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## RESEARCH ARTICLE

# Sustainable inventory model with environmental factors using permissible delay in payments

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#### **Abstract**

Provision of permissible delay in payments is a good business tactic resorted to in almost all types of businesses. Such a practice also exists in inventory management companies. Many articles have been presented on the subject of permissible delay in payments within the economic order quantity (EOQ) framework with the ultimate aim of minimizing total cost. In general, such articles focused on the supplier offering the retailer a fully permissible delay in payments only if a certain minimum quantity (as specified by the supplier) is ordered. A variation in the above idea has been used to develop a model with the objective of a cost minimization problem to determine the retailers' optimal inventory cycle time and optimal order quantity. The objective of this paper is to analyze the abovementioned model in detail and provide use case scenarios in which the effect of the variation of quantity of selected variables on the cycle time and the optimal order quantity are presented. The effect of these variables on the optimum and transportation costs is also analyzed. Finally, improvements in the existing model have been suggested with numerical examples for more clarity.

**Keywords:** Sustainable inventory, Environmental factors, Permissible delay in payments, Cycle time, Order quantity, Optimum cost, Transportation cost.

### Introduction

This paper introduces a new method for incorporating sustainability into inventory models. While sustainability research in operations management has grown, quantitative models are lacking. This work addresses this gap by adapting traditional inventory models to include environmental sustainability. Instead of simplifying sustainability into a single goal, the classic economic order quantity (EOQ) model is reformulated as a multi-objective problem. This new model, which considers environmental factors, is called the sustainable order quantity model with permissible payment delays.

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Traditional inventory models focus on minimizing costs related to overstocking, understocking, and the cost of the inventory itself. These models are used to optimize inventory parameters. However, they often fail to address certain inventory challenges, including the need for environmentally responsible systems. A key issue is that current cost accounting rarely reflects the true environmental impact of activities. This can lead to flawed decision-making. We are thus faced with the challenge of either accurately determining environmental costs or using estimated values. This paper explores these issues and discusses how inventory models can be adapted to better address environmental concerns.

Suppliers often offer payment delays, a fixed period before interest accrues. Traditional EOQ models assume immediate payment upon delivery, which isn't always the case. This delay acts as an interest-free loan for the buyer, allowing them to sell goods and earn interest before settling the account. This incentivizes delaying payment until the end of the allowed period. Numerous studies have explored inventory problems with varying conditions under such payment terms.

#### Literature Review

(Goyal, 1985) first modeled a single-item scenario for determining optimal order quantity with supplier-offered payment delays. (Chung,1998) simplified the solution to Goyal's problem. (Aggarwal *et al.*, 1995) incorporated

exponential deterioration into the model, while (Jamal et al.,1997) further included shortages. (Huang et al.,1997) simultaneously optimized price and lot size, considering price-elastic demand. (Jamal et al., 1997) examined scenarios where retailers could pay at the end of the credit period or later with interest charges. They focused on deteriorating items and optimal cycle/payment times. (Teng, 2002) modified Goyal's model by distinguishing selling and purchase prices, finding that smaller, more frequent orders can be advantageous for established retailers to maximize payment delay benefits. (Chung et al., 2003) extended Goyal's work to finite replenishment rates. (Huang, 2003) introduced a two-level trade credit, where the retailer also extends credit to their customers, reflecting real-world supply chain dynamics.

(Khouja, 2003) demonstrated that complete supply chain synchronization can be a disadvantage to some members. (Huang et al., 2003) considered cash discounts and payment delays, focusing on minimizing average total cost from the retailer's perspective. They explored how suppliers might use cash discounts to encourage earlier payments (Arcelus et al.,2003). Modeled profit-maximizing retail promotion strategies under vendor-offered credit or price discounts. (Abad et al., 2003) analyzed seller-buyer relationships, determining optimal policies under cooperative and noncooperative scenarios. (Huang, 2004) extended Chung and Huang's model, allowing varied retailer payment policies and differing purchase/selling prices, developing a solution for optimal cycle time and order quantity.

Existing research on EOQ models with payment delays typically assumes the supplier offers a fixed, order-quantityindependent delay. However, (Huang et al., 1997) explored a scenario where the credit period is tied to the order size, and demand depends on the selling price, optimizing both price and order size. (Chung et al., 2004) examined order quantity determination for deteriorating items with order-size-dependent payment delays. Critically, these and other studies on EOQ with payment delays often operate under the assumption that the supplier only grants the full payment delay if the retailer orders a minimum quantity; otherwise, no delay is permitted.

