The Scientific Temper (2025) Vol. 16 (4): 4012-4016
E-ISSN: 2231-6396, ISSN: 0976-8653
Doi: 10.58414/SCIENTIFICTEMPER.2025.16.4.02
https://scientifictemper.com/

RESEARCH ARTICLE

A study on energy sum of dominating sets in East Indian states

N. Anbarasi[1], K. Anitha[2], S. Hemalatha[1]

Abstract

This article explores the spectral properties of dominating sets in a map network for the East Indian states, with a focus on computing various energy sums using the eigenvalues of matrices depending on the Laplacian. Dominating sets—subsets of vertices that ensure each node of a graph is either included in or close to a node in the set—are critical for network optimization, resource allocation, and regional planning. This study uses three types of Laplacian matrices: The Laplacian Matrix, the Laplacian Matrix of Vertex Dominance, and the Laplacian Matrix of Edge Domination. The structural and dominating characteristics of the graph are characterized by calculating the eigenvalues for those matrices and examining the energy sums associated with them. The results are confirmed using computational coding in the MATLAB application, ensuring correctness and providing a consistent framework for spectral graph study. The findings contribute to our understanding of the network's resilience, connectivity, and optimization potential, as well as give important details for East India's growth in infrastructure and regional planning. This paper explains why spectral graph theory can be used to investigate map-based networks and provides a versatile approach for future research in related disciplines.

Mathematics Classification Code: 05C050

Keywords: Total Dominating set, Laplacian matrix, Laplacian matrix of total dominating set for vertex domination, Laplacian matrix of total dominating set for edge domination, Energy sum, East India.

Introduction

The map or graph of Eastern Indian states is the main topic of this essay. States are represented as nodes (Catherine Mary A, Maria Sujitha Y, Meena S, 2023) in an East India graph (Figure 1), and if the states have a border, edges are drawn between the nodes (Figures 2 and 3). We may learn more about this graph's structure and possible uses in resource management and infrastructure building by examining its dominating sets.

Relationships between items are represented by graphs, which are strong mathematical structures. A dominant set (Ivan Gutman, Harishchandra S. Ramane, 2020) is a key idea in graph theory. In a graph, it is a subset of vertices where each vertex is either part of the set or related directly to another node in the set. Real-world issues like resource allocation, network optimization, and regional planning can be resolved with the use of dominating sets.

A dominant set, as defined in graph theory (José M. Sigarreta, 2021), is a subset $\mathbb{D}$ in a graph's vertices such that every vertex in $\mathbb{G}$ is in $\mathbb{D}$ or has a neighbour in $\mathbb{D}$. A subset S of a graph's edges (Kiyoshi Yoshimoto, 2008) is called a dominant set if every edge of $\mathbb{G}$ is in S or has a neighbour in S. The number of nodes in a minimal dominating set (Malathy K, Meenakshi S, 2021, Manjula C. Gudgeri , Varsha, 2020, Meenakshi S, Lavanya S, 2017) for $\mathbb{G}$ is known as the domination number $\Upsilon(\mathbb{G})$.

The number of edges occurring to a vertex in a graph is its degree, also known as its valency. In a graph, an edge's degree (Kiyoshi Yoshimoto, 2008) is equal to the number of its neighbours, or $|\tilde{N}(u) \cup \tilde{N}(v)| - 2$. The number of links that connect a vertex in a graph is its degree.

We apply spectral graph theory to examine the characteristics of these dominating sets. This entails researching the graph's matrices, especially the Laplacian matrix (Anne Marsden, Rajesh Kanna M R, Dharmendra B N, Sridhara G, 2013). One essential tool for comprehending the connectivity and structure of graphs is the Laplacian matrix. We use three different kinds of Laplacian matrices in this study:

[1]Department of Mathematics, SDNB Vaishnav College for Women, Shanthi Nagar, Chromepet, 600044, Tamil Nadu, India.

[2]Department of Mathematics, Sri SaiRam Engineering College, Sai Leo Nagar, West Tambaram, 600045, Tamil Nadu, India.

***Corresponding Author:** N.Anbarasi, Department of Mathematics, SDNB Vaishnav College for Women, Chromepet, Chennai - 600 044, Tamil Nadu, India., E-Mail: anbarasi.maths.digiclass@gmail.com

How to cite this article: Anbarasi, N., Anitha, K., Hemalatha, S. (2025). A study on energy sum of dominating sets in East Indian states. The Scientific Temper, **16**(4):4012-4016.

