



RESEARCH ARTICLE

Green inventory model for growing items with constraints under demand uncertainty

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Abstract

An economic order quantity model for fast-growing animals is a mathematical or statistical framework used to analyze and forecast the financial aspects of maintaining and rearing animals that grow quickly while adhering to sustainable and environmentally friendly breeding practices. This model generally considers several variables and aspects involved in the production and management of these animals, such as the cost of acquisition, retention, and disposal, cost of feeding, as well as taxes on the emission of carbon dioxide and cost of shortage. Carbon dioxide production can be expressed through a functional polynomial equation, wherein the variables are impacted by both the age of the animals and the mortality function. This study proposes an economic growth quantity model for rapidly growing animals with discrete ordering, slaughter, and service level constraints where the shortage is permitted and is back ordered under uncertain demand. When an animal reaches the consumption age, it is prepared for processing and eventual slaughter to make meat products. The model aims to find the ideal age for slaughter and the most efficient quantity of newly hatched chicks procured from the supplier, aiming to minimize the overall expenses. We used spherical triangular fuzzy numbers to represent uncertain demand. Finally, we employ numerical examples to elucidate the envisaged model.

Keywords: Sustainability, Spherical Triangular Fuzzy numbers, Economic Order Quantity, Discrete Ordering, Slaughter age, growing items, Constraints.

Introduction

In recent times, the livestock farming industry has been experiencing rapid growth. When a farm animal is introduced into this system, it undergoes a feeding and maturation process, leading to an increase in its weight,

essentially categorizing it as a growing inventory. This inventory level continues to rise until the point of slaughter, which marks the culmination of the breeding period. It's worth noting that excessive growth of the animal can have a negative impact on the quality of the meat. Furthermore, the care and handling of animals before slaughter are just as crucial as the breeding process in ensuring the quality of the end product (Składanowska-Baryza and Stanisz, 2019). Subsequently, the consumption period begins, during which the inventory of slaughtered items gradually decreases due to both customer demand and natural deterioration. Economic Order Quantity (EOQ) inventory model applied to growing items, such as livestock, fish, and poultry; a notable distinction lies in the fact that these items increase in both value and size as they mature over time. Unlike traditional inventory systems, the unique aspect here is that the weight of the products increases during the stocking period without the need for additional purchases. Specifically, it focuses on the scenario of ordering newborn animals, nurturing them until they reach the desired weight for consumers, and then slaughtering them to meet demand. These birds would then be raised until they reached the desired market slaughter weight before being sold. Subsequently, (A. H. Nobil et al., 2019) expanded model by introducing an Economic Growing

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Quantity (EGQ) model that incorporated full backorders. They explicitly allowed for shortages in the inventory system, with any deficits being promptly addressed upon the arrival of replenishment with an adequate quantity. In the same year, (A. Nobil & Taleizadeh, 2019) presented a solution method for the model (A. H. Nobil et al., 2019) that eliminated shortages. An EOQ inventory model with budget-capacity constraint for growing items when a portion of the items are of imperfect quality. This method achieved discrete optimal solutions for both the ordered items and the slaughter date. Findings of (Macleod et al., 2019) The breeding of animals like chickens results in the release of greenhouse gases (GHGs), notably carbon dioxide. This occurs due to various factors, including the processing of animals, their transformation, and emissions from housing and manure management. As a result, it is of paramount importance for companies to address and mitigate GHG emissions, as most governments impose taxes on those who produce GHGs. Additionally, another significant concern for businesses is the occurrence of mortality during the breeding process, which can happen due to various reasons such as diseases, injuries, and inactivity. The primary objective of this research is to formulate an economic growth quantity model that accounts for discrete ordering and slaughter age constraints, while also incorporating service level constraints in the face of uncertain demand. In this study, we specifically address demand uncertainty by modeling it using spherical triangular fuzzy numbers (Yadav & Kaur, 2022). To evaluate the model's performance and applicability, we conduct a comprehensive numerical analysis. Through a detailed numerical example, we aim to illustrate the practical utility and benefits of our model.