Suppliers often use payment delays to encourage larger orders. While the common scenario is a 100% delay for sufficiently large orders and no delay otherwise, this is an extreme case. In reality, suppliers might offer partial payment delays for smaller orders. This means the retailer makes a partial payment upon delivery and pays the remaining balance later. For example, a supplier might offer a 100% delay for large orders but only a (between 0 and 100) delay for smaller ones. This flexible approach allows suppliers to better manage demand stimulation. This more realistic scenario is the focus of this study. Therefore, like many previous studies, we will not consider item deterioration, inflation, or finite time horizons.

A mathematical model is developed to determine the optimal inventory cycle time given these conditions. By incorporating additional costs, such as transportation (including fuel and road construction), the retailer's inventory system is modeled as a cost minimization problem to find the optimal cycle time and order quantity. Three cases, representing a more general framework for optimal replenishment policies, are analyzed, within which some prior research can be seen as special instances and numerical examples are used to illustrate these cases and provide managerial insights.

#### Materials and Methods

(Ritha et al., 2013) have formulated a model that takes into account the effect of the supplier offering a certain time to fulfill his payment commitments. The retailer can take advantage of the delayed payments and try to maximize his earnings till the allowed time for repayment. The model explains the various parameters in detail and their relationship with the optimal cycle time and the optimum quantity.

The main objectives of this paper are as follows:

- Perform use case analysis on the selected parameters of the model, analyzing their impact on the optimum cycle time, optimum quantity and the optimum cost. For each of the parameters, the quantities are varied in a sequence and their effect on the objectives is captured. The parameters considered for such analysis are Demand & Replenishment rate, Order Cost, Unit Purchase price, Unit Holding Stock, Interest that can be earned, Interest charges to be paid and Permissible Delay Period.
- Suggest improvement in the model pertaining to the transportation portion, for which examples have been provided for more clarity.

### Description of the model

The existing model is explained below. For the sake of brevity, the extensive calculations used for arriving at the final objective functions are not presented here and only the objective functions are provided.

### **Notation and assumptions**

#### Notation

D -Demand rate per year

P -Replenishment rate per year,  $P \ge D$ 

A -Cost of placing one order

 $1 - \frac{D}{P} \ge 0$ 

C -Unit Purchasing Price per item

H -Unit Stock Holding Cost per item / year

excluding interest charges

Interest which can be earned per year

Interest charges per investment in inventory

M -Permissible delay period

Road construction cost per trip

fuel cost

Distance travelled

Cycle time

### **Assumptions**

- 1. Demand rate, D is known and constant
- 2. Replenishment rate, P is known and constant
- 3. Shortages are not allowed
- 4. Time period is infinite
- 5.  $I_{k} I_{e}$
- 6. During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account; when  $T \le M$ , the account is settled at T = Mand we start paying for the interest charges on the item in stock.
- 7. When  $T \le M$ , the account is settled at T = M and we do not need to pay any interest charge.

### **Model formation**

The annual total cost consists of the following:

- 1. Annual ordering cost is not dependent upon size and it is calculated for cycle time A/T.
- 2. Annual Stock Holding cost depends upon the size and the storage space

$$= \frac{hT(p-D)/(DT/P)}{2T} = \frac{DTh}{2} (1-\frac{D}{P})$$
$$= \frac{DThp}{2}$$

3. There are 3 cases to occur in costs of interest charges for the items kept in stock per year.

Case (i):  $M \le PM/D \le T$ 

Annual interest payable for the goods by the retailer

$$= CI_{k} \frac{DT2p}{2} - \frac{P-D)M2}{2} = CI_{k}p \frac{DT2}{2} - \frac{PM2}{2} \dots (1)$$

Case (ii):  $M \le T \le PM/D$ 

Annual interest payable for the goods by the retailer

$$= CI_{k} \frac{DT2P}{2} - \frac{P - D)M2}{2} = CI_{k}p \frac{DT2}{2} - \frac{PM2}{2} \dots (2)$$

Case (iii):  $T \le M$ 

In this case, no interest is charged for the items

4. There are 3 cases to occur in interest earned per year

Case (i): 
$$M \le PM/D \le T$$

Annual interest earned = 
$$CI_e \frac{DM2}{T}$$
 ...(3)

Case (ii):  $M \le T \le PM/D$ 

Annual interest earned = 
$$CI_e \frac{DM2}{T}$$
 ...(4)

Case (iii):  $M \le T \le PM/D$ 

Annual interest earned = 
$$CI_e$$
  $\frac{DT2}{2} + DT(M-T)$  ... (5)

- 5. Transportation cost consists of
  - Road Construction Cost / Cycle Time = x/T

ii. Fuel Cost / Cycle Time = 
$$f \frac{d}{T}$$

From the above arguments, the total relevant cost for the retailer can be expressed as

TVC(T) = Ordering Cost + Stock Holding Cost + Interest Payable – Interest Earned + Transportation Cost

