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RESEARCH ARTICLE

A green inventory model for deteriorating items while producing overtime with nonlinear cost and stock-dependent demand

Vinodini R and Ritha W*

Abstract
Carbon emission alludes to the release of greenhouse gases into the atmosphere and the land or water as a result of human activities. These emissions are caused by climate change, biodiversity loss, global warming, and the degradation of ecosystems, which affects our human health and environment. To overcome this problem, this study presents a green inventory model for deteriorating items while producing overtime with nonlinear cost and stock-dependent demand. In this paper, the Production process allows for overtime operations with nonlinear expenses, and the inventory level influences the demand. The mathematical formulation is designed to optimize inventory management and to develop an eco-friendly model for deteriorating items. The model was extended with two new costs: opportunity cost and Emission cost. To evaluate this green inventory model a numerical example was examined. The results are discussed with different values of the variables. As the final outcome of this model, the gross cost was determined for environmental benefits.

Keywords: Green Inventory Model, nonlinear cost, Stock-dependent demand, Deterioration items, Opportunity cost, and Emission cost.

Introduction
A Green Inventory Model alludes to the system that integrates environmental friendly methods into inventory management and its optimization. The main concept is to strike a balance between inventory management and developing eco-friendly methods to dispose of the wastages, including waste reuse, reducing carbon emissions, and using renewable energy. This idea can be used in advanced sustainability, like the variety of supply chains and businesses, without sacrificing operational effectiveness. The authentic green inventory model was developed under the fundamental assumption that the rate of production is constant. In reality, it is noticed that the rate of production is not always constant because of the heavy competition in the trading and advertising situation, then the demand for the product as well. Now the innovative manufacturing method is more flexible for fulfilling the market demand.

This green inventory model includes two costs that are environmentally friendly and helps us to choose the best option among a couple of options. The opportunity cost is the difference between the remaining option and the chosen option. The remaining option may considered as the best option, so we have to find the difference between the two options. Finding a value of opportunity cost that evaluates the best time to produce the inventory. Emission cost is used to reduce carbon emissions. This cost was calculated for the goodness of the environment. The opportunity cost and the emission cost are derived and included in this model. The numerical example is examined to evaluate the developed model with the two new costs.

The rest of the paper is formulated as follows: Section 2 presents the literature review. Section 3 explains the mathematical formulation of the proposed model. Section 4 illustrates a numerical example. Section 5 concludes the

PG & Research Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirapalli-620002, India.

***Corresponding Author:** Ritha W, PG & Research Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirapalli-620002, India., E-Mail: ritha_prakash@yahoo.co.in

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paper, and Section 6 includes the references.

Literature Review

The initial study was carried out by Goyal (1976) on joint lot-sizing in a supply chain system. Goyal's model was extended by Banerjee (1986), removing the assumption of infinite production rate. A Goswami et al. (1991) formulated an EOQ model for deteriorating items with shortages and a linear trend in demand. BC Giri et al. (1996) developed an economic order quantity model for deteriorating items with time-varying demand and costs. Hariga M (1996) created Optimal EOQ models for deteriorating items with time-varying demand. YP Lee et al. (2012) constructed an inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate. S Tiwari et al. (2018) developed Sustainable inventory management with deteriorating and imperfect quality items considering carbon emissions. AHM Mashud et al. (2021) explained sustainable inventory management in supply chains. BS Mohanty (2024) described green inventory and carbon emissions. Green technology is a recent trend in all industries. Many people have extended their model from inventory to green inventory. J Marklund et al. (2017) explained about the Green inventory management. P. Becerra et al. (2021) constructed green supply chain quantitative models for sustainable inventory management. Mohammad Abdul Halim et al. (2021) developed an overtime production inventory model for deteriorating items with nonlinear price and stock-dependent demand. This research work also intends to create an inventory model which consists of the opportunity cost and the emission cost included with the gross cost.

