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STABILITY IN THE EQUILIBRIUM POSITION OF AN EXTENSIBLE CABLE-CONNECTED TWO SATELLITE SYSTEM UNDER PERTURBATIVE FORCE IN CIRCULAR ORBIT

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ABSTRACT

One stable position of equilibrium must exist for a system of two satellites connected by extensible string whether perturbative forces act on it or not. In this paper we have obtained an equilibrium point for the system when perturbative force like air resistance acts on it and has been shown to be stable in the sense of Liapunov.

INTRODUCTION

This paper is devoted to examine the stability of the equilibrium point of the centre of mass of the system when air resistance as perturbative force acts on it. Singh; R.B. and Sharma; B are the pioneer workers in this fields.

EQUATIONS OF MOTION

The equation of motion of one of two satellites moving along Keplerian elliptical orbit in mechvill's coordinates with air resistance can be obtained by using Lugranges equations of motion of first kind in the form :

$$x'' - 2y' - 3\rho x = -\bar{\lambda}_\alpha \rho^4 \left[1 - \frac{\ell_0}{\rho_r}\right] x$$

$$y'' + 2x' + f = -\bar{\lambda}_\alpha \rho^4 \left[1 - \frac{\ell_0}{\rho r}\right] y$$

and $z'' + z = -\bar{\lambda}_\alpha \rho^4 \left[1 - \frac{\ell_0}{\rho r}\right] z$, $f =$ Air resistance force parameter

Where $\rho = \frac{1}{1+e \cos v}$; $r = \sqrt{x^2 + y^2 + z^2}$

v = True anomaly of the centre of mass of the system.

$\bar{\lambda}_\alpha = \frac{p^3}{\mu} \lambda_\alpha$, $\ell_0 =$ Natural length of the string.

Here dashes denote differentiation w.r. to true anamoly v. The condition of constraint is given by

$$x^2 + y^2 + z^2 \le \frac{\ell_0^2}{\rho^2} \qquad (2)$$

For circular orbit of the centre of mass of the system, we have

e = 0

$$\rho = \frac{1}{1+ecosv} = 1 \; and \; \rho^1 = 0$$

Putting $\rho = 1$ in (1), we get the equations of motion for two dimensional case in the form

$$x'' - 2y' - 3x = -\bar{\lambda}_\alpha \left[1 - \frac{\ell_0}{r}\right] x$$

$$y'' + 2x' + f = -\bar{\lambda}_\alpha \left[1 - \frac{\ell_0}{r}\right] y \qquad (3)$$

where $r = \sqrt{x^2 + y^2}$

The condition of constraint given by (2) takes the form

$$x^2 + y^2 \leq \ell_0^2 \qquad (4)$$

We find that the equations of motion (3) do not contain time to explicitly. Hence Jacobian integral for the problem must exist. Multiplying the first equation of (3) by 2x' and the 2nd equation of (3) by 2y' and adding them together and then integrating, we get Jacabian integral as

$$x'^2 + y'^2 - 3x^2 + 2fy + \bar{\lambda}_\sigma \left[(x^2 + y^2) - 2\ell_0\sqrt{x^2 + y^2}\,\right] = h \qquad (5)$$

$$x' = 0 = x'' \; and \; y' = 0 = y''$$

Where n is the constant of integration.

Equation (5) can be rewritten as

$$x'^2 + y'^2 = 3x^2 - 2fy - \bar{\lambda}_\alpha \left[(x^2 + y^2) - 2\ell_0\sqrt{x^2 + y^2}\,\right] + h \qquad (6)$$

$$a = \sqrt{\left(\frac{\lambda_\alpha \ell_0}{\lambda_\alpha - 3}\right)^2} - \frac{f_2}{9} \; and \; b = \frac{-f}{3}$$

The curve of zero velocity can be obtained in the form

$$3x^2 - 2fy - \bar{\lambda}_\alpha \left[(x^2 + y^2) - 2\ell_0\sqrt{x^2 + y^2}\,\right] + h = 0 \qquad (7)$$

Hence, we conclude that the satellite of mass m, will move inside the boundances of different curves of zero velocity represented by (7) of (6) for different values of Jacobian constant h.

EQUILIBRIUM POINT OF THE PROBLEMS

We have obtained the system of equations given by (3) for the motion of the system in rotating frame of reference. It has been assumed that the system is moving with effective constraints and the connecting cable of two satellites of masses m_1 and m_2 respectively will always remain tight.

The equilibrium position of the system is given by the constant values of the coordinates in the rotating frame of reference.