The annual total relevant cost, TVC(T) is given by

$$TVC_1$$
, if  $T \ge \frac{PM}{D}$ 

$$TVC(T) = TVC_{2'} \text{ if } M \le T <= \frac{PM}{D} \qquad \dots (6)$$

$$TVC_{3'}$$
, if  $0 \le T \le M$ 

$$TVC_{1} = \frac{A}{T} + \frac{DTHp}{2} + CI_{k}(\frac{DT^{2}}{2} - \frac{PM^{2}}{2}) / T - CI_{e}(\frac{DM^{2}}{2})$$

$$/T + \frac{x}{T} + \frac{fd}{T}$$
 ... (7)

$$TVC_2 = \frac{A}{T} + \frac{DTHp}{2} + CI_k(\frac{D(T-M)^2}{2} / T - CI_e(\frac{DM^2}{2}) / T$$

$$+\frac{x}{T}+\frac{fd}{T}$$
 ... (8)

$$TVC_3 = \frac{A}{T} + \frac{DTHp}{2} + CI_e(\frac{DT^2}{2} + DT(M-T)/T - CI_e(\frac{DM^2}{2})$$

$$/T + \frac{x}{T} + \frac{fd}{T}$$
 ... (9)

### The determination of the Optimum Cycle Time T<sup>0</sup>

Case-1

If 
$$T \ge \frac{PM}{D}$$
, the optimum cycle time is  $T_1^0$ 

Case-2

If M 
$$\leq$$
 T  $\leq \frac{PM}{D}$ , the optimum cycle time is  $T_2^0$ 

Case-3

If  $0 \le T \le M$ , the optimum cycle time is  $T_3^0$ 

$$T_{1}^{0} = \sqrt{\frac{2A + DM^{2}C(Ik - Ie) - PM^{2}CIk + 2x + 2fd}{Dp(h + CIk)}} \quad ... (10)$$

$$T_{2}^{0} = \sqrt{\frac{2A + DM^{2}C(Ik - Ie) + 2x + 2fd}{Dp(h + CIk)}} \dots (11)$$

$$\mathsf{T}^{0}_{3} = \sqrt{\frac{2(A+x+fd)}{Dp(h+C\mathsf{Ie})}} \qquad \dots (12)$$

## **Working Methodology**

- 1. For the given set of values for all the parameters, the values of  $T_{1}^{0}$ ,  $T_{2}^{0}$  and  $T_{3}^{0}$  are calculated based on the equations (10), (11) and (12) given above.
- 2. The optimum cycle time T<sub>0</sub> is derived (from one of the three values of T) based on the three cases discussed above.
- 3. The optimum quantity is derived by multiplying Demand D by the optimum cycle time. ie  $Q_0 = D \times T_0$
- 4. The total variable cost is derived based on the equations (7), (8) and (9) given above based on the equation (6).

### **Observation/Results**

# Effect of small variation of demand on the optimum time, quantity and cost

Given

D = 3000; P = 3200; A = 100; C = 35; H = 5; 
$$I_e = 0.12$$
;  $I_K = 0.15$ ; M = 0.10; x = 75; f = 51; d = 50

The demand **D** is varied from 3000 to 12000 units in steps of 1000 units keeping the values of all other variables the same and the values of optimum time, quantity and cost are calculated and given below.

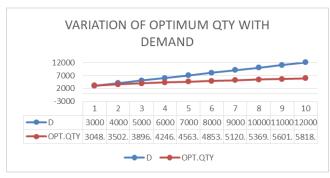


Figure 1: Opt. Qty (vs) Demand

From Table 1, Figures 1, 2 and 3, the following can be inferred

- 1.  $T^0$ , is the calculated optimum time for all the values of D.
- 2. The optimum time is decreasing with increasing values of D.
- 3. There exists a direct relationship between demand with optimum quantity and optimum cost.
- 4. The percentage of transportation costs is quite high, ranging from 50.2 to 57.5%. If the optimum cost is