Mathematical Formulation

To construct the proposed model, the following notations and assumptions are used. The proper explanation of these notations are given below:

Notations

$S(t)$	The stock amount at any moment t
P_0	Production rate per unit time
σ	Deterioration rate $(0 < \sigma < 1)$
C_s	The setup cost of the production system
C_p	Per unit time cost of normal production
C_φ	Per unit time cost of overtime production
C_d	Per unit time cost of deterioration
C_α	Per unit time opportunity cost
C_E	Per unit time emission cost
C_h	Per unit time holding cost
t_1	Time at which the stock amount arrives at maximum
T	The time at which the stock amount vanishes
π	The average cost of the system
p	Selling price per unit of an item
$D\{p,S(t)\}$	Nonlinear price and stock-dependent demand

Assumptions

- The production rate is constant and persistent.
- The demand rate $D\{p,S(t)\}$ is dependent on the product's trading cost and the inventory's storage amount. It is denoted by. $D\{p,S(t)\} = ap^{-b} + cS(t), a,b,c > 0$
- Reworking or refunding for decay products is not allowed.
- This approach handles a single product along with one stocking moment and the project operates for an infinite time horizon.
- Replacement is continuous when products are needed. i.e., the lead time is negligible.
- Shortages are impermissible.
- Overtime production is considered.

Mathematical Model

Green inventory model for production is constructed with the assumption of persistent production rate and depends on storage amount also demand for a decay item.
Defining the demand rate,

$$D\{p,S(t)\} = ap^{-b} + cS(t), a,b,c > 0$$

and the production rate is,

$P(t) = P_0 + \{-\gamma.S(t) + \delta.D\{p,S(t)\}$, where, $P_0, \delta > 0$ and $0 \le \gamma \le 1$

Initially, the production rate starts at $t = 0$ and produces to the maximum level $t = t_1$, which is. When the time reaches the maximum level, production gradually decreases and is reduced to zero. The time is denoted by $t = T$. The Graphic explanation of this Production System is designed in Figure 1.

The required green inventory model are formulated as differential equations

$$\frac{dS(t)}{dt} + \sigma.S(t) = P(t) - D\{p,S(t)\}, \quad 0 < t \le t_1 \tag{1}$$

$$\frac{dS(t)}{dt} + \sigma.S(t) = -D\{p,S(t)\}, \quad t_1 < t \le T \tag{2}$$

With the supportive constraints
The storage amount of the inventory $S(t) = 0$ at $t = 0$ and $t = T$. (3)
The production is perpetual at $t = t_1$
According to the condition (3), the solutions of the equations

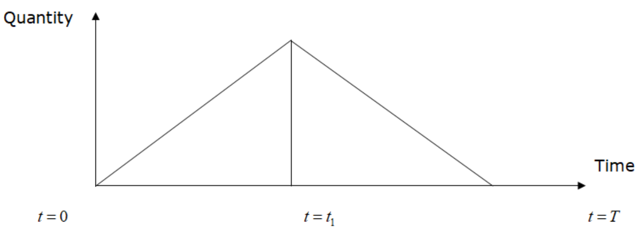

Figure 1: Graphic representation of the production system

(1) and (2) are found.

$$S(t)=\frac{\beta(1-e^{\alpha t})}{\alpha},0<t\le t_1 \quad (4)$$

where $\alpha=\sigma+\gamma-c(\delta-1)$ and $\beta \aleph D_0 \quad {}^{-b} \text{Đ}S$,

and $S(t)=\dfrac{ap^{-b}\{e^{(\sigma+c)(T-t)}-1\}}{(\sigma+c)},t_1<t\le T$ (5)

Making use of the continuity condition at $t=t_1$, equating the equations (4) and (5)

$$\frac{\beta(1-e^{\alpha t})}{\alpha}=\frac{ap^{-b}\{e^{(\sigma+c)(T-t_1)}-1\}}{(\sigma+c)} \quad (6)$$

