The Scientific Temper (2025) Vol. 16 (4): 4095-4101
E-ISSN: 2231-6396, ISSN: 0976-8653
Doi: 10.58414/SCIENTIFICTEMPER.2025.16.4.12
https://scientifictemper.com/

RESEARCH ARTICLE

Inventory model considering trade discounts and scrap disposal with sustainability

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Abstract

Inventory management in today's scenario is very complex involving many factors exercising influence on each other. Studies are being made continuously to find the relationship among these factors to arrive at the most optimum solutions. This paper develops a mathematical model for inventory management, incorporating factors such as ordering, holding, screening, and disposal costs, along with quantity discounts, interest payable/earned, and transportation costs. The model considers three scenarios based on the relationship between the cycle time and the trade credit period.

The analysis determines the optimal order quantity and cycle time for each case. Case (ii), where interest is earned on revenue while avoiding interest charges, yields the lowest total cost. Conversely, case (i), which only accounts for inventory holding costs without any offsetting earnings, is the costliest.

Numerical examples illustrate the model's application and validate the findings. The results provide insights for businesses to optimize their inventory management strategies, reduce costs, and improve overall efficiency within the supply chain.

Keywords: Sustainable inventory, Environmental factors, Quantity discounts, Cycle time, Order quantity, Optimum cost, Transportation cost, Screening cost, Scrap disposal cost.

Introduction

An inventory model that incorporates trade discounts aims to optimize ordering quantities by considering price reductions offered by suppliers for larger purchases. This complexity arises because the unit cost of inventory is no longer constant but varies based on the order size, potentially leading to multiple price break points. The model must balance the cost savings from these discounts against the increased holding costs associated with larger inventory levels. Furthermore, it needs to determine the optimal order quantity that minimizes the total inventory cost, which includes purchasing costs, ordering costs, and holding costs, while taking into account the tiered pricing structure offered by trade discounts.

The integration of scrap disposal into the inventory model adds another layer of complexity. Over time, some inventory may become obsolete, damaged, or otherwise unusable, resulting in scrap. The model needs to consider the timing and quantity of scrap disposal, as well as the associated costs or potential revenues. Therefore, the inventory model must not only optimize the inflow and storage of goods but also strategically manage the outflow of scrap in an economic manner. This might involve determining optimal times for scrap removal, evaluating different disposal methods, and potentially adjusting ordering policies to minimize future scrap generation.

Sustainable transportation in inventory management focuses on minimizing the environmental impact of moving goods. This involves strategies like optimizing delivery routes to reduce mileage and fuel consumption, utilizing fuel-efficient or alternative fuel vehicles (such as electric or hybrid), and consolidating shipments to maximize vehicle capacity. Embracing intermodal transportation, like combining road and rail, can also lower emissions. Furthermore, sustainable packaging choices that reduce weight and volume contribute to more efficient and eco-

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How to cite this article: Nalini S., Ritha W. (2025). Inventory model considering trade discounts and scrap disposal with sustainability. The Scientific Temper, 16(4):4095-4101.

Doi: 10.58414/SCIENTIFICTEMPER.2025.16.4.12

Source of support: Nil

Conflict of interest: None.

Received: 24/03/2025 **Accepted:** 19/04/2025 **Published** : 25/04/2025

friendly transportation. The goal is to create a greener supply chain by reducing the carbon footprint associated with inventory movement.

Literature Review

A fundamental model analyzing the effect of permissible delay in payments offered by the supplier to the retailer on the optimal order quantity was developed (Goyal, 1985). This model serves as a cornerstone for subsequent research in the field and was extended to incorporate the scenario where shortages in inventory are allowed, providing a more realistic representation of inventory management in certain contexts (Aggarwal *et al.*, 1995). Focusing on the seller's perspective, the optimal unit price and the length of the credit period were jointly determined, recognizing that end demand is sensitive to price (Abad *et al.*, 2003). The retailer's optimal replenishment policy was examined, considering the non-instantaneous receipt of goods and the impact of both trade credit and cash discounts offered by the supplier (Huang, 2007). The optimal economic order quantity (EOQ) under the conditions of date-terms supplier credit was derived, providing insights into how credit terms influence ordering decisions (Carlson *et al.*, 1989). A decision-making procedure for a vendor who aims to dispose of excess stock was formulated, evaluating the options of offering either a price discount or a credit period to incentivize additional purchases (Arcelus *et al.*, 1993).