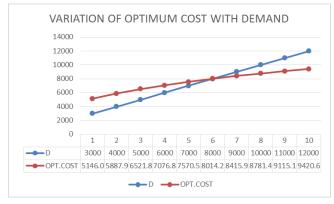


Figure 2: Opt. Cost (vs) Demand



Figure 3: Transp. Cost (vs) Opt. Cost

Table 1: Demand effect

Table 1. Demand effect						
D	Selected time	Opt. time	Opt. qty	Opt. cost	Transp. cost	%Transp. cost with respect to opt. cost
3000	T <sup>0</sup> <sub>1</sub>	1.02	3049	5146	2583	50.2
4000	T <sup>0</sup> <sub>1</sub>	0.88	3503	5888	2998	50.9
5000	T <sup>0</sup> <sub>1</sub>	0.78	3897	6522	3368	51.6
6000	T <sup>0</sup> <sub>1</sub>	0.71	4247	7077	3709	52.4
7000	T <sup>0</sup> <sub>1</sub>	0.65	4564	7571	4026	53.2
8000	T <sup>0</sup> <sub>1</sub>	0.61	4853	8014	4327	54.0
9000	T <sup>0</sup> <sub>1</sub>	0.57	5121	8416	4614	54.8
10000	T <sup>0</sup> <sub>1</sub>	0.54	5369	8781	4889	55.7
11000	T <sup>0</sup> <sub>1</sub>	0.51	5601	9115	5155	56.6
12000	T <sup>0</sup> <sub>1</sub>	0.48	5819	9421	5414	57.5

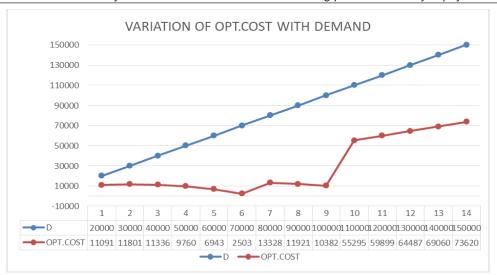


Figure 4: Opt. Cost (vs) Demand

to be decreased, ways and means of reducing the transportation cost is to be explored.

# 2 Effect of large variation of demand on the optimum time, quantity and cost

The demand  $\bf D$  is varied from 20000 to 150000 units in steps of 10000 units, keeping the values of all other variables the same and the values of optimum time, quantity and cost are calculated and given below.

- 1. From Table 2 and Figure 4, the following can be inferred
- 2. The calculated optimum time for the values of D from 20000 to 70000 is  $T_{1,}^0$  from 80000 to 10000 is  $T_{2}^0$  and from 110000 to 150000 is  $T_{3}^0$ .
- 3. The optimum time is decreasing with increasing values of D.

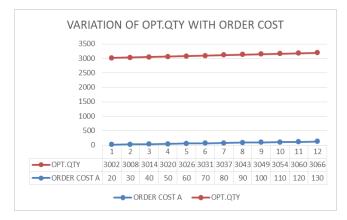


Figure 5: Opt. Qty (vs) Order Cost

Table 2: Demand effect

Table 2. Demand effect							
D	Selected	Opt.	Opt.	Opt. cost	Transp. cost		
	time	time	qty				
20000	T <sup>0</sup> <sub>1</sub>	0.36	7177	11091	7315		
30000	T <sup>0</sup> <sub>1</sub>	0.27	8249	11801	9546		
40000	T <sup>0</sup> <sub>1</sub>	0.22	8857	11336	11856		
50000	T <sup>0</sup> <sub>1</sub>	0.18	9093	9760	14434		
60000	T <sup>0</sup> <sub>1</sub>	0.15	8988	6943	17524		
70000	T <sup>0</sup> <sub>1</sub>	0.12	8528	2503	21546		
80000	$T^0_{2}$	0.11	9095	13328	23090		
90000	$T^0_{2}$	0.11	9727	11921	24289		
100000	$T^0_{2}$	0.10	10337	10382	25395		
110000	T <sup>0</sup> <sub>3</sub>	0.10	10914	55295	26458		
120000	T <sup>0</sup> <sub>3</sub>	0.09	11399	59899	27634		
130000	T <sup>0</sup> <sub>3</sub>	0.09	11864	64487	28763		
140000	T <sup>0</sup> <sub>3</sub>	0.09	12312	69060	29849		
150000	T <sup>0</sup> <sub>3</sub>	0.08	12744	73620	30896		

Table 3: Order Cost effect

Order cost A	Selected time	Opt. time	Opt. qty	Opt. cost	Transp. cost
20	T <sup>0</sup> <sub>1</sub>	1.001	3002	5066	2623
30	T <sup>0</sup> <sub>1</sub>	1.003	3008	5076	2618
40	T <sup>0</sup> <sub>1</sub>	1.005	3014	5086	2613
50	T <sup>0</sup> <sub>1</sub>	1.007	3020	5096	2608
60	T <sup>0</sup> <sub>1</sub>	1.009	3026	5106	2603
70	T <sup>0</sup> <sub>1</sub>	1.010	3031	5116	2598
80	T <sup>0</sup> <sub>1</sub>	1.012	3037	5126	2593
90	T <sup>0</sup> <sub>1</sub>	1.014	3043	5136	2588
100	T <sup>0</sup> <sub>1</sub>	1.016	3049	5146	2583
110	T <sup>0</sup> <sub>1</sub>	1.018	3054	5156	2578
120	T <sup>0</sup> <sub>1</sub>	1.020	3060	5166	2573
130	T <sup>0</sup> <sub>1</sub>	1.022	3066	5176	2569