The Holding cost is calculated in the two time periods. The time periods are $[0,t_1]$ and $[t_1,T]$. Integrating the terms,

$$C_{hol}=C_h\left\{\int_0^{t_1}S(t)dt+\int_{t_1}^{T}S(t)dt\right\}$$

Substitute the value of $S(t)$ from (4) and (5) according to the time period and solving,

$$C_{hol}=C_h\left[\frac{\beta}{\alpha}\left\{t_1+\frac{\left(e^{-\alpha t_1}-1\right)}{\alpha}\right\}-\frac{ap^{-b}}{(\sigma+c)}\left\{\frac{1-e^{(\sigma+c)(T-t_1)}}{(\sigma+c)}+(T-t_1)\right\}\right] \quad (7)$$

The Deterioration cost is calculated in the two time periods. The periods are $[0,t_1]$ and $[t_1,T]$ Integrating the terms,

$$DC=C_d\left[\int_0^{t_1}\sigma.S(t)dt+\int_{t_1}^{T}\sigma.S(t)dt\right]$$

Substitute the value of $S(t)$ from (4) and (5) according to the period and solving,

$$DC=C_d\sigma\left[\begin{array}{l}\frac{\beta}{\aleph}\left\{t_1+\frac{\left(e^{-\alpha t_1}-1\right)}{}\right\}-\frac{ap^{-b}}{(\quad+c)}\\ \left\{\frac{\left(1-e^{(\sigma+c)(T-t_1)}\right)}{(\sigma+c)}+(T-t_1)\right\}\end{array}\right]. \quad (8)$$

The general formula for calculating Opportunity cost is,
Opportunity cost = Value of the Next Best Alternative option-value of chosen option

- *Value of the Next Best Alternative*

It defines value of the option among the two which is not chosen.

- *Value of the chosen option*

It defines the value of the remaining option which is chosen. The Opportunity cost of the two time periods are calculated as follows:

$$C_{oc}=\left[(C_p+C_{op})\left(\int_0^{t_1}S(t)dt\right)-(C_d\sigma)\left(\int_{t_1}^{T}S(t)dt\right)\right] \quad (9)$$

Substituting the value of $S(t)$ for the corresponding period in (9),

$$C_{oc}=\left[\begin{array}{l}(C_p \aleph C_{op})\frac{\beta}{\alpha}\left\{t_1 \quad \frac{\left(e^{-\alpha t_1}-1\right)}{\alpha}\right\}\\ C_d\sigma\frac{ap^{-b}}{(\sigma+c)}\left\{\frac{1-e^{(\sigma+c)(T-t_1)}}{\left(\sigma+c\right)}+\left(T-t_1\right)\right\}\end{array}\right] \quad (10)$$

The general formula for emission cost is,

$$C_E=\sum_{i=1}^{n}(Q\times EF_i)$$

Q - Quantity of the material

E_i - Emission Factor of the material

The Emission cost of this model is calculated in time periods $[0,t_1]$ and $[t_1,T]$.

$$C_E=EF_i\left[\int_0^{t_1}S(t)dt+\int_{t_1}^{T}S(t)dt\right] \quad (11)$$

Substituting the value of $S(t)$ for the corresponding time period in (11),

$$C_E = EF_i \begin{bmatrix} \dfrac{\beta}{\alpha}\left(t_1 + \dfrac{\left(e^{-\alpha t_1} - 1\right)}{\alpha} \right) - \\ \dfrac{ap^{-b}}{(\sigma + c)} \left(\dfrac{\left(1 - e^{(\sigma+c)(T - t_1)}\right)}{(\sigma + c)} + (T - t_1) \right) \end{bmatrix} \tag{12}$$

The total Production cost for all the products that are manufactured in the given time period. The production rate is increasing in the time period $[0, t_1]$ and decreasing in the time period $[t_1, T]$. It leads to be zero. So now we only considered the time period from 0 to t_1 .The Total Production cost is,

$$PC = C_p P_0 t_1 + C_{op} \int_0^{t_1} \{-\gamma . S(t) + \delta . D(p, S(t)\}.dt$$

$$PC = C_p P_0 t_1 + C_{op} \left\{ K_1 t_1 - K_2 (e^{-\alpha t_1} - 1) \right\} \tag{13}$$

$K_1 = ap^{-b}\delta + \dfrac{\beta}{\alpha}(c\delta - \gamma)$ Where and $K_2 = \dfrac{\beta}{\alpha^2}(\gamma - c\delta)$.