Doi: 10.58414/SCIENTIFICTEMPER.2025.16.4.02

Source of support: Nil

Conflict of interest: None.

© *The Scientific Temper. 2025*

Received: 19/11/2024 **Accepted:** 13/01/2025 **Published** : 25/04/2025

The graph's fundamental structure is reflected in the Standard Laplacian Matrix. The Laplacian matrix $Ł_{m \times m}$Ln×n of a simple graph Ǥ with its vertices $\{u_1, u_2, u_3, \ldots, u_m\}$ is defined element-wise as

$$Ł_{i,j}(Ǥ) = \begin{cases} deg(u_i), & if\ i = j \\ -1, & if\ i \neq j\ and\ u_i\ is\ adjacent\ to\ u_j \\ 0, & otherwise \end{cases}$$

or similarly by the matrix Ł = Đ− Å, where Å is the graph's adjacency matrix and Đ is its degree matrix. Since Ǥ represents a simple graph, all of its diagonal members are zeros, and Å only contains one or zeros.

The Laplacian Matrix for Vertex Domination emphasizes the dominating function of particular nodes. The Laplacian matrix $Ł_{m \times m}$ for the vertex dominance set of a simple graph Ǥ = (Û, Ě) with nodes $\{u_1, u_2, u_3, \ldots, u_m\}$ is defined element-wise by

$$Ł_{i,j}(\mathbb{D}(Û)) = \begin{cases} deg(u_i), & if\ i = j\ and\ i \in D \\ -1, & if\ i\ dominates\ j\ or\ i\ is\ dominated\ by\ j \\ 0, & otherwise \end{cases}$$

The dominance interactions between edges are taken into account by the Laplacian Matrix for Edge Domination. The Laplacian matrix $Ł_{m \times m}$Ln×n for the edge dominance set of a simple graph ǤGǤ with vertices $\{u_1, u_2, u_3, \ldots, u_m\}$ is defined as

$$Ł_{i,j}(S(Ě)) = \begin{cases} deg(e_i), & if\ i = j\ and\ i \in S \\ -1, & if\ i\ dominates\ j\ and\ i\ is\ dominated\ by\ j \\ 0, & otherwise \end{cases}$$

We can measure crucial aspects of the graph, including resilience and connectedness, by computing the eigen values for those matrices and examining the energy sums that arise. The characteristic equation, often known as the characteristic polynomial, is the equation that must be solved in order to determine the eigen values of a matrix.

The characteristic expression in variable η for a general m by m matrix M is defined by $|A - \eta I| = 0$, where $|A - \eta I|$ is the matrix's determinant and I is its identity matrix. The eigen values are the roots for the characteristic polynomials. The scalar η that satisfies the equation $A\upsilon = \eta\,\upsilon$, where υ is a non-zero column vector and A is the square matrix, is a matrix's eigen value.

A graph's energy sum (Ivan Gutman, Harishchandra S. Ramane, 2020, Keerthi G. Mirajkar and Baghyashri R. Doddamani, 2019, Kolamban S, Kamal Kumar M, 2018) is the total of the given matrix's eigen values' absolute values. i.e.

$$\grave{E}(Ǥ) = \sum_{i=1}^{n} \eta_i \ ,$$

Where $\eta_i, i = 1, 2, 3, \ldots, n$ are the eigen values of the graph Ǥ's adjacency matrix.

In addition to improving our comprehension of the eastern India map graph, this work shows how spectral graph theory can be applied more broadly to map-based networks. The results can be used to plan regional infrastructure, create effective networks, and allocate resources as efficiently as possible in other areas. This work establishes the groundwork for further investigations into the spectral study of graphs in other domains.

An overview of Eastern India

The relationships between the states of Jharkhand, Bihar, West Bengal, and the Union Territory of Chhattisgarh are depicted in the eastern India states graph. Each territory or state is shown as a node in this graph, and the edges denote connections like common borders, transit connections, or socioeconomic exchanges.

The interconnectedness of the states within the area is shown in this graph, which is usually linked and undirected. The eastern India states graph is a useful tool for network dynamics research, resource allocation optimization, and connectivity analysis. It is an essential structure for both

Figure 1: East India Map

Figure 2: East India states in India map

theoretical research and real-world applications in the area since it offers insights into communication tactics, crisis management, transportation efficiency, and regional planning.