Now, let $x = x_o$ and $y = y_o$ give the equilibrium position where x_o and y_o are constants-

Hence $x' = 0 = x''$ and $y' = 0 = y''$

Thus, equations given by (3) take the form

$$-3x_0 = -\bar{\lambda}_\alpha \left[1 - \frac{\ell_0}{r_0}\right] x_o$$

$$f = - - \bar{\lambda}_\alpha \left[1 - \frac{\ell_0}{r_0}\right] y_o \qquad (8)$$

Where

$$r_0 = \sqrt{x_o^2 + y_o^2}$$

From (8), we get the equilibrium point as

$$[a, b] = \left[\sqrt{\left(\frac{\bar{\lambda}_\alpha \ell_0}{\bar{\lambda}_\alpha - 3}\right)^2} - \frac{f_2}{9}, \frac{-f}{3}\right] \qquad (9)$$

Stability of the equilibrium point [a,b] of the system

We have $a = \sqrt{\left(\frac{\lambda_\alpha \ell_0}{\lambda_\alpha - 3}\right)^2} - \frac{f_2}{9} \; and \; b = \frac{-f}{3}$

Let ξ and η denote variations in the coordinates for this position of equilibrium. Then

$$x = a + \xi \;\; ; \;\; y = b + \eta$$

$$\therefore x' = \xi' \;\; ; \;\; y' = \eta' \qquad (10)$$

$$x'' = \xi'' \;\; ; \;\; y'' = \eta''$$

Using (10) in (3) , we get

$$\xi'' - 2\eta' - 3(a + \xi) = -\bar{\lambda}_\alpha \left(1 - \frac{\ell_0}{r_1}\right)(a + \xi)$$

$$\eta'' + 2\xi' + f = -\bar{\lambda}_\alpha \left(1 - \frac{\ell_0}{r_1}\right)(b + \eta) \qquad (11)$$

Where $r_1 = \sqrt{(a + \xi)^2 + (b + \eta)^2}$ (12)

As the original system of equations admits Jacobi's integral, so there must exist a Jacobi's integral for the system of equation(11). Moreover, it is not different to deduce the Jaobi's integral for (11) in the form :

$$\xi'^2 + \eta'^2 - 3(a + \xi)^2 + 2f(b + \eta) + \bar{\lambda}_\alpha[(a + \xi)^2 + (b + \eta)^2]$$

$$-2\ell_0 \bar{\lambda}_\alpha[(a + \xi)^2 + (b + \eta)^2]^{\frac{1}{2}} = h_1 \qquad (13)$$

Where h_1 is the constant of integration.

Now, (13) can be put in the form :

$$\xi'^2 + \eta'^2 + \xi^2\left[\bar{\lambda}_\sigma - 3 - \frac{\bar{\lambda}_\alpha a^2 \ell_0}{(a^2+b^2)^{3/2}} - \frac{\bar{\lambda}_\alpha \ell_0}{(a^2+b^2)^{1/2}}\right]$$

$$+\eta^2\left[\bar{\lambda}_\alpha - \frac{\bar{\lambda}_\alpha \ell_0}{(a^2+b^2)^{1/2}} + \frac{\bar{\lambda}_\alpha b^2 \ell_0}{(a^2+b^2)^{3/2}}\right]$$

$$+\frac{4\xi\eta\ell_0\bar{\lambda}_\alpha ab}{(a^2+b^2)^{3/2}} + \xi\left[2\bar{\lambda}_\alpha a - 6a - \frac{2\bar{\lambda}_\alpha a\ell_0}{(a^2+b^2)^{1/2}}\right]$$

$$+\eta\left[2f + 2\bar{\lambda}_\alpha b - \frac{2\bar{\lambda}_\alpha b\ell_0}{(a^2+b^2)^{1/2}}\right] + 0(3) = h_1 \quad (14)$$

Where 0(3) stands for 3rd and higher of terms in ξ *and* η. To test the stability in the sense of Liapunov, we take Jacobian integral given by (14) as Liapunov's function $V(\xi', \eta', \xi, \eta)$ and applying Liapunov's theorem on stability it follows that the only anterior for given equilibrium position [a,b] to be stable is that V must be positive definite and for this the following conditions must be satisfied :

(i) $2\bar{\lambda}_\alpha a - 6a - \frac{2\bar{\lambda}_\alpha a\ell_0}{(a^2+b^2)^{1/2}} = 0$

(ii) $2\bar{\lambda}_\alpha b + 2f - \frac{2\bar{\lambda}_\alpha \ell_0}{(a^2+b^2)^{1/2}} = 0$

(iii) $\bar{\lambda}_\alpha - 3 + \frac{\bar{\lambda}_\alpha a^2\ell_0}{(a^2+b^2)^{3/2}} - \frac{\bar{\lambda}_\alpha \ell_0}{(a^2+b^2)^{1/2}} > 0$

(iv) $\bar{\lambda}_\alpha - \frac{\bar{\lambda}_\alpha \ell_0}{(a^2+b^2)^{1/2}} + \frac{\bar{\lambda}_\alpha b^2\ell_0}{(a^2+b^2)^{3/2}} > 0$

Putting the values of a and b for (9) in (15), it can be easily seen that all the four conditions in (15) are identically satisfied. Hence the equilibrium position [a,b] of the system is stable in the sense of Liapunov.

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