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RESEARCH ARTICLE

The economic order quantity model for sustainable green inventory considers deterioration impact on the real-time replacement and various reorder points with imperfect quality items

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Abstract

In real-life situations, every entrepreneur targets instantaneous replenishment of inventories to the customers. It is not always possible when deterioration occurs in the inventories. There are different cases specified based on lead time, cycle length and order size for the successive replenishment with imperfect quality items. In this economic order quantity (EOQ) model, the cycle length was derived with the deterioration items. The objective of this research is to optimize the length of each cycle for the sustainable green inventory. To examine the proposed model, a numerical example is obtained and to get various results, apply the cycle length value in different cases of reorder point.

Keywords: Economic order quantity, Sustainable green inventory, Deterioration rate, Reorder point, Imperfect items.

Introduction

Inventory models are the mathematical structure that assists businesses in managing their inventory in an efficient, economical and environmentally friendly. Green technology is used to optimize the cycle length and reduce the defective items. While reducing the defective items simultaneously the pollution was reduced. These models are used to determine the stock level of inventory, when to reorder, manage the defective items and their loss, control the overflowing arrival of inventories, consider the demand rate and production rate, minimize the lead time and regulate the length of the cycle, handling the overall costs of the inventory. Inventory models with different constraints and parameters are accessible in the world of operation research. The classical Economic order quantity (EOQ) model is the most common model that is considered as a traditional model. Its formula includes the parameters demand rate, ordering cost and holding cost per unit. Reorder point is the inventory level that a new order should be placed to replenish stock before it runs out. Assure that the stock that is available can satisfy the demand during lead time. Lead time is the time interval between the placing of an order and the actual arrival of the inventory. In this economic order quantity model, expression of the reorder point is obtained for the various cases based on the lead time and the cycle length. Cycle length is the time interval between the two successive orders. To optimize the cycle length for creating sustainable green inventory and, it is derived from the two expressions that are formulated with order size, demand rate and the deterioration rate. To evaluate this derived cycle length expression, a numerical example is calculated and the value of cycle length is in various cases of reorder point.

The rest of the paper is formulated as follows: Section 2 presents the literature review. Section 3 explains the mathematical formulation of the proposed model. Section 4 illustrates a numerical example and Section 5 explains the results and discussion. Finally, Section 6 concludes the outcome of this work and sorts out the references.

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Literature Review

More researchers have attracted inventory models with imperfect quality items to optimize the parameters of the model in recent years. The created models are based on the parameter's constraints of the inventory level in the storage. Previously, some researchers created and developed the inventory model that is related to this research work. Salameh, M. K *et al*. (1999) created the effect of deteriorating items on the instantaneous replenishment model. Salameh, M. K., and Jaber, M. Y (2000) created an economic production quantity model for items with imperfect quality. Lama Moussawi–Haidar *et al*. (2014) developed the effect of deterioration on the instantaneous replenishment model with imperfect quality items. Cheng Y *et al*. (2018) formulated An integrated lot-sizing model for imperfect production with multiple disposals of defective items. Maddah B and Jaber M.Y (2008) developed Economic order quantity for items with imperfect quality. Saxena N *et al*. (2017) modified A green supply chain model of vendor and buyer for remanufacturing. Dave, U. and Patel, L. K. (1981) created a policy inventory model for deteriorating items with time-proportional demand. Jaber, M. Y *et al*. (2013) constructed An entropic economic order quantity (EnEOQ) for items with imperfect quality. Voros, J (2013) formulated Economic order and production quantity models without constraint on the percentage of defective items. Moussawi-Haidar *et al*. (2013) modified An instantaneous replenishment model under the effect of a sampling policy for defective items. Amir Hossein Nobil *et al*. (2020) are created reorder point for the EOQ inventory model with imperfect quality items.

Mathematical Formulation

To develop the proposed model, the following notations and assumptions are used.

Notations

λ - Time for inspecting received items
Q - Optimum batch size
r - reorder point
D - Demand rate
L - Lead time
T - Cycle length
p - Percentage of defective items
K - Ordering cost per order
h - holding cost per item
x – Inspection rate

Assumptions

- Every supplied goods may be of imperfect quality and needs to be inspected.
- System has an inspection ratio (x) for received items, this ratio is greater than or equal to demand rate *(D)*, i.e. $x \geq D$.
- Defective items are sold in another batch with a lower price.
- Each lot contains p fraction of defective items with a known probability density function.