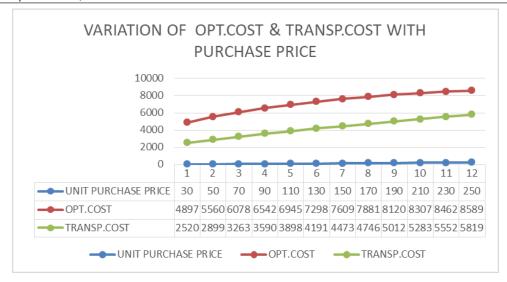


Figure 7: Opt. and Transp. Cost (vs) Price

- 4. The optimum quantity is increasing with increasing values of D.
- 5. The optimum cost values are decreasing up to D = 70000 and increasing thereafter.
- 6. In other words, the retailer can safely accept the demand of up to 70000 units (the cost is the least at D = 70000) since the cost is decreasing.

# 3 Effects of variation of order cost on the optimum time, quantity and cost

The order cost A is varied from Rs 20 to Rs 130 in steps of Rs 10, keeping the values of all other variables the same and the values of optimum time, quantity and cost are calculated and given below.

From Table 3 and Figures 5 the following can be inferred

- 1. The calculated optimum time for all the values of A is  $T_{i}^{0}$
- 2. The optimum time is increasing with increasing values of A. But, the increase is small.
- 3. The optimum quantity is also increasing with increasing values of A, the increase is within a smaller range.

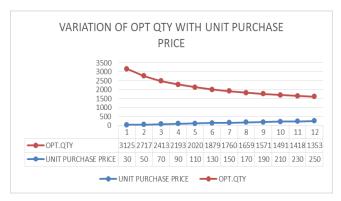


Figure 6: Opt. Qty (vs) Price

- 4. The optimum cost values are increasing for all values of A, but the increase is marginal.
- 5. The transportation cost is also increasing in the same manner as opt. cost
- 6. The increase in order cost does not have any major impact on the optimum time, optimum cost and transportation cost.

# Effect of variation of unit purchase price on the optimum time, quantity and cost

The purchase unit price C is varied from Rs 30 to Rs 250 in steps of Rs 20, keeping the values of all other variables the same and the values of optimum time, quantity and cost are calculated and given below.

From Table 4 and Figures 6 & 7, the following can be inferred

- 1. The calculated optimum time for all the values of C from 30 to 19 is  $T_1^0$
- 2. The optimum time is decreasing with increasing values of *C*
- 3. The optimum quantity is also decreasing with increasing values of C.
- 4. The optimum cost values are increasing for all values of C, but the increase is marginal up to C = 190.
- 5. So, the threshold limit for the value of C is 190 after which the opt. cost values are increasing drastically.
- 6. The transportation cost is also increasing in the same manner as opt. cost

# Effect of variation of holding cost on the optimum time, quantity and cost

The holding cost **H** is varied from Rs 5 to Rs 60 in steps of Rs 5, keeping the values of all other variables the same and the values of optimum time, quantity and cost are calculated and given below.

Table 4: Price effect							
Unit purchase price C	Selected time	Opt. time	Opt. qty	Opt. cost	Transp. cost		
30	T <sup>0</sup> <sub>1</sub>	1.04	3125	4897	2520		
50	T <sup>0</sup> <sub>1</sub>	0.91	2717	5560	2899		
70	T <sup>0</sup> <sub>1</sub>	0.80	2413	6078	3263		
90	T <sup>0</sup> <sub>1</sub>	0.73	2193	6542	3590		
110	T <sup>0</sup> <sub>1</sub>	0.67	2020	6945	3898		
130	T <sup>0</sup> <sub>1</sub>	0.63	1879	7298	4191		
150	T <sup>0</sup> <sub>1</sub>	0.59	1760	7609	4473		
170	T <sup>0</sup> <sub>1</sub>	0.55	1659	7881	4746		
190	T <sup>0</sup> <sub>1</sub>	0.52	1571	8120	5012		
210	T <sup>0</sup> <sub>1</sub>	0.50	1491	8307	5283		
230	T <sup>0</sup> <sub>1</sub>	0.47	1418	8462	5552		
250	T <sup>0</sup> <sub>1</sub>	0.45	1353	8589	5819		