The states are represented as vertices in this graph, and the lines connecting any two vertices are called edges [Catherine Mary A, Maria Sujitha Y, Meena S, 2023]. The vertices are named u_1 - Bihar, u_2 – West Bengal, u_3 - Jharkhand, u_4 – Chhattisgarh, u_5 – Orissa. The edges are named e_1 – the line connecting Bihar and West Bengal, e_2 – Bihar and Jharkhand, e_3 – West Bengal and Jharkhand, e_4 – West Bengal and Orissa, e_5 – Jharkhand and Chhattisgarh, e_6 – Jharkhand and Orissa, e_7 –Chhattisgarh and Orissa.

Main Results

In this section, we used a MATLAB program to confirm the correctness of our calculations and calculate the energy sum for the eastern India graph. The accuracy of our findings was guaranteed by this validation.

Theorem

Assume that Ǥ =EI is a simple graph with a dominant set D. This inequality is true if an energy sum E_L(Ǥ) is obtained in a different way:

$$\grave{E}_{Ł_{i,j}(Ɖ(\hat{U}))}(Ǥ) < \grave{E}_{Ł_{i,j}(S(\check{E}))}(Ǥ) < \grave{E}_{Ł_{i,j}}(Ǥ)$$

where the energy sum of Ǥ under various graph characteristics is represented by $\grave{E}_{Ł_{i,j}(Ɖ(\hat{U}))}$(Ǥ), $\grave{E}_{Ł_{i,j}(S(\check{E}))}$(Ǥ), and $\grave{E}_{Ł_{i,j}}$(Ǥ).

Proof

Three different kinds of matrices were used in this proof to bolster our findings. Laplacian Matrix, Laplacian Dominant Matrix of Vertex Domination, and Laplacian Dominant Matrix of Edge Domination are some examples of these. These matrices collectively were essential to our computations and findings.

i. Let's examine the Laplacian matrix for the Eastern India map graph to get the energy sum. Let EI be the graph representing Eastern India, where nodes stand in for the states and edges indicate the relationships between them according to how close they are to one another.

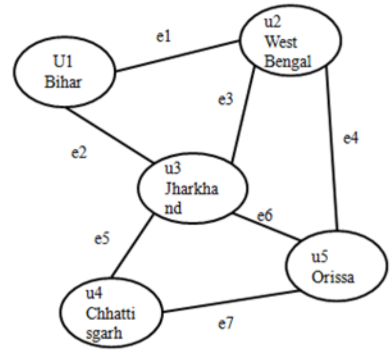

Figure 3: East India graph

Consequently, Eastern India's Laplacian matrix is

$$Ł_{i,j}(EI) = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & -1 & 3 \end{bmatrix}$$

The matrix's characteristic equation is

$$\begin{vmatrix} 2-\eta & -1 & -1 & 0 & 0 \\ -1 & 3-\eta & -1 & 0 & -1 \\ -1 & -1 & 4-\eta & -1 & -1 \\ 0 & 0 & -1 & 2-\eta & -1 \\ 0 & -1 & -1 & -1 & 3-\eta \end{vmatrix} = 0$$

$$\Rightarrow \; -\eta^5 + 14\eta^4 - 70\eta^3 + 146\eta^2 - 105\eta = 0$$

The eigen values are 0, 1.5858, 3, 4.4142, 5.

Then the energy sum of EI is $\grave{E}_{Ł_{i,j}}$(EI) = 14.

ii. Let EI stand for the Eastern India graph, in which states are represented by nodes and links between them are represented by edges based on their proximity to one another. The EI graph's vertex domination set is {u_3 (Jharkhand)}. The EI graph's dominant number is 1.

For the vertex dominance set, the Laplacian matrix is

$$Ł_{i,j}(Ɖ(\hat{U})) = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ -1 & -1 & 4 & -1 & -1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

The matrix's characteristic equation is

$$\begin{vmatrix} -\eta & 0 & -1 & 0 & 0 \\ 0 & -\eta & -1 & 0 & 0 \\ -1 & -1 & 4-\eta & -1 & -1 \\ 0 & 0 & -1 & -\eta & 0 \\ 0 & 0 & -1 & 0 & -\eta \end{vmatrix} = 0$$

$$\Rightarrow -\eta^5 + 4\eta^4 + 4\eta^3 = 0$$

The eigen values are: -0.8284, 0, 0, 0, 4.8284.