Combining all the costs will get the gross cost $\pi(t_1, T)$ or π^* that is,

$$\pi(t_1, T) = \frac{1}{T} \begin{bmatrix} C_s + C_p P_o t_1 + C_{op} \left\{ K_1 t_1 - K_2 \text{Đ} e^{-\alpha t_1} - \quad \right\} \\ + (C_h + C_d \sigma) \begin{bmatrix} \dfrac{\beta}{\alpha} \left\{ t_1 + \dfrac{\left(e^{-\alpha t_1} - 1\right)}{\alpha} \right\} - \\ \dfrac{ap^{-b}}{(\sigma + c)} \left\{ \dfrac{1 - e^{(\sigma+c)(T - t_1)}}{(\sigma + c)} + (T - t_1) \right\} \end{bmatrix} + \\ \begin{bmatrix} (C_p + C_{op}) \dfrac{\beta}{\alpha} \left\{ t_1 + \dfrac{\left(e^{-\alpha t_1} - 1\right)}{\alpha} \right\} + \\ C_d \sigma \dfrac{ap^{-b}}{(\sigma + c)} \left\{ \dfrac{1 - e^{(\sigma+c)(T - t_1)}}{(\sigma + c)} + (T - t_1) \right\} \end{bmatrix} + \\ EF_i \begin{bmatrix} \dfrac{\beta}{\alpha} \left(t_1 + \dfrac{\left(e^{-\alpha t_1} - 1\right)}{\alpha} \right) - \\ \dfrac{ap^{-b}}{(\sigma + c)} \left(\dfrac{\left(1 - e^{(\sigma + \text{Đ})(\; - \; _1)}\right)}{(\sigma + c)} + (T - t_1) \right) \end{bmatrix} \end{bmatrix} \tag{14}$$

where $\alpha = \sigma + \gamma - c(\delta - 1)$, $\beta \aleph \text{Đ}_0 \quad {}^{-b}\text{Đ}\delta$,

$K_1 = ap^{-b}\delta + \dfrac{\beta}{\alpha}(c\delta - \gamma)$ and $K_2 = \dfrac{\beta}{\alpha^2}(\gamma - c\delta)$.

Now the goal is to achieve optimal values of t_1 and T for optimizing the gross cost $\pi(t_1, T)$. Assume the values of t_1 and T for the unit time, and then you will find the value of the gross cost.

Numerical Example

Consider sthe following parameters to illustrate the proposed model:

P_0	110 units	p	15
C_s	50/cycle	σ	0.1
C_p	10/unit	δ	0.1
C_φ	2.7/unit	γ	0.03
C_d	17/unit/unit time	a	100
C_E	0.6/unit	b	0.5
C_h	0.25/unit/unit time		

Substitute all the above values in (14) in order to unit time, then the value of the gross cost found as minimum and maximum. Substituting the values and $\gamma = 0$, $\delta = 0$ with the unit time, then solution of the minimum gross cost is, $t_1 = 0.3123$ and $T = 1.2431$ then the gross cost is $\pi^* = 685.23$. Substituting the above values as it is with the unit time, then the solution of the maximum gross cost is, $t_1 = 1.0047$ and $T = 1.6727$. Then the Gross cost is $\pi^* = 1418.45$.