	Table 5: Holding Cost effect							
Hola cost	9	,	time Opt.	. qty Opt. o	cost Transp. cost			
5	T <sup>0</sup> <sub>1</sub>	1.04	312	5 4897	2520			
10	T <sup>0</sup> <sub>1</sub>	0.76	228	7 6552	3443			
15	T <sup>0</sup> <sub>1</sub>	0.63	188	1 7795	4186			
20	T <sup>0</sup> <sub>1</sub>	0.54	163	0 8803	4831			
25	T <sup>0</sup> <sub>1</sub>	0.48	145	4 9649	5416			
30	T <sup>0</sup> <sub>1</sub>	0.44	132	2 1037	4 5958			
35	T <sup>0</sup> <sub>1</sub>	0.41	121	7 1100	2 6468			
40	T <sup>0</sup> <sub>1</sub>	0.38	113	2 1155	0 6955			
45	T <sup>0</sup> <sub>1</sub>	0.35	106	1 1202	8 7424			
50	T <sup>0</sup> <sub>3</sub>	0.23	700	7387	0 11257			
55	T <sup>0</sup> <sub>3</sub>	0.22	669	8893	5 11766			
60	T <sup>0</sup> <sub>3</sub>	0.21	643	9642	8 12250			

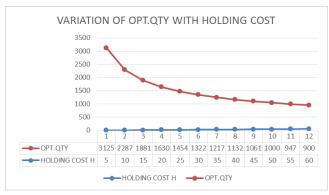


Figure 8: Opt. Qty (vs) Holding Cost

From Table 5 and Figures 8 & 9, the following can be inferred

- 1. The calculated optimum time for the values of H from Rs 5 to Rs 45 is T<sup>0</sup>, and from 50 to 60 is T<sup>0</sup>,
- 2. The optimum time is decreasing with increasing values of C.
- 3. The optimum quantity is also decreasing with increasing values of C.
- 4. The optimum cost values are increasing for all values of C
- 5. The transportation cost is also increasing in the same manner as opt. cost

# Effect of variation of permissible delay period on the optimum time, quantity and cost

The permissible delay period M is varied from 0.1 to 1 year in steps of 0.1, keeping the values of all other variables the same and the values of optimum time, quantity and cost are calculated and given below.

From Table 6 and Figures 10 & 11, the following can be inferred

1. The calculated optimum time for the values of M from 0.1 to 0.5 is  $T^0$ , for M=0.6 is  $T^0$ , and from M=0.7 up to 1.0 is  $T^0$ ,

- 2. The optimum time is decreasing with increasing values of M up to M=0.6 and thereafter remains constant.
- 3. The optimum quantity is decreasing with increasing values of M up to M=0.6 and thereafter remains constant.
- 4. The optimum cost values are decreasing for values of M up to 0.6 and increasing significantly from M=0.7 onwards.
- 5. The transportation cost is also increasing with increasing values of M up to M=0.6 and thereafter remains constant.
- 6. For the value of M=0.5, the optimum cost is the lowest.

### Summary of observations

Variation in the method of computing transportation cost In the existing model, the transportation cost is derived as follows:

i. Road Construction Cost / Cycle Time = x/T

ii. Fuel Cost / Cycle Time = 
$$\frac{fd}{T}$$

In the fuel cost portion, the number of trips made by the trucks used for transportation has not been considered. The existing equation is modified considering the number of trucks as follows.

Fuel cost / Cycle Time = 
$$\frac{fdn}{T}$$

Where n is the number of trips made and

$$\mathbf{n} = \frac{D}{tc}$$
 where Demand is D and Truck

Capacity is to

Accordingly, the revised equations are given below

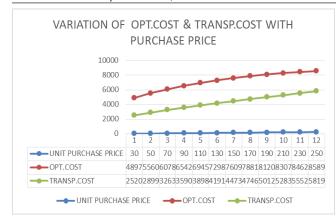


Figure 9: Opt. and Transp. Cost (vs) Holding Cost

Table 6: Delay Period effect							
Delay period M	Selected time	Opt. time	Opt. qty	Opt. cost	Transp. cost		
0.10	T <sup>0</sup> <sub>1</sub>	1.66	4988	3158	1579		
0.20	T <sup>0</sup> <sub>1</sub>	1.60	4792	2912	1643		
0.30	T <sup>0</sup> <sub>1</sub>	1.48	4446	2466	1771		
0.40	T <sup>0</sup> <sub>1</sub>	1.30	3911	1732	2014		
0.50	T <sup>0</sup> <sub>1</sub>	1.03	3089	449	2549		
0.60	$T^0_{\ 2}$	0.62	1884	-2569	4179		
0.70	$T^0_{\ 3}$	0.63	1903	9415	4137		
0.80	$T^0_{3}$	0.63	1903	10675	4137		
0.90	$T^0_{3}$	0.63	1903	11935	4137		
1.00	T <sup>0</sup> <sub>3</sub>	0.63	1903	13195	4137		