Methodology

Assumptions

- At the outset of the breeding period, there exists an initial supply of newly acquired birds, denoted as χ , each with an initial weight of ' I_w '
- During the breeding period, these birds are nurtured until they attain the desired market weight W_{I_t}
- Mortality occurs during the breeding period, and it is characterized by the function $Cm(I_t)$ which quantifies the cumulative daily mortality percentage.
- When the designated slaughter date arrives, a portion of the live birds, specifically $\div(1 - Cm(I_t))$ is processed for consumption, while the remaining $\chi \cdot Cm(I_t)$ is disposed of.
- Shortages are permitted, and backorders are allowed.

- The growth is considered to be time-varying and is formulated as $W_{I_t} = w_A \left(1 + C_b e^{-G_r I_t}\right)^{\frac{-1}{n}}$, where w_A is the maximum possible weight of the item, C_b is the Integration constant, G_r is a constant rate that determines the spread of the growth Curve.
- A substantial number of birds do not survive the breeding process, so we employ a Polynomial function $Cm(I_t) = cm_0 + cm_1 I_t + cm_2 I_t^2 + cm_3 I_t^3$ to estimate the Cumulative daily mortality percentage, which establishes a connection between the Daily mortality rate and the age of the birds.
- Birds undergo growth and nurturing through a feeding process that is characterized by a polynomial function. The parameters of this function are established by analyzing data gathered during this period, and the estimation process is outlined as follows:

$$PF = pf_0 + pf_1 I_t + pf_2 I_t^2 + pf_3 I_t^3$$

- Furthermore, for the carbon dioxide production function, we opt for a polynomial function commonly observed in real-world applications, and this function is contingent upon the age of the birds. It is determined and estimated as shown

$$Cd(I_t) = cd_0 + cd_1 I_t + cd_2 I_t^2 + cd_3 I_t^3$$

Notations

Parameters Description

- | | |
|-----------|---|
| W_{I_t} | Weight of a single unit item at time ' I_t ' (measured in weight units). |
| s | Shape parameter of the growth function. |
| M_u | Maximum allowable breeding period duration (in days). |
| M_l | Minimum allowable breeding period duration (in days). |
| D_r | Constant demand rate per weight unit (measured in weight units per year). |
| P_c | Cost of purchase per weight unit (measured in monetary units per year). |
| S_c | Setup cost per growing cycle (measured in monetary units per setup). |
| G_z | Cost of production (feeding) per unit item during the growing cycle (measured in monetary units per unit item). |
| C_h | Cost of holding per weight unit (measured in |

- monetary units per weight unit per year).
- t_a Tax on carbon dioxide production (measured in monetary units per liter per day per weight unit).
- T The duration of one complete cycle.
- $Cm(I_t)$ Polynomial function representing the percentage of dead items during the growing cycle.
- $Pf(I_t)$ Polynomial function representing production (feeding) consumption (measured in unit items).
- $Cd(I_t)$ Polynomial function representing carbon dioxide production (measured in liters per day per weight unit).
- G_r Growth rate.
- d_c Cost of carcass disposal (measured in monetary units per unit carcass).
- w_A Asymptotic weight.
- C_b Integration constant of the growth function.
- θ_i The ratio of defective products that were manufactured and subsequently discarded.
- Q_w Total weight of inventory (measured in weight units).
- BC Cost of back ordering.
- I_{t1} Breeding period duration (measured in days).
- I_{t2} Consumption period duration (measured in years).
- AI_c Annual installation cost (measured in monetary units).
- AP_c Annual acquisition cost (measured in monetary units).
- AC_h Annual holding cost (measured in monetary units).
- Ad_c Annual clearance cost (measured in monetary units).
- APF_c Annual production (feeding) cost (measured in monetary units).
- ACd_c Annual carbon dioxide production tax (measured in monetary units).
- TC Total cost in a year (measured in monetary units).
- I_t Slaughter age (measured in days).
- $\div$ Total number of growing items ordered at the beginning of a cycle (measured in unit items).
- ϵ_s The buffer for the total acceptable deficit.
- π_b The cost incurred per unit of time for backorder.

