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AN INFLATED PROBABILITY MODEL FOR INFECTION

UMA SHANKAR SHUKLA
[1]Dept of Mathematics, LBSS PG College,
Anand Nagar, Maharajganj (UP)

ABSTRACT

The present paper deals with the projected probability model in inflated form for the infection. There parameters of the model was estimated by suitable estimation technique. The suitability of the proposed model was tested by observed set of data. The study showed inflation trend or over-dispersion in the case of community level, while delayed exponential increase in the case of preventive care towards infection. This research might be useful to elucidate strategy against infection in proper time.

Key wards: probability distribution, inflated estimation technique.

INTRODUCTION

A person infected with malaria may be re infected before recovering if bitten again by an infectious mosquito. Even third and higher order infections are possible (Brumpt, 1949). Here the best data available consist of planet studies are used for the present study, where blood samples were observed from a fixed set of individuals at discrete points in time (Johnson and Kutz, 1969). Both *Plasmodium vivax* and *Plasmodium falciparum* occur in abundance, but *Plasmodium flaciparum* (the killer parasite) is accounting more as compared to eighties and nineties. Let us consider N objects and each subject is observed at discrete point of time in the interval (o,T). Let $f_X(x)$ be the probability function of infections, where x is a discrete random variable, representing the number of infections. Poission model in case of both method of moments and likelihood are good fit for both of the data sets (Clifford and Cohen, 1961).

Malaria is one of the most widespread diseases in the world today. Malaria is the most important parasitic disease in the world. It kills 3,000 children every day and more than one million each year. The majority of these deaths occur among children under five years of age and pregnant women is sub-saharan Africa Earlier about 80% of malaria disease was due to plasmodium vivax parasite. Now malaria disease due to Plasmodium falciparum is also increased. This is the most virulent of the four malaria parasites of humans. It is estimated that Plasmodium falciparum kills between 1.5 and 2.7 million children per year (Good 1999). Also women are at increased risk during pregnancy (Fried, 1998), Despite fifty years of world experience in malaria control more people are dying of malaria now (David and Elizabeth, 1998). Malaria is caused by the plasmodium parasite transmitted by the infective female Anopheles mosquito. So, the infected malaria patient can transmit the disease to many susceptible.

A person infected with malaria may be re-infected before recovering if bitten again by an infectious mosquito. Even third and higher order infections are possible. Two lines of evidence support these contentions. First individuals frequently harbor in their blood two or more of the four species of malaria parasite, which can infect humans (Sharma, 1995). Other evidence implicates multiple infections implicates multiple infections with a single species of parasite. The process of acquiring multiple infections of a single species called super infection was first modeled mathematically by the malariologist George Macdonald (1950). Macdonald- Dietz modeled (1970) describes the flow of infections through an individual as an infinite serve queue. The continuous observation of state is obviously practically impossible. Hence the best data available consist of panel studies are used to fit some probability models, where blood samples were observed from affixed set of individuals at discrete points.

Probability Model: Let α be the risk of infected population in the society in specific area and 1- α be the risk of no infections. Under this situation the proposed model takes the following inflated form (Nedelman, 1985):-

$P[x=o] = 1-\alpha, \quad k=o$

$$P[X=k] = \frac{\alpha\theta^k (e^\theta - 1)^{-1}}{k!} \qquad k = 1, 2, 3, ...$$

Where, θ denotes average infection in population. The above model has two parameters α and θ.

Estimation of Parameters: The proposed model has two parameters α and θ. The parameters are estimated with the help of Maximum likelihood method in following way.

A sample consisting of N observations of random variables x with probability function has been considered (Nedelman, 1985). The value chosen as estimates of and are those which maximize the expression

$$P(x_1, x_1 ......x_n, \alpha, \theta) = (1-\alpha)^{fo}[\alpha\theta(e^\theta - 1)]^{f1}. [\alpha\{1-\theta(e^\theta - 1)\}]^{f-f1-fo} \qquad ....(1)$$

Now, taking logarithmic of above equation and partially differentiating w.r. to α and θ in term and equating to zero yield the estimating equation.

$$\frac{\partial \log L}{\partial \alpha} = -\frac{f_0}{1-\alpha} + \frac{f_1}{\alpha} + \frac{f - f_1 - f_0}{\alpha} = 0 \qquad .....(2)$$

$$\frac{\partial \log L}{\partial \alpha} = \frac{f_1\left[-\theta\, e^\theta (e^\theta - 1)^{-2} + (e^\theta - 1)^{-1}\right]}{\theta\,(e^\theta - 1)^{-1}} +$$

$$\frac{(f - f_1 - f_0)\left[\theta\, e^\theta (e^\theta - 1)^{-2} - (e^\theta - 1)^{-1}\right]}{\left[1 - \theta\,(e^\theta - 1)^{-1}\right]} = 0 \qquad .....(3)$$

Now, solving the above two equation we get the value of α and θ as.