Inventory models with imperfect quality items and analyzed repair and disposal policies were considered (Taleizadeh *et al.*, 2013). Both ordering and disposal decisions were integrated, recognizing the importance of coordinating these two aspects of inventory management, including potential scrap (Nahmias, 1982). Economic order quantity models for items with imperfect quality were analyzed, addressing the issue of managing defective or flawed products within the inventory system, which can lead to scrap (Teunter *et al.*, 2004). A model that explicitly incorporates transportation costs into the inventory management framework was developed (Ertogral *et al.*, 2007). A joint economic-lot-size model was the focus, considering the perspectives of both the purchaser and the vendor to optimize the overall supply chain efficiency, including transportation (Banerjee, 1986).

Optimal inventory policies in a general sense were discussed (Woolsey, 1963). An inventory model that incorporates both trade credit was investigated (Lu *et al.*, 2010). A comprehensive analysis of inventory models was provided (Hadley *et al.*, 1963). The relationship between network design and inventory theory was examined (Friesz *et al.*, 1984). The impact of freight consolidation on inventory costs was analyzed (Blumenfeld *et al.*, 1987). Planning for inbound logistics was discussed (Bramel *et al.*, 1997). A periodic review inventory model with a return policy was considered (Song *et al.*, 2005). Reverse logistics for end-of-life computers was explored (Blackburn *et al.*, 1999). The Vendor-Buyer's integrated inventory model with quantity discount, delay in payments, and trade credit policy was explained (Ritha *et al.*, 2016).

Materials and Methods

The model focusses on the optimization of the total cost of the inventory with particular reference to the relationship of trade credit with the cycle time. The impact of the transportation cost on the overall cost scenario is also captured.

Description of the model

The model identifies the individual cost components and the relationship of the components with each other in the form of individual cost functions. The critical parameters are the optimum quantity and the optimum cycle time. These are calculated and their impact on the total cost function is determined.

Notation and Assumptions

Notation

D Annual demand
K Setup costs per order
h Holding costs %
p Delivered unit price paid by the buyer
Q Order quantity
C Unit purchase cost
Cs Unit disposal cost for scrap items
S Scrap quantity to be disposed
Vs Screening cost per item
d Per unit rupee discount to the buyer
I Initialization cost
W Minimum order quantity at which the delay in payments is permitted
c Unit purchasing price per item
s Unit selling price per item
M The trade credit period
T The cycle time
Ie Interest which can be earned per year
Ip Interest charges per investment in inventory per year
B Backordering ratio
F Truck charge per km
z distance in km
t_c truck capacity

Assumptions

- Demand is known and constant.
- Shortages are not allowed.
- Time period is infinite.
- The buyer does not return the damaged products instead make arrangement for screening or disposed

for damaged products.

- If Q < W, i.e. T < W / D, the delayed payment is not permitted. Otherwise, fixed trade credit M is permitted. Hence, if Q < W, cQ is paid when the order is received. If Q > W, cQM time periods after the order is received.
- During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. When T >= M, the account is settled at T = M, the buyer pays off all units sold and keeps profits and starts paying for the higher interest charges on the items in stock. When T <= M, the account is settled at T = M and the buyer does not need to pay any interest charge.
- $s >= c$, $I_p >= I_e$

Model formation

The annual total cost consists of the following:

Ordering cost = $\frac{DK}{2}$

Holding cost = $\frac{h(1-B)^2 QC}{2}$

Screening cost = $\frac{V_s Q}{2}$

Disposal cost = $\frac{C_s Q}{2}$

Quantity discount given by vendor = pQd

Cost of interest charges for the items kept in stock per year.

Case-(i): 0 < T < W/D

Cost of interest charges per year = $\frac{cI_p DT}{2}$

Cost of interest earned per year = 0

Case-(ii): W/D < T < M

Cost of interest charges per year = 0

Cost of interest earned per year = $DsI_e(M-\frac{T}{2})$

Case-(iii): T > M

Cost of interest charges per year = $\frac{cI_p D(T-M)^2}{2T}$

Cost of interest earned per year = $\frac{DM^2 sI_e}{2T}$

Transportation cost = $\frac{fzq}{t_c}$

TOTAL COST TC = Ordering cost + Holding cost + Screening cost + Disposal cost + Quantity discount given by vendor + interest Payable - Interest earned + Transportation cost.