σ_i The fraction of imperfect quality products produced.

Model Formulation

These are the methods used to obtain each component of the overall cost:

Installation Cost

Every production cycle begins with certain responsibilities and procedures, such as cleaning and maintenance, required to start the breeding process. To determine the annual installation cost, it can be divided by I_t , where I_t is the utilization interval.

$$AI_c = \frac{D_r S_c}{\chi W_{I_t} (1 - Cm(I_t))}$$

Warehousing Cost

The cost of holding per cycle is calculated as

$$\frac{C_h I_t \chi W_{I_t} (1 - Cm(I_t))}{2I_t}, \text{ where } C_h \text{ is the annual inventory}$$

cost per unit item. Therefore, the annual holding cost is

$$\text{determined as follows } AC_h = \frac{C_h I_t W_{I_t} (1 - Cm(I_t))}{2}.$$

Acquisition Cost

The annual acquisition cost for purchasing new-born birds

$$\text{from the supplier is given as } AP_c = \frac{D_r P I_w}{W_{I_t} (1 - Cm(I_t))}.$$

Clearance Cost

During the breeding process, various challenges and reasons can arise for disposing of birds, including some birds losing their ability to walk, which leads to death. Therefore, the annual clearance cost can be expressed as follows:

$$Ad_c = \frac{D_r d_c Cm(I_t)}{W_{I_t} (1 - Cm(I_t))}.$$

Backordering cost

It involves effectively managing the supply and demand of birds. This approach helps balance customer needs with bird growth rates and production cycles while maintaining a commitment to deliver birds as they become available.

The shortage cost is calculated as $S^* = \frac{C_h D_r T}{C_h + f}$, where T is

the duration of one complete cycle, and it is formulated as

$$T = \frac{(1 - \theta_i) Q_w}{D_r}$$

Backordering cost with shortage can be expressed as

$$CB = \frac{1}{2T} \left(\frac{\pi_b P_c (1 - \sigma_i)}{D_r} \right) S^2$$

Production Cost

To calculate the annual feeding (Production) cost for a bird, you need to consider the feeding function per weight unit, which is dependent on the age of the chicken. Thus the annual production cost is determined as

$$Apf_c = \frac{D_r G_z \gamma(I_t)}{W_t (1 - Cm(I_t))} \text{ where}$$

$$\gamma(I_t) = Pf_0 I_t + \frac{Pf_1 I_t^2}{2} + \frac{Pf_2 I_t^3}{3} + \frac{Pf_3 I_t^4}{4} - cm_0 Pf_0 I_t - \frac{cm_0 Pf_1 I_t^2}{2} -$$

$$\frac{cm_0 Pf_2 I_t^3}{3} - \frac{cm_0 Pf_3 I_t^4}{4} - \frac{cm_1 Pf_0 I_t^2}{2} - \frac{cm_1 Pf_1 I_t^3}{3}$$

$$- \frac{cm_1 Pf_2 I_t^4}{4} - \frac{cm_1 Pf_3 I_t^5}{5} - \frac{cm_2 Pf_0 I_t^3}{3} - \frac{cm_2 Pf_1 I_t^4}{4}$$

$$- \frac{cm_2 Pf_2 I_t^5}{5} - \frac{cm_2 Pf_3 I_t^6}{6} - \frac{cm_3 Pf_0 I_t^4}{4}$$

$$- \frac{cm_3 Pf_1 I_t^5}{5} - \frac{cm_3 Pf_2 I_t^6}{6} - \frac{cm_3 Pf_3 I_t^7}{7}$$