Then the energy sum of EI is $`\mathrm{E}_{\text{Łi,j(Đ(Û))}}(\mathrm{EI}) = 5.6569$.

iii. Let EI stand for the Eastern India graph, in which states are represented by nodes and links between them are represented by edges based on their proximity to one another. The EI graph's edge domination set is $\{e_1, e_7\}$. (Note: the boundaries between Chhattisgarh and Orissa and Bihar and West Bengal are denoted by e_1 and e_7, respectively). The edge dominance set's Laplacian matrix is

$$Ł_{i,j}(S(\check{E})) = \begin{bmatrix} 3 & -1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & -1 & -1 & 3 \end{bmatrix}$$

The matrix's characteristic equation is

$$\begin{bmatrix} 3-\eta & -1 & -1 & 0 & 0 & 0 & 0 \\ -1 & -\eta & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -\eta & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -\eta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\eta & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -\eta & -1 \\ 0 & 0 & 0 & -1 & -1 & -1 & 3-\eta \end{bmatrix} = 0$$

$$\Rightarrow -\eta^7 + 6\eta^6 - 5\eta^5 - 12\eta^4 - 4\eta^3 = 0$$

The eigen values are: 3.5616, 3.5616, 0.5616, -0.5616, 0, 0, 0.

Then the energy sum of EI is $`\mathrm{E}_{\text{Łi,j(S(Ě))}}(\mathrm{EI}) = 8.2462$.

The least value ($`\mathbf{EŁ}_{i,j}(Đ(\hat{U}))(\mathrm{EI}) = 5.6569$) among all three energy sums computed for the eastern portion of India map graph is regarded as the recommended and ideal approach. This is due to the fact that it efficiently streamlines the graph while maintaining the most crucial spectral characteristics. It is the recommended technique for processing or analyzing the graph since the smaller energy sum denotes less computational complexity. The diagram below will serve as a representation of this.

Conclusion from the Bar Chart

The eastern India map graph's energy sums are compared using three distinct approaches in the bar chart. The values indicate that:

This result emphasizes how crucial it is to select techniques for real-world graph theory applications that maximize simplicity while maintaining important spectral properties.

Verification of Results Using MATLAB

A MATLAB program was used to evaluate and validate the aforementioned results in order to guarantee their accuracy.

1. Lapalacian Matrix MATLAB Code

```
A = [2, -1, -1, 0, 0;
  -1, 3, -1, 0, -1;
  -1, -1, 4, -1, -1;
   0, 0, -1, 2, -1;
   0, -1, -1, -1, 3];
% Calculate the eigenvalues of A
eigenvalues = eig(A);
% Calculate the energy sum (sum of the absolute values of eigenvalues)
energy_sum = sum(abs(eigenvalues));
% Display the energy sum
disp(['Energy sum of the graph matrix: ', num2str(energy_sum)]);
```

MATLAB: The Lapalacian matrix code for vertex domination is as follows

```
A = [0, 0, -1, 0, 0;
     0, 0, -1, 0, 0;
    -1, -1, 4, -1, -1;
     0, 0, -1, 0, 0;
     0, 0, -1, 0, 0];
```

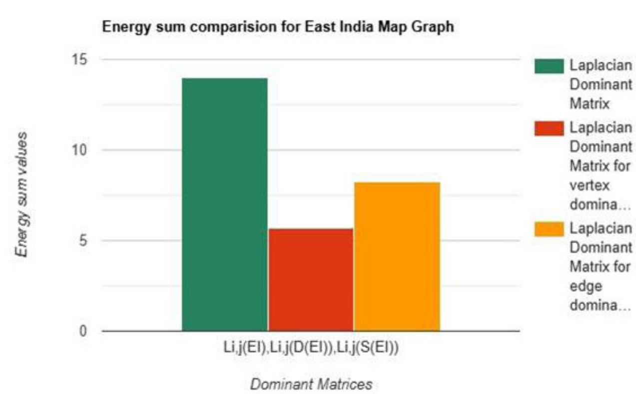

Figure 4: Bar chart representation of energy sum of East India graph

```
% Calculate the eigenvalues of A
eigenvalues = eig(A);
% Calculate the energy sum (sum of the absolute values of eigenvalues)
energy_sum = sum(abs(eigenvalues));
% Display the energy sum
disp(['Energy sum of the graph matrix: ', num2str(energy_sum)]);
```