The total costs for the 3 cases are:

$$TC_1 = \frac{DK}{Q} + \frac{h(1-B)^2 QC}{2} + \frac{V_s Q}{2} + \frac{C_s S}{2} + pQd + \frac{cI_p DT}{2} + \frac{fzQ}{t_c}$$

$$TC_2 = \frac{DK}{Q} + \frac{h(1-B)^2 QC}{2} + \frac{V_s Q}{2} + \frac{C_s S}{2} + pQd - DsIe\left(M - \frac{T}{2}\right) + \frac{fzQ}{t_c}$$

$$TC_3 = \frac{DK}{Q} + \frac{h(1-B)^2 QC}{2} + \frac{V_s Q}{2} + \frac{C_s S}{2} + pQd + \frac{cI_p D(T-M)^2}{2T} - \frac{DM^2 sI_e}{2T} + \frac{fzQ}{t_c}$$

Differentiation of TC with respect to Q and T and setting it to 0 are done separately for all the three cases and the optimum values of Q* and T* are found.

Case-(i): 0 ≤ T ≤ W/D

$$\frac{dTC_1}{dQ} = 0$$

Hence, $Q^* = \sqrt{\dfrac{2DK}{h(1-B)^2 C + V_s + C_s + 2pd + 2\dfrac{fz}{t_c}}}$

$$\frac{dTC_1}{dT} = 0$$

$$\frac{dTC_1}{dT} = \frac{d}{dT}\left(\frac{CI_p DT}{2}\right) = \frac{CI_p D}{2} = 0$$

But this cannot be zero unless all of the parameters C, I_p, D are zero — which is typically not the case in a real-world scenario

There is no minimum or optimal point for T from this equation within this cost function in the current range (Case i: 0<T<W/D), because:

- The function is strictly increasing in T since $\frac{dTC_1}{dT} > 0$
- Hence, lower values of T are more favorable in this case.

This means that TC_1 increases as T increases, so the function is monotonically increasing with respect to T in this interval.

The optimal value of T is as close to zero as practical or feasible in operations — meaning shorter cycle times are better for minimizing cost in case (i).

Examples for calculating $TC_1(T) = \frac{CI_p DT}{2}$

$\frac{W}{D} = \frac{4000}{5000} = 0.8$ years

Valid range of case-(i) = 0 ≤ T ≤ 0.8

Now calculate $TC_1(T)$ for a few values of T

$$TC_1(T) = \frac{CI_p DT}{2} = \frac{100*0.1*5000*T}{2} = 2500T$$

Table 1: Case-(i) Input

Description	*Parameter*	*Value*
Unit purchase price	c	Rs 100
Interest charges per investment per year	I_p	10% = 0.1
Annual Demand	D	5000 units
Minimum order qty	W	4000 units
Cycle Time in years	T	Variable

Table 2: Case-(i) TC_1(T) (vs) T

T (years)	*Total cost TC_1(T) Rs*
0.1	250
0.3	750
0.5	1250
0.7	1750
0.79	1975

From the above data, it is observed that the TC_1(T) is least at the value of T = 0.1 years. Hence, **T* = 0.1 years**

Cost increases linearly with T (due to interest charges only).

For this case, the total cost function TC1 increases linearly with T due to interest charges only, and is therefore minimized at the smallest practical value of T.

As shown in Table 1, the input parameters include a unit purchase price of Rs 100 and an annual demand of 5000 units. The corresponding total costs are presented in Table 2, and the relationship between cycle time and cost is graphically illustrated in Figure 1.

Cost increases linearly with T (due to interest charges only).

Case-(ii): W/D ≤ T ≤ M

$$\frac{dTC_2}{dQ} = 0$$

Hence, $Q^* = \sqrt{\dfrac{2DK}{h(1-B)^2 C + V_s + C_s + 2pd + 2\dfrac{fz}{t_c}}}$

$TC_2(T) = pDI_e(M - \frac{T}{2})$ $\quad \frac{dTC_2}{dQT} = \frac{d}{dT}\left(_{pDIeM} - \frac{pDIe}{2}\right)$

$$\frac{-pDI_e}{2} = 0$$

Table 3: Case-(ii) Input

Description	*Parameter*	*Value*
Unit selling price	c	Rs 150
Interest earned rate	I_e	8% = 0.08
Annual Demand	D	5000 units
Minimum order qty	W	3000 units
Trade credit period	M	0.6 years

This is only possible if p, D, or Ie = 0, which is not realistic in most practical cases.