Carbon Emissions Tax

Carbon dioxide emissions in animals are directly linked to their metabolic heat production, and this relationship is also influenced by their metabolic body weight, which, in turn, can be affected by the activity level and environmental temperature of the birds. The breeding process naturally results in the production of carbon dioxide. The annual tax

$$\text{of carbon dioxide is calculated as } Acd_c = \frac{D_r t_a \beta(I_t)}{W_t (1 - Cm(I_t))}$$

$$\text{where } \beta(I_t) = cd_0 I_t + \frac{Pf_0 cd_1 I_t^2}{2} + \frac{cd_2 I_t^3}{3} + \frac{cd_3 I_t^4}{4}$$

$$- cm_0 cd_0 I_t - \frac{cm_0 cd_1 I_t^2}{2} - \frac{cm_0 cd_2 I_t^3}{3} - \frac{cm_0 cd_3 I_t^4}{4}$$

$$- \frac{cm_1 cd_0 I_t^2}{2} - \frac{cm_1 cd_1 I_t^3}{3} - \frac{cm_1 cd_2 I_t^4}{4} - \frac{cm_1 cd_3 I_t^5}{5}$$

$$- \frac{cm_2 cd_0 I_t^3}{3} - \frac{cm_2 cd_1 I_t^4}{4} - \frac{cm_2 cd_2 I_t^5}{5} - \frac{cm_2 cd_3 I_t^6}{6}$$

$$- \frac{cm_3 cd_0 I_t^4}{4} - \frac{cm_3 cd_1 I_t^5}{5} - \frac{cm_3 cd_2 I_t^6}{6} - \frac{cm_3 cd_3 I_t^7}{7}$$

Annual Total Cost is expressed as,

$$ATC = \frac{D_r S_c}{W_t (1 - Cm(I_t))} + \frac{C_h I_t W_t (1 - Cm(I_t))}{2}$$

$$+ \frac{D_r P I}{W_t (1 - Cm(I_t))} + \frac{D_r d Cm(I_t)}{W_t (1 - Cm(I_t))} + \frac{D_r G_z \gamma(I_t)}{W_t (1 - Cm(I_t))}$$

$$+ \frac{1}{2T} \left(\frac{\pi_b P_c (1 - \sigma_i)}{D_r} \right) S^2 + \frac{D_r t_a \beta(I_t)}{W_t (1 - Cm(I_t))}$$

Definition

A Spherical Triangular Fuzzy Number (STFN), denoted as

$$\tilde{X}_s = (t, i, f) = (t_1, t_2, t_3; i_1, i_2, i_3; f_1, f_2, f_3) \text{ is a mathematical}$$

representation characterized by three sets of values (t, i, f) , each containing three components, such that

these values fall within the closed interval $[0, 1]$. The

membership function for t, i, f can be defined as,

$$t_{\tilde{X}_s}(x) = \begin{cases} \frac{x - t_1}{t_2 - t_1} & \text{if } t_1 \leq x \leq t_2 \\ \frac{t_3 - x}{t_3 - t_2} & \text{if } t_2 \leq x \leq t_3 \\ 0 & \text{otherwise} \end{cases}$$

$$i_{\tilde{X}_s}(x) = \begin{cases} \frac{x-i_1}{i_2-i_1} & \text{if } i_1 \leq x \leq i_2 \\ \frac{i_2-x}{i_3-i_2} & \text{if } i_2 \leq x \leq i_3 \\ 0 & \text{otherwise} \end{cases}$$

$$f_{\tilde{X}_s}(x) = \begin{cases} \frac{x-f_1}{f_2-f_1} & \text{if } f_1 \leq x \leq f_2 \\ \frac{f_2-x}{f_3-f_2} & \text{if } f_2 \leq x \leq f_3 \\ 0 & \text{otherwise} \end{cases}$$

- **Accuracy Function**

The accuracy function of $\tilde{X}_s$ is represented and defined as follows:

$$A(\tilde{X}_s) = \frac{(3t_1+t_2+3t_3) + (3i_1+i_2+3i_3) + (3f_1+f_2+3f_3)}{18}$$

- **Constraints**

Constraints serve as integral elements in both problem-solving and project management, providing the framework within which actions and decisions are confined. The two constraints of the model detailed in the subsequent sections encompass the discrete ordering, slaughter and service level constraints.