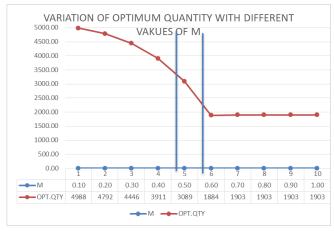


Figure 10: Opt. Qty (vs) Delay Period

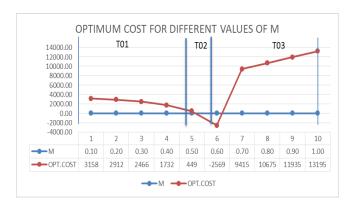


Figure 11: Opt. Cost (vs) Delay Period

Table 7: Summary Observations

Variable	Range of values	Effect on optimum quantity	Effect on optimum cost	Effect on transportation cost
DEMAND <b>D</b>	3000 to 12000	Increases as demand increases	Increases as demand increases	Increases as demand increases. % of transportation cost to the total cost is significant.
DEMAND <b>D</b>	20000 to 150000	Increases as demand increases	Decreases for D up to 70000 and increases thereafter. The lowest cost is for D=70000	Increases as demand increases.
DIFFERENTIAL INTEREST	0.05 to 0.60	Decreases as differential interest increases	Increases as differential interest increases	Increases as differential interest increases
ORDER COST A	20 to 130	Increases over a narrow range as order cost increases. No significant impact.	Increases over a narrow range as order cost increases. No significant impact.	Increases over a narrow range as order cost increases. No significant impact.
UNIT PURCHASE PRICE <b>C</b>	30 to 250	Decreases as unit purchase price increases	Increases as unit purchase price increases	Increases as unit purchase price increases
HOLDING COST <b>H</b>	5 to 60	Decreases as unit purchase price increases	Increases as unit purchase price increases	Increases as holding cost increases
DELAY PERIOD <b>M</b>	0.1 TO 1.0	Decreases for M up to 0.6 and then remains constant	Decreases for M up to 0.5 and then increases	Increases as delay period increases

Table 8: Transp. Cost						
Attribute	Quantity	Opt. cost	Opt. cost		ost	
		Existing model	Revised model	Existing model	Revised model	
Demand <b>D</b>	3000	5146	9717	2583	4864	
Differential interest rate	0.15	5972	10450	2992	5227	
Order cost <b>A</b>	30	5076	8162	2618	4117	
Unit purchase price <b>C</b>	50	5560	7929	2899	3990	
Holding cost <b>H</b>	10	6552	7283	3443	3646	
Delay period <b>M</b>	0.2	2912	4178	1643	2171	

Total cost

$$\begin{split} & \mathsf{TVC_1} = \frac{A}{T} + \frac{DTHp}{2} + \mathsf{CI_k}(\frac{DT^2}{2} - \frac{PM^2}{2}) \ / \, \mathsf{T} - \mathsf{CI_e}(\frac{DM^2}{2}) \\ & / \, \mathsf{T} + \frac{x}{T} + \frac{fdn}{T} \qquad \dots (7) \\ & \mathsf{TVC_2} = \frac{A}{T} + \frac{DTHp}{2} + \mathsf{CI_k}(\frac{D\big(T-M\big)^2}{2} \ / \, \mathsf{T} - \mathsf{CI_e}(\frac{DM^2}{2}) \ / \, \mathsf{T} \\ & + \frac{x}{T} + \frac{fdn}{T} \qquad \dots \qquad (8) \\ & \mathsf{TVC_3} = \frac{A}{T} + \frac{DTHp}{2} + \mathsf{CI_e}(\frac{DT^2}{2} + \mathsf{DT(M-T)} \ / \, \mathsf{T} - \mathsf{CI_e}(\frac{DM^2}{2}) \\ & / \, \mathsf{T} + \frac{x}{T} + \frac{fdn}{T} \quad \dots (9) \end{split}$$

Optimum time

$$T_{1}^{0} = \sqrt{\frac{2A + DM^{2}C(Ik - Ie) - PM^{2}CIk + 2x + 2fdn}{Dp(h + CIk)}} \quad ... (10)$$

$$T_{2}^{0} = \sqrt{\frac{2A + DM^{2}C(Ik - Ie) + 2x + 2fdn}{Dp(h + CIk)}} \dots (11)$$

$$\mathsf{T^0}_3 = \sqrt{\frac{2(A+x+fdn)}{Dp(h+CIe)}} \qquad \dots (12)$$

### **Numerical Examples**

In Table 8, given below, the optimum and the transportation costs are compared based on the existing and revised model. Given

D = 3000; P = 3200; A = 100; C = 35; H = 5; 
$$I_e = 0.12$$
;  $I_K = 0.15$ ; M = 0.10; x = 75; f = 51; d = 50

Truck capacity tc = 1000; Number of trucks = D/tc

The revised model includes the number of trucks  $\underline{\mathbf{n}}$  in the equation

From the above table, it is evident that in the revised model, the transportation costs and the optimum costs are increasing with respect to the existing model because of the inclusion of a number of trips in the calculation of the transportation cost.