Therefore, $\frac{dTC_2}{dQT} < 0$, means that TC2 decreases with increasing T in this range.

To maximize interest earned (i.e., minimize cost)

T* = M (just below M)

Examples for calculating $TC_2(T) = pDI_e(M - \frac{T}{2})$

Cycle Time T is in the range W/D < T < M

Total cost includes interest earned $TC_2(T) = pDI_e(M - \frac{T}{2})$

$\frac{W}{D} = \frac{5000}{3000}$ = 0.6 Let us assume T $\in$ (0.6, 0.9)

$TC_2(T) = 60000 \times (0.6 - \frac{T}{2})$

As T increases, cost (TC2) decreases. So, the function is monotonically decreasing in T.

T* = M = 0.6 (just below the upper limit)

This case benefits from earned interest without incurring any interest charges, making it the most cost-effective scenario. The values used are provided in Table 3 and the computed total costs at various cycle times are displayed in Table 4. The decreasing trend in cost with increasing T is visually represented in Figure 2.

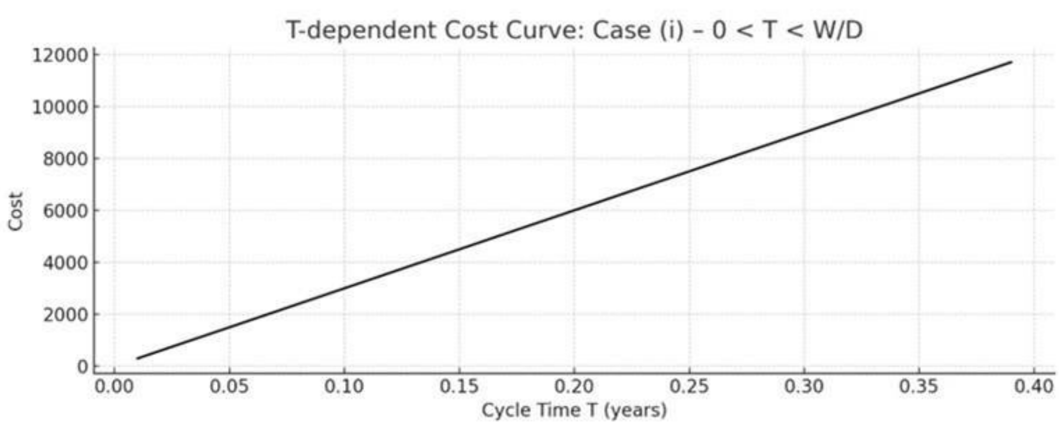

Figure-1: Case-(i) Cycle time T (vs) cost

Table 4: Case-(ii) $TC_2(T)$ (vs) T

T(years)	$TC_2(T) = 60000 \times (0.6 - \frac{T}{2})$	*Cost Rs*
0.61	60000 x (0.60 – 0.305)	17,700
0.7	60000 x (0.60 – 0.35)	15,000
0.8	60000 x (0.60 – 0.4)	12,000
0.9	60000 x (0.60 – 0.445)	9,300

Table 5: Case-(iii) Input

Description	*Parameter*	*Value*
Unit purchase price	c	Rs 100
Interest charges per investment per year	I_p	10% = 0.1
Interest charges earned	I_e	8% = 0.08
Annual Demand	D	5000 units
Unit selling price	W	Rs 150
Trade credit period	M	0.6