- **Discrete Ordering and Slaughter Constraint**

When a company intends to purchase new-born birds, it's essential to consider certain constraints. These constraints involve ensuring that the number of birds ordered is a whole number, as live birds are not sold in fractions. Additionally, the time frame for the birds growth, from the beginning of the breeding period (M_1) to its maximum allowable duration (M_u), must also be an integer, reflecting the real-world constraints related to market demand, the breeding process, and the natural growth of these animals. Hence, this constraint can be expressed as follows: $M_1 \leq t \leq M_u$.

- **Service Level Constraint**

The service level constraint is determined by considering the total shortage quantity (S) and the safety factor (ϵ_s).

It is expressed as follows: $\frac{N \times S}{D_r} \leq \epsilon_s$.

In this context, N stands for the number of items needed to achieve the desired performance standard. The ideal quantity of new-born items to be ordered is calculated based on the given constraints,

$$\div = \left[-0.5 + \sqrt{0.25 + \frac{2D S}{w_A \left(1 + C_b e^{-G_r I_t}\right)^{-2} \left(1 - \text{Cm}(I_t)\right)^2}} \right]$$

Results and Discussion

Numerical Discussion

We provide a numerical illustration for a particular inventory system involving rapidly maturing birds, specifically male broilers, to demonstrate the suggested model. In industrial agriculture settings, broiler chickens have very brief lifespans. To create a practical model, we establish the minimum (M_l) and maximum (M_u) acceptable durations for the breeding period as 21 and 55, respectively.

$$s=0.0087, C_b=0.043, w_A=6870.2, G_r=0.036, pf_0=532.2, pf_1=67.15,$$

$$pf_2=-0.651, pf_3=0.0018, cm_0=0.0126, cm_1=0.00174, cm_2=-0.0000556, cm_3$$

$$=0.000000753, cd_0=8.16, cd_1=-0.9768, cd_2=0.13416, cd_3$$

$$=-0.0016392, I_w=45, C_h=0.002,$$

$$D_r = (56299850, 100000000, 110000000;$$

$$51366670, 100000000, 115000000;$$

$$47333350, 100000000, 120000000),$$

$$G_z=0.0001, S_c=5000, P_c=0.01, t_a=0.001, d_c=1, \sigma_i$$

$$=0.135, \epsilon_s=0.09, \pi_b=5$$

We consider the demand as a Spherical Triangular Fuzzy Number. Using the solution approach presented, we calculate the optimal slaughter age $I_t^* = 44$, $\chi = 417.06$

and $ATC^* = 859906.78\$$.

Conclusion

The emission of carbon dioxide gas into the air due to a range of activities and procedures connected with raising animals,

cultivating crops, or managing the breeding process. These emissions may come from various sources, including heating systems, transportation, energy consumption, and other practices associated with the breeding phase. It is assumed that carbon dioxide emissions follow a polynomial function that correlates them with the age of the birds. This practical model makes it easy to estimate a company's carbon dioxide production, enabling managers to compute the associated expenses, as governmental authorities typically levy taxes on each ton of carbon dioxide generated. Furthermore, to enhance the feasibility of the Economic Order Quantity (EOQ) problem for items that grow, this study devises a procedure for determining a discrete number of newborn chicks. Hence, the presented mathematical model is structured as an integer nonlinear programming challenge for an expanding inventory system that factors in mortality and carbon dioxide emissions. The primary goal of this model is to minimize the overall inventory cost, thereby identifying the most efficient age for bird slaughter and the ideal quantity of new-born chicks to order. To verify these findings, a numerical approach is employed. The model for sustainable economic growth quantity pertains to rapidly growing animals, particularly in situations of unpredictable demand, where the demand rate is represented as a triangular fuzzy number with a spherical distribution and is subject to discrete ordering, slaughter as well as service level constraints. Furthermore, this model can be extended by multi-item inventory, shipment of slaughter animals, trade credit, and warehousing capacity.

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Conflict of interest:

The authors declare no conflict of interest.

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