2. MATLAB: The Lapalacian matrix code for edge domination is as follows:

```
A = [3, -1, -1, 0, 0, 0, 0;
    -1,  0,  0, 0, 0, 0, 0;
    -1,  0,  0, 0, 0, 0, 0;
    -1,  0,  0, 0, 0, 0, 0;
     0,  0,  0, 0, 0, 0, -1;
     0,  0,  0, 0, 0, 0, -1;
     0,  0,  0, -1, -1, -1, 3];
% Calculate the eigenvalues of A
eigenvalues = eig(A);
% Calculate the energy sum (sum of the absolute values of eigenvalues)
energy_sum = sum(abs(eigenvalues));
% Display the energy sum
disp(['Energy sum of the graph matrix: ', num2str(energy_sum)]);
```

Conclusion

The Eastern India (EI) graph's energy sum was examined in this work under various spectral circumstances (Figure 4). The difference in energy values according to the graph matrix selection. This illustrates how a graph's spectral characteristics and matrix-based calculations affect its energy sum. These discoveries are essential for furthering the study of graph theory and its usage in dominant set energy calculations.

Acknowledgments

The authors are grateful to management and principals for their support of our research.

Conflict of Interest

Regarding the release of this study on the energy sum of the West India graph, the author states that there is no conflict of interest.

References

Anne Marsden, (2013). Eigen values of the Laplacian and their Relationship to the Connectedness of a Graph, The University of Chicago, Department of Mathematics. https://math.uchicago.edu/~may/REU2013/REUPapers/Marsden.pdf

Catherine Mary A, Maria Sujitha Y, Meena S, (2023). Four Colouring Problem Applying in Tamilnadu (District), International Journal For Innovative Research in Multidisciplinary Field, Volume - 9, Issue-3. https://acadpubl.eu/jsi/2017-115-5/5/5.pdf

Ivan Gutman, Harishchandra S. Ramane,(2020). Research on Graph Energies in 2019, MATCH Commun. Math. Comput. Chem. 84, 277- 292 ISSN 0340 – 6253. https://match.pmf.kg.ac.rs/electronic_versions/Match84/n2/match84n2_277-292.pdf

José M. Sigarreta, (2021). Total Domination on Some Graph Operators, MDPI, Mathematics, 9, 241. https://ideas.repec.org/a/gam/jmathe/v9y2021i3p241-d487339.html.

Keerthi G. Mirajkar and Baghyashri R. Doddamani, (2019). On energy and spectrum of degree product adjacency matrix for some class of graphs, International Journal of Applied Engineering Research, vol. 14, no. 7, pp. 1546–1554. https://www.ripublication.com/ijaer19/ijaerv14n7_16.pdf

Kiyoshi Yoshimoto, (2008). Edge degrees and dominating cycles, Discrete Mathematics Volume 308, Issue 12, Pages 2594-2599. https://www.sciencedirect.com/science/article/pii/S0012365X07003251

Kolamban S, Kamal Kumar M,(2018). Various Domination Energies in Graphs, Int. J. Math. Combin. 3, 108–124. https://www.researchgate.net/publication/344227957_Various_Domination_Energies_in_Graphs

Malathy K, Meenakshi S,(2021). Minimum Total Dominating Hyper energetic Graphs", Advances and Applications in Mathematical Sciences Volume 21, Issue 2, December 2021, Pages 539-553, Mili Publications, India. https://www.mililink.com/upload/article/950364099aams_vol_212_december_2021_a3_p539-553_k._malathy_and_s._meenakshi.pdf

Manjula C. Gudgeri , Varsha, (2020). Total Dominating Energy of Some Graphs, International Journal of Recent Technology and Engineering (IJRTE), ISSN: 2277-3878, Volume-9 Issue-1. https://www.researchgate.net/publication/363765384_Total_Dominating_Energy_of_Some_Graphs

Meenakshi S, Lavanya S, (2017). Minimum Dom Strong Dominating Energy of Graph, International Journal of Pure and Applied Mathematics, Volume 115, No. 5. https://www.ijirmf.com/wp-content/uploads/IJIRMF202303027-min.pdf

Muhuo Liu, Bolian, (2011). A note on sum of powers of the Laplacian eigen values of graphs, ELSEVIER, Applied Mathematics Letters, 249 – 252. https://www.sciencedirect.com/science/article/pii/S0893965910003356

Rajesh Kanna M R, Dharmendra B N, Sridhara G, (2013). The Minimum Dominating Energy of a Graph, International Journal of Pure and Applied Mathematics, Volume 85 No. 4. https://www.ijpam.eu/contents/2013-85-4/7/7.pdf