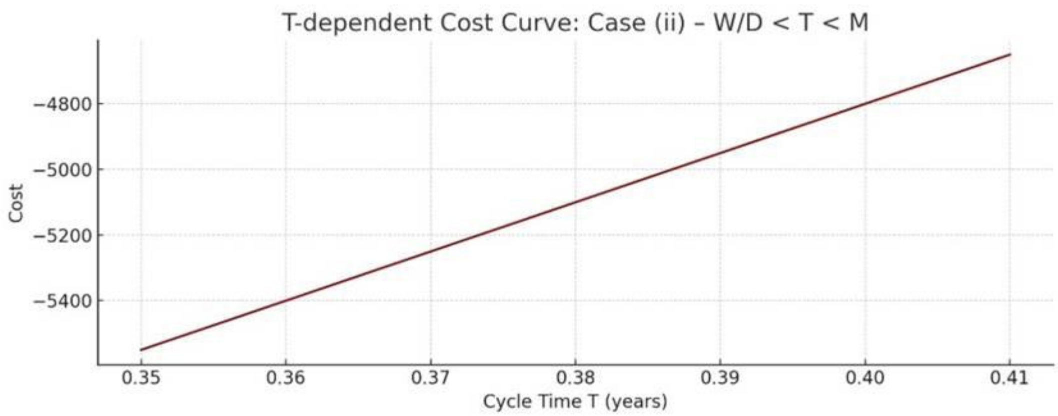

Figure-2: Case-(ii) Cycle time T (vs) cost

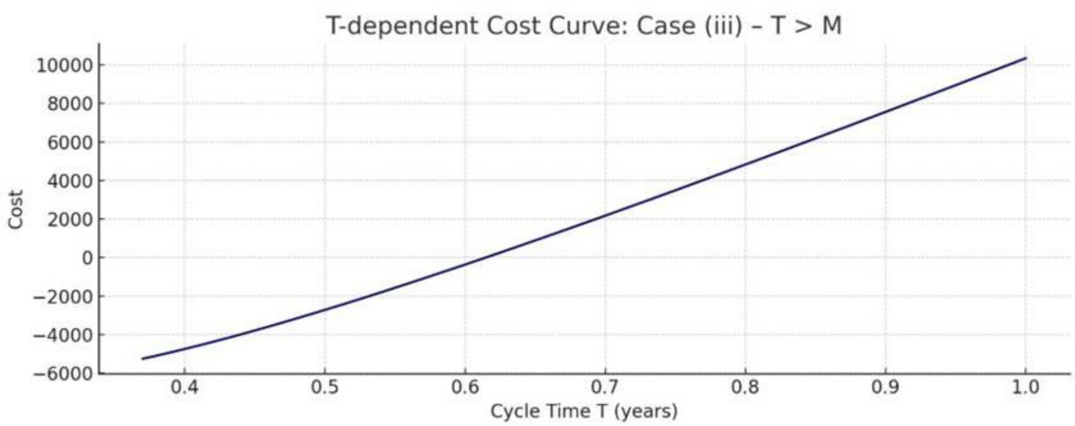

Figure-3: Case-(iii) Cycle time T (vs) cost

Cost decreases with T (due to interest earned on delayed payments)

Case-(iii): T > M

When T > M, total cost includes

Interest charges (for late payments beyond the trade credit period)

Interest earned (on revenue within the trade credit period)

$TC_3(T) = cI_pD(T-M)^2 - DM^2sI_e$

The optimality equation is $cI_pD(T-M)(T+M)+DM^2sI_e = 0$

Find the value of T* (in the range of T > M)

On simplifying the equation $cI_pD(T-M)(T+M)+DM^2sI_e = 0$, we get

$cI_p(T^2-M^2)+M^2sI_e = 0$

Here, interest is both earned and paid. While better than Case-(i), this scenario is not as efficient as Case-(ii). Input data are shown in Table 5. The cost behavior with respect to cycle time is shown in Figure 3.

Solving the above equation for T^2 with the values, we get $T^2 = -0.072$

But T^2 cannot be negative.

Table 6: Optimal T* and Costs for all cases

Case	*T* (Theoretical)*	*T* (Numerical)*	*Minimum Cost (Rs)*
Case (i)	→ 0 or 0.1	0.1	300.00
Case (ii)	≈ M (0.6)	0.35	-5,550.00
Case (iii)	≈ 0.424	0.37	-5,245.95

Table 7: Input data for Numerical Example

D=5000	*K=50*	*H=0.1*	*p=110*	*C=100*	*C_s=20*	*S=75*	*V_s=20*
d=0.1	I=5	W=4000	C=100	s=150	M=0.36	I_e=0.08	I_p=0.1
B=0.5	f=5	z=20	t_c=10				