Mathematical Model

The reorder point is the inventory level at which a new order should be placed to replenish stock before it runs out. It is used in inventory management to ensure that stock levels are maintained without overstocking or running out of inventory. The reorder point may have one of the following cases based on the on-hand inventory. Then m is the floor function which is defined by $m = \left\lfloor \frac{L}{T} \right\rfloor$.

Case 1

$L < T$ and $L - mT \leq T - \lambda$:

The reorder point is, $r = DL$

Case 2

$L \geq T$ and $L - mT \leq T - \lambda$:

Define the floor function m is, $m = \left\lfloor \frac{L}{T} \right\rfloor$

The reorder point is, $r = DL - mE(1-p)Q^*$

Where $m = \left\lfloor \frac{L}{T} \right\rfloor$ the floor function and E (.) is expected value.

Case 3

$L < T$ and $L - mT > T - \lambda$

The reorder point is, $r = DL + E(p)Q^*$

Case 4

$L \geq T$ and $L - mT > T - \lambda$

The reorder point is, $r = DL + E(p)Q^* - mE(1-p)Q^*$

The above cases are synopsized as follows:

1. $L - mT \leq T - \lambda$
   - $L < T$ then $r = DL$
   - $L \geq T$ then $r = DL - mE(1-p)Q^*$
2. $L - mT > T - \lambda$
   - $L < T$ then $r = DL + E(p)Q^*$
   - $L \geq T$ then $r = DL + E(p)Q^* - mE(1-p)Q^*$

Salameh and Jaber derived the optimum order size,

$$Q^* = \sqrt{\frac{2KD}{h(E[(1-p)^2] + 2DE(p)/x)}}$$

Time for inspecting received items,

$$\lambda^* = \frac{Q^*}{x}$$

Derivation of cycle length

The cycle length refers to the time gap between two consecutive inventory replenishment cycles. The lead time is time gap between placing of an order and actual arrival of ordered inventory.

$I_1(t)$ is the stock amount of inventory at time t, where $0 \le t \le x$ and

$I_2(t)$ is the stock amount of inventory at time t, where $x \le t \le T$

Then the demand rate D, lot size Q and the deterioration rate α. The corresponding expression for $I_1(t)$ is,

$$I_1(t) = -\frac{D}{1-p} + \left(Q + \frac{D}{1-p}\right)e^{-(1-P)t}, \quad 0 \le t \le x \qquad \dots (1)$$

Then the demand rate D, lot size Q and the deterioration rate α. Solve the expression with the initial condition $I_2(x) = I_2(x) - pQ$ substitute $I_1(t)$ by its expression in (1),

$$I_2(t) = -\frac{D}{1-p} + \left(Q + \frac{D}{1-p} - \alpha Q e^{(1-p)x}\right)e^{-(1-p)T}, \quad x \le t \le T \ . \quad ..(2)$$

To determine the cycle length, manipulate the equation (2) as $I_2(t) = 0$,

$$-\frac{D}{1-p} + \left(Q + \frac{D}{1-p} - \alpha Q e^{(1-p)x}\right)e^{-(1-p)T} = 0$$

$$\left(Q + \frac{D}{1-p} - \alpha Q e^{(1-p)x}\right)e^{-(1-p)T} = \frac{D}{1-p}$$

$$e^{-(1-p)T} = \frac{\frac{D}{1-p}}{\left(Q + \frac{D}{1-p} - \alpha Q e^{(1-p)x}\right)}$$

$$\frac{1}{e^{(1-p)T}} = \frac{\frac{D}{1-p}}{\left(Q + \frac{D}{1-p} - \alpha Q e^{(1-p)x}\right)}$$

$$e^{(1-p)T} = \frac{\left(Q + \frac{D}{1-p} - \alpha Q e^{(1-p)x}\right)}{\frac{D}{1-p}} \qquad \dots (3)$$

Taking log on both the sides of equation (3),

$$(1-p)T = \log\left(\frac{\left(Q + \frac{D}{1-p} - \alpha Q e^{(1-p)x}\right)}{\frac{D}{1-p}}\right)$$

The cycle length is,

$$T = \frac{1}{(1-p)}\left[\log\left(Q + \frac{D}{1-p} - \alpha Q e^{(1-p)x}\right) - \log\left(\frac{D}{1-p}\right)\right] \qquad \dots (4)$$

Numerical example

Consider the following parameters to illustrate the proposed model:

D	50,000 units/year
K	100 $/order
h	5 $/year
x	175200 units/year
σ_1	0
σ_2	0.04

Consider the deterioration rate as a random variable α. It is uniformly distributed on

α, where $\sigma_1 = 0$ and $\sigma_2 = 0.04$.

