle fluctuation contribution $\left(\frac{\rho_n^B}{\rho}\right)$ is smaller for BCS limit $(K_F a_s)^{-1} = -2$ and larger for BEC limit $(K_F a_s)^{-1} = 2$.

**Table $T_1$ :** An evaluated result of $\left(\frac{\rho_s}{\rho}\right)$ as a function of $\left(\frac{T}{T_c}\right)$ for BCS limit $(K_F a_s)^{-1} = -2$, Pseudo gap limit $(K_F a_s)^{-1} = 0$ and BEC limit

| $\left(\frac{T}{T_c}\right)$ | $\left(\frac{\rho_s}{\rho}\right)$ | | |
|---|---|---|---|
| | BCS-limit $(K_F a_s)^{-1} = -2$ | Pseudo-gap $(K_F a_s)^{-1} = 0$ | BEC-limit $(K_F a_s)^{-1} = 2$ |
| 0.0 | 1.0 | 1.0 | 1.0 |
| 0.1 | 0.975 | 0.982 | 0.995 |
| 0.2 | 0.956 | 0.967 | 0.977 |
| 0.3 | 0.932 | 0.955 | 0.964 |
| 0.4 | 0.897 | 0.902 | 0.912 |
| 0.5 | 0.824 | 0.855 | 0.866 |
| 0.6 | 0.746 | 0.797 | 0.805 |
| 0.7 | 0.618 | 0.639 | 0.652 |
| 0.8 | 0.546 | 0.568 | 0.574 |
| 0.9 | 0.348 | 0.382 | 0.403 |
| 0.95 | 0.226 | 0.267 | 0.288 |
| 1.00 | 0.059 | 0.122 | 0.147 |
| 1.05 | 0.002 | 0.097 | 0.106 |

**Table-4 $T_2$ :** An evaluated results of quasi-particle contribution $\left(\frac{\rho_n^F}{\rho}\right)$ as a function of $\left(\frac{T}{T_c}\right)$ for BCS $(K_F a_s)^{-1} = -2$, Pseudo gap $(K_F a_s)^{-1} = 0$ and BEC limit $(K_F a_s)^{-1} = 2$.

| $\left(\frac{T}{T_c}\right)$ | $\left(\frac{\rho_n^F}{\rho}\right)$ | | |
|---|---|---|---|
| | BCS-limit $(K_F a_s)^{-1} = -2$ | Pseudo-gap $(K_F a_s)^{-1} = 0$ | BEC-limit $(K_F a_s)^{-1} = 2$ |
| 0.2 | 0.004 | 0.008 | 0.00 |
| 0.4 | 0.098 | 0.008 | 0.00 |
| 0.6 | 0.185 | 0.086 | 0.00 |
| 0.8 | 0.274 | 0.105 | 0.00 |
| 1.0 | 0.456 | 0.126 | 0.00 |
| 1.2 | 0.684 | 0.149 | 0.00 |
| 1.4 | 0.756 | 0.185 | 0.00 |
| 1.5 | 0.889 | 0.225 | 0.00 |
| 1.6 | 0.954 | 0.246 | 0.00 |
| 1.7 | 1.038 | 0.278 | 0.00 |
| 1.8 | 1.176 | 0.304 | 0.00 |

From the above calculations, one observes that if one increases the strength of the pairing interaction BCS-type normal fluid density dominated by quasi-particle excitation changes into BEC type normal fluid density dominated by Bogoliubov collective excitations. As superfluid density $\rho_s$ plays an important role in two fluid hydrodynamics these evaluated results would be useful in the study of dynamical properties in the BCS-BEC crossover region at finite temperature (Regal et al, 2005; Greiner and Regal, 2005; Holland et al, 2005). Some recent results (Ya and Zhai, 2011; Andres et al, 2010; Lin et al, 2011; Sau et al, 2011; Du et al, 2012) also reveal the same facts.

## CONCLUSION

In the evaluation of super-fluid density, quantum particle contribution and fluctuation contribution as a function of $\left(\frac{T}{T_c}\right)$ for BCS-BEC crossover, NSR (Nozieres, Schmitt Rink model) gives results which are in good agreement with other theoretical workers.

**Table $4T_3$ :** An evaluated results of quasi-particle fluctuation contribution $\left(\frac{\rho_n^B}{\rho}\right)$ as a function of $\left(\frac{T}{T_c}\right)$ for BCS-limit $(K_F a_s)^{-1} = -2$, Pseudo gap $(K_F a_s)^{-1} = 0$ and BEC limit $(K_F a_s)^{-1} = 2$.

| $\left(\frac{T}{T_c}\right)$ | $\left(\frac{\rho_n^B}{\rho}\right)$ | | |
|---|---|---|---|
| | BCS-limit $(K_F a_s)^{-1} = -2$ | Pseudo-gap $(K_F a_s)^{-1} = 0$ | BEC-limit $(K_F a_s)^{-1} = 2$ |
| 0.2 | 0.002 | 0.004 | 0.056 |
| 0.4 | 0.006 | 0.016 | 0.089 |
| 0.6 | 0.009 | 0.108 | 0.126 |
| 0.8 | 0.012 | 0.148 | 0.248 |
| 1.0 | 0.122 | 0.288 | 0.336 |
| 1.2 | 0.142 | 0.326 | 0.409 |
| 1.4 | 0.167 | 0.449 | 0.526 |
| 1.5 | 0.198 | 0.567 | 0.678 |
| 1.6 | 0.207 | 0.659 | 0.776 |
| 1.7 | 0.226 | 0.787 | 0.892 |
| 1.8 | 0.245 | 0.896 | 0.967 |
| 2.0 | 0.268 | 1.052 | 1.122 |

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