Table 8: Optimum values of Q* and T*

Case	*Q**	*T**
Case-(i)	77	0.1
Case-(ii)	77	0.35
Case-(iii)	77	0.37

Calculation of Total Cost (Rs)

Table 9: Total cost for all cases

Cost Component	*Case-(i)*	*Case-(ii)*	*Case-(iii)*
Ordering Cost	125000	125000	125000
Holding Cost	96	96	96
Screening Cost	769	769	769
Disposal Cost	750	750	750
Quantity Discount	846	846	846
Interest-To be paid	5000	0	1
Interest-Earned	0	11100	1439
Transportation	769	769	769
Total Cost	133231	117131	126793

This means that the interest earned is not enough to balance the interest charges, even when T is just above M. In this scenario, it's better to operate at or just below T=M , which places us back into case-(ii)

So, the minimum point is not in case-(iii) at all.

Plugging different values for Ip and Ie, we finally arrive at the optimum values

When I_p = 12% I_e = 4%, T* = 0.424 years

The curve shows a minimum point around T≈0.424T years. To the right of that point, the cost increases rapidly due to high interest charges. To the left, cost decreases until just after T=M=0.36T = M = 0.36T=M=0.36, confirming that T∗∈(M,1).

Optimal T and Costs in Each Case*

The theoretical and numerical optimum values of cycle time (T*) and associated costs for all cases are summarized in Table 6.

Theoretical values are derived from mathematical analysis (e.g., differentiation). Numerical values are obtained from cost evaluations at multiple T points.

For practical applications, it is better to use the numerically tested T* values from the cost table for practical applications (i.e., 0.1, 0.35, 0.37).

Case (ii)

gives the lowest cost: Best when you can earn interest on revenue and avoid interest charges.

Case (i)

is costlier, since you're only paying inventory holding cost with no offsetting earnings.

Case (iii)

is still better than (i), but not as efficient as (ii), because you're paying interest for the delay beyond the trade credit period.

Why are the costs negative in Cases-(ii) and (iii)?

In case-(ii), this is the interest earned on the revenue. Since there's no interest paid in this case, and no other T-dependent cost, the function is purely negative — it reflects a financial gain due to favourable credit terms.

So, a negative cost here doesn't mean you're "making money overall" — just that this portion of the cost (interest) is being offset or reduced due to interest earned.

In case-(iii), if the interest earned is larger than the interest paid, the net result could be negative. This again reflects a cost offset — not a profit.

Numerical Examples

As outlined in Table 7, parameter values used for the numerical example include demand (D=5000), setup cost (K=50), and holding cost rate (H=0.1).

The optimum values of Q* and T* for each case are presented in Table 8. The detailed breakdown of total costs per component across the three cases is given in Table 9, clearly showing Case-(ii) as the most cost-effective option.

Given

From the above data, it is observed that opting for case-(ii) is the least cost option, since in this case, interest to be paid is 0 and interest earned is Rs 1439. After case-(ii), case-(iii) is preferable. This has already been corroborated earlier.

Discussions

A fundamental model analysing the effect of permissible delay in payments offered by the supplier to the retailer on the optimal order quantity was developed (Goyal, 1985). The Vendor-Buyer's integrated inventory model, incorporating quantity discount, delay in payments, and trade credit policy, was explained (Ritha *et al*., 2016).

The proposed model effectively evaluates inventory costs under varying trade credit conditions. Among the three cases, case-(ii) consistently yielded the lowest total cost, as it allows firms to earn interest on revenues without paying any interest charges. Case-(i), with only holding and interest charges, was the costliest. Case-(iii) showed intermediate results where, earned interest partly offsets the charges incurred after exceeding the credit period.

Incorporating screening, disposal, and transportation costs adds practical relevance, especially for industries dealing with defective or perishable items. The results validate that aligning cycle time with credit terms significantly reduces overall costs, emphasizing the role of smart credit utilization in sustainable inventory management.

Conclusion

The study demonstrates that integrating trade credit, disposal, and transportation into inventory modeling leads to more accurate and cost-efficient strategies. Case-(ii) provides the best balance, helping businesses reduce costs by maximizing interest earnings within the credit period. These findings guide inventory planners in optimizing order timing and leveraging supplier credit to improve financial and operational efficiency.

Acknowledgments

Nil

Conflicts of Interest

The authors declare no conflict of interest.

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