$$\alpha = \frac{f - f_0}{f}$$

$$\theta(e^\theta - 1)^{-1} = \frac{f_1}{f - f_0}$$

The second partial derivative of log L w.r. to α and weget as

$$\frac{\partial^2 \log L}{\partial \alpha^2} = -\frac{f_0}{(1-\alpha^2)} + \frac{f_1}{\alpha^2} + \frac{(f - f_1 - f_0)}{\alpha^2} \qquad ....(4)$$

Now before second partial derivative of log L and taking approximation at one place up to third term and then differentiating, we obtained as

$$\frac{\partial^2 \log L}{\partial \theta^2} = -f\left[-e^{2\theta}(e^\theta - 1)^{-2} + (e^\theta - 1)^{-1} e^\theta\right] - \frac{f_1}{\theta^2} +$$

$$\left(f - f_0 - f_1\right)\frac{\left[\left(\frac{\theta^2}{2}+\theta\right) - (1+\theta)^2\right]}{\left[\frac{\theta^2}{2}+\theta\right]^2} \quad ....(5)$$

Now partial derivative of and w.r. to θ and α respectively we get as,

$$\frac{\partial^2 \log L}{\partial\alpha\,\partial\theta} = \frac{\partial^2 \log L}{\partial\theta\,\partial\alpha} = 0 \quad \frac{\partial^2 \log L}{\partial\alpha\,\partial\theta} = \frac{\partial^2 \log L}{\partial\theta\,\partial\alpha} = 0 \quad ....(6)$$

Using the fact

$E[f_o] = f(1-\alpha)$

$E[f_1] = f\alpha\,[\theta\,(e^{\theta}-1)^{-1}]$

$E[f - f_o - f_1] = f\alpha\,[1-\theta\,(e^{\theta}-1)^{-1}]$

Where E denote for the expectation, The expected value of second partial derivative of log L can be obtained as

$$\phi_{11} = \frac{E\left[-\frac{\partial^2 \log L}{\partial\alpha^2}\right]}{f} = \left[\frac{1}{1-\alpha}+\frac{1}{\alpha}\right] \quad .....(7)$$

$$\phi_{22} = \frac{\left[-\frac{\partial^2 \log L}{\partial\theta^2}\right]}{f} = \alpha\,\theta\,e^{\theta}(e^{\theta}-1)^2\left[1-e^{\theta}(e^{\theta}-1)^{-1}+\frac{\alpha(e^{\theta}-1)^{-1}}{\theta}\right]+$$

$$+\frac{\alpha\left[1-\theta\,(e^{\theta}-1)\right]\left[\frac{\theta^2}{2}+\theta+1\right]}{\left[\frac{\theta^2}{2}+\theta\right]^2} \quad ....(8)$$

$$\phi_{12} = \frac{E\left[-\frac{\partial^2 \log L}{\partial\alpha\,\partial\theta}\right]}{f} = 0 \quad ......(9)$$

$$\phi_{21} = \frac{E\left[-\frac{\partial^2 \log L}{\partial\theta\,\partial\alpha}\right]}{f} = 0 \quad ——(10)$$

Therefore, by inverting the information matrix, expression for asymptotic variances of the estimates can be obtained as :

$$V(\hat{\alpha}) = \frac{1}{f}\left[\frac{\phi_{22}}{\phi_{11}\,\phi_{22}-\phi_{12}^2}\right] ———— (11)$$

$$V(\hat{\theta}) = \frac{1}{f}\left[\frac{\phi_{11}}{\phi_{11}\,\phi_{22}-\phi_{12}^2}\right] ———— (12)$$

Here f_o be Number of observation in zeroth cell

f_1 – Number of observation in first cell.

f – Total number of observations.

– Observed Mean

Application: We used the data collected by the WHO in Garki, Nigeria in particular data of infection with Plasmodium falciparum only. The value of α from the Table (1) is 0.77 and Table (2) is 0.64. And the value of θ is 0.64 from Table (1) and 2.2045 from Table (2). The χ^2 shows that proposed model found to be better approximation to the both set of the observed data.

Table 1: Observed and expected values from WHO in Garki, Nigeria (Source: Molineaux and Grammicia, 1980).

Infections	Observed infants	Expected infants	Estimated Values(M.L.E.)
0	7	7.13	
1	3	2.99	
2	5	4.95	=4.92
3	10	5.47	=0.77
4	4	10.13	=2.7975
5	1	10.53	=0.0082
6	0	0	=0.0013
7	1	Df=2	
Total	N=31	N =31	

Table 2: Observed and expected values from WHO in Garki, Nigeria (Source: Molineaux and Grammicia, 1980) .

Infections	Observed infants	Expected infants (By M.L.E.)	Estimated Values
0	6	5.99	
1	3	3.01	= 5.65
2	1	3.31	= 0.77
3	3	2.43	= 2.7975
4	3	1.34	= 0.013
5	1	0.92	= 0.071

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