According to the Continuous Probability distribution,

$$E(p) = \frac{\sigma_1 + \sigma_2}{2} = \frac{0 + 0.04}{2} = 0.02$$

$$E(p) = 0.02$$

$$E(1-P) = \frac{1}{\sigma_2 - \sigma_1}\int_{\sigma_1}^{\sigma_2}(1-p)dp$$

$$= 1 - \frac{(\sigma_2 + \sigma_1)}{2}$$

$$= 1 - \frac{(0.04 + 0)}{2}$$

$$= 1 - 0.02$$

$$E(1-p) = 0.98$$

and

$$E[(1-p)^2] = \frac{1}{\sigma_1 - \sigma_2}\int_{\sigma_1}^{\sigma_2}(1-p)^2 dp$$

$$= \frac{\sigma^2{}_1 + \sigma_1\sigma_2 + \sigma^2{}_2}{3} + 1 - \sigma_1 - \sigma_2$$

$$= \frac{0 + 0(0.04) + (0.04)^2}{3} + 1 - 0 - 0.04$$

$$= \frac{0.0016}{3} + 0.96$$

$$= 0.00053 + 0.96$$

$$E[(1-p)^2] = 0.96053$$

The optimum batch size is,

$$Q^* = \sqrt{\frac{2KD}{h(E[(1-p)^2] + 2DE(p)/x)}} \cong 1434.$$

Table 1: reorder point

S. No.	*L*	*m*	*case*	*r*
1.	0.001	0	1	50
2.	0.002	0	1	100
3.	0.003	0	1	150
4.	0.009	0	3	450
5.	0.010	0	3	500
6.	0.011	0	3	550

The cycle length is,

$$T = \frac{1}{(1-p)}\left[\log\left(Q + \frac{D}{1-p} - \alpha Q e^{(1-p)x}\right) - \log\left(\frac{D}{1-p}\right)\right]$$

Substitute the respective values in the above equation and get the value of cycle length.

$$T \cong 0.012$$

Results

In this sustainable green inventory model the value of cycle length is optimized. This optimization was need because of regulate the supply, demand, arrival of the inventory and the lead time. Through this regulation, can minimize the defective items and maximize the profit. The optimized value of cycle length is $T \cong 0.012$ .Here are some different lead times to calculate the reorder point. 0.001, 0.002, 0.003, 0.009, 0.010, 0.011 (Table 1).

Discussion

Salameh, M. K *et al*. (1999) created effect of deteriorating items on the instantaneous replenishment model. Lama Moussawi – Haidar *et al*. (2014) developed effect of deterioration on the instantaneous replenishment model with imperfect quality items. Maddah B and Jaber M.Y (2008) developed Economic order quantity for items with imperfect quality. Saxena N *et al*. (2017) modified A green supply chain model of vendor and buyer for remanufacturing. Amir Hossein Nobil *et al*. (2020) are created Reorder point for the EOQ inventory model with imperfect quality items. The cycle length value which is calculated in this model less than the Amir Hossein Nobil *et al*. value. The value was reduced because of the optimization. The derived cycle length has the characteristics as replenishment is instantaneous. So that the lead time is zero or negligible. Now, can conclude lead time cannot be more than cycle length. The four cases that are specified above, presents the relation between lead time and cycle length.

Conclusion

The Economic Order Quantity (EOQ) model for sustainable green inventory emphasis on the deterioration impact of real-time replacement and various reorder point with imperfect quality items. The model has various cases of the reorder point based on demand rate, lead time, expression of order size and derived cycle length. Green techsnology is used to optimize the cycle length and reduce the defective items. While reducing the defective items simultaneously the pollution was reduced. The cycle length is formulated by using the assumptions and the continuous uniform probability distribution. To evaluate the mathematical model a numerical example is solved. The calculated value of cycle length is reduced compare with the existing value. Examine which reorder point scenarios apply to the calculated cycle length. This results conclude that the inventory ordering period is regulated for the successive orders of the inventory.

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None

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