Discussion

Mohammad Abdul Halim et al. (2021) developed an overtime production inventory model for deteriorating items with nonlinear price and stock-dependent demand, and J Marklund et al. (2017) explained Green inventory management. S Tiwari et al. (2018) developed Sustainable inventory management with deteriorating and imperfect quality items considering carbon emissions. AHM Mashud et al. (2021) explained sustainable inventory management in supply chains. P. Becerra et al. (2021) constructed green supply chain quantitative models for sustainable inventory management. BS Mohanty (2024) described about green inventory and carbon emissions. This research work is based on extending the inventory model into the green inventory model, and it helps to reduce carbon emissions. The mathematical model introduces two new costs that are used for manufacturing a product without affecting the environment. The new costs, which are opportunity costs and emission costs. Derive the two new costs and contemplate it in relation to the total gross cost. The new costs, which are opportunity cost and emission cost, and the values of the variables have changed. The values are calculated as both minimum and maximum levels. From the calculated values and the total gross cost, we the developed model was environmentally friendly and also reduced environmental pollution.

Conclusion

This paper focus on the deterioration items, nonlinear cost, stock-dependent demand, and two new costs that are the Opportunity cost and the Emission cost. These costs are included for reducing the emissions and aid in making the right decision at the right time. The model was created in a simple way to understand the assumptions and the formulations. Through the numerical example the value of the total gross cost was found in both minimum and maximum. This paper concludes the results that the new costs are helping us to manufacture and sustain a product that is environmentally friendly.

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Conflict of interest

None

References

A Goswami., KS Chaudhuri., (1991) An EOQ model for deteriorating items with shortages and a linear trend in demand, *Journal of Operational Research Society*, 42(12), 1105-1110. https://doi.org/10.1057/jors.1991.204

AHM Mashud., M Pervin., and et al., (2021) A Sustainable inventory model with controllable carbon emissions in green-warehouse farms, *Journal of Cleaner Production*, 298, 126777. https://doi.org/10.1016/j.jclepro.2021.126777

BC Giri., A Goswami., KS Chaudhuri., (1996) An EOQ model for deteriorating items with time-varying demand and costs, *Journal of the Operational Research Society*, 47(11), 1398-1405. https://doi.org/10.1057/jors.1996.175

BS Mohanty., (2024) Green Inventory and Carbon Emissions: A Review, *Advanced Green Technology for Environmental Sustainability and Circular Economy*, 208-222.

Goyal, S.K., A Gunasekaran, (1995) An integrated production-inventory-marketing model for deteriorating items, *Computers and Industrial Engineering,* 28, 755-762. https://doi.org/10.1016/0360-8352(95)00016-T

Hariga, M., (1996) Optimal EOQ models for deteriorating items with time-varying demand, *Journal of the Operational Research Society*, 47(10), 1228-1246. https://doi.org/10.1057/jors.1996.151

J Marklund., P Berling., (2017) Green inventory management, *Sustainable supply chains: a research-based textbook on operations and strategy*, 189-218.

Mohammad Abdul Halim., Paul, A., and et al., (2021) An overtime production inventory model for deteriorating items with nonlinear price and stock dependent demand, *Alexandra Engineering journal*, 60, 2779-2786. https://doi.org/10.1016/j.aej.2021.01.019

P Becerra., J Mula., R Sanchis., (2021) Green supply chain quantitative models for sustainable inventory management: A review, *Journal of cleaner production*, 328, 129544. https://doi.org/10.1016/j.jclepro.2021.129544

P Becerra., J Mula., R Sanchis., (2022) Sustainable inventory management in supply chains: Trends and further research, *Sustainability*, 14(5), 2613. **https://doi.org/10.3390/su14052613**

S Tiwari., Y Daryanto., (2018) HM Wee., Sustainable inventory management with deteriorating and imperfect quality items considering carbon emission, *Journal of Cleaner Production*, 192, 281-292. https://doi.org/10.1016/j.jclepro.2018.04.261

YP Lee., CY Dye., (2012) An inventory model for deteriorating items under stock dependent demand and controllable deterioration rate, *Computers and Industrial Engineering*, 63(2), 474-482. https://doi.org/10.1016/j.cie.2012.04.006