#### Discussion

The optimization of both the price and order size was advocated by (Huang et al., 1997), wherein the credit period is tied to the order size, and demand depends on the selling price. The determination of order quantity for deteriorating items with order-size-dependent payment delays was suggested by (Chung et al., 2004). A model that takes into account the effect of the supplier offering a certain time to fulfill his payment commitments was introduced by (Ritha et al., 2013). The effect of the various input parameters on the optimum cycle time, optimum quantity and optimum cost has been examined in detail. While for a small range of variation in the values of demand from 3000 to 12000 units, the variation in optimum cost is linear to that of the demand, but for the large variation of demand from 20000 to 150000 units, the behavior is totally different. A decrease in the optimum cost is observed for the values of demand up to 70000 units, after which the increase in cost is observed, signifying the threshold value of demand of 70000 units. The relationship between the variation in the differential interest, unit purchase price and holding cost is found to be linear with respect to the optimum cost. It is also observed that the variation of order cost does not affect the optimum cost in a significant way. The variation in the delay period results in a decrease in the optimum cost up to the value of 0.6 years for the delay period. These observations can be considered as a useful tool by the retailer for optimizing the cost.

The introduction of the new variable (number of trips made by the trucks) in the determination of the transportation cost has resulted in an increase in the values of transportation cost. It is also observed in the calculation of the optimum cost that the transportation cost component is quite significant. Hence, any reduction in the optimum cost is feasible only by the reduction in the transportation cost.

### Conclusion

The option of providing a certain time to the retailers for the payments due to the suppliers is one of the very good options available under the trade credit scenario, which can be very well used under different use cases.

### **Acknowledgments**

Nil

### **Conflicts Of Interest**

The authors declare no conflict of interest.

#### References

- Abad, P.L., Jaggi, C.K. (2003). A Joint Approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. International Journal of Production Economics, 83: 115-122.
- Aggarwal, S.P., Jaggi, C.K. (1995). Ordering policies of deteriorating items under permissible delay in payments. Journal of the Operational Research Society, 46: 658-662.
- Arcelus, F.J., Shah, N.H., Srinivasan, G. (2003). Retailer's Pricing, Credit and Inventory Policies for deteriorating items in response to temporary price / credit incentives. International Journal of Production Economics, 81-82: 153- 162.
- Chung, K.J. (1998). Theorem on the deterioration of economic order quantity under conditions of permissible delay in payments, Computers and Operations Research, 25: 49-52.
- Chung, K.J., Huang, Y.F. (2003). The Optimal Cycle Time for EPQ Inventory Model under Permissible delay in payments. International Journal of Production Economics, 84: 307-318.
- Chung, K.J., Liao, J.J. (2004). Lot-sizing Decisions under Trade Credit Depending on the Ordering Quantity. Computers and Operations Research, 31: 909-928.
- Goyal, S.K. (1985). Economic Order Quantity under Conditions of Permissible Delay in Payments. Journal of the Operational Research Society, 36: 335-338.

- Huang, H., Shinn, S.W. (1997). Retailer's Pricing and Lotsizing Policy for Exponentially Deteriorating Products under the Condition of Permissible Delay in Payments. Computers and Research, 24: 539-547.
- Huang, Y.F., (2003). Optimal Retailer's Ordering Policies in the EOQ Model under Trade Credit Financing. Journal of the Operational Research Society, 54: 1011-1015.
- Huang, Y.F., Chung, K.J. (2003). Optimal Replenishment and Payment Policies in the EOQ Model under Cash Discount and Trade Credit, Asia-Pacific Journal of Operational Research, 20, 177-190.
- Huang, Y.F., (2004). Optimal Retailer's Replenishment Policy for the EPQ Model under Supplier's Trade Credit Policy. Production Planning and Control, 15: 27-23.
- Jamal, A.M.M., Sarker, B.R., Wang, S. (1997). An Ordering Policy for Deteriorating Items with Allowable Shortages and Permissible Delay in Payment. Journal of the Operational Research Society.,48: 826-833.
- Khouja, M. (2003). Synchronization in Supply Chains, Implication for Design and Management. Journal of the Operational Research Society, 54: 984-994.
- Teng, J.J. (2002). On the Economic Order Quantity under condition of Permissible Delay in Payments. Journal of the Operational Research Society. 53: 915-918.
- W. Ritha, I. Antonitte Vinoline,. (2013), Environmentally Sustainable Inventory Model Under Permissible Delay in Payments, International Journal Of Computers & Technology, Vol 11, No.10