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RESEARCH ARTICLE

Impregnable inventory stewardship for a closed loop supply chain manoeuvring pentagonal fuzzy number

R. Saarumathi[1] and W. Ritha[2]

Abstract

To enhance the inventory level and investments under a carbon tax policy, a mathematical inventory model for a manufacturer-multi-retailer system is recommended. Manoeuvring production rate adjustment, the energy engrossed by the production and rework, and emissions due to supply chain operations are scrutinized and administered. To toe the line with the carbon policy and to reduce emissions, the manufacturer subsidizes green production technologies. This model focuses on curtailing the joint total cost by ascertaining the shipment frequencies, shipment lot, safety factor, production rate, collection rate, and investment concurrently. Reduction in total cost can be achieved through emission control, energy consumption, and the number of defects by adjusting the production rate. The green investment reduces emissions and increases returns on used products, resulting in an effective environmental performance of the supply chain.

Keywords: Closed loop supply chain, Consignment lashing, Energy usage, Green investment, Pentagonal fuzzy Number, Safety cushion, Werner fuzzy and operator approach.

Introduction

For a few decades, an increase in greenhouse gas emissions and the crisis of energy sources have had a major impact on climate change, leading to a great global issue. To save us and the Earth from existing environmental disasters, remanufacturing plays an important role in saving energy and raw materials, reducing CO_2 emissions, and reducing the amount of solid waste disposed of. To conserve natural resources, remanufacturing demands less energy than manufacturing products from raw materials.

Joint efforts and coordination between parties are obligatory when considering a closed-loop supply chain to reduce the rate of emissions produced. In this competitive global market, in order to enhance profits and compete with order industries, a closed-loop supply chain has to be more effective in establishing environmental sustainability, considering carbon emissions, energy use, and recovery of used production.

Formal research on closed-loop supply chains focused on some types of carbon policies to check emissions and did not consider the benefits of managing a green investment. To limit emissions, apart from using carbon taxes, green investment is also integrated into their model to curtail emissions from storage, rework, and production.

In this model, the extent of energy expended by production and rework can be sustained employing production rate adjustment. This model also permits the production rate adjustment to regulate emissions and defective items. The embodiment of multi-retailers on a closed-loop supply chain model will increase the convolution of the issue exclusively for reconciling the manufacturer's production cycle with the retailer's ordering cycle.

Literature Review

A reappraisal of some research rivulets in relation to the closed-loop supply chain inventory model is given. This

[1]Research Scholar, PG & Research Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirappalli – 620002, India.

[2]Associate Professor, PG & Research Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirappalli – 620002, India.

***Corresponding Author:** W. Ritha, Associate Professor, PG & Research Department of Mathematics, Holy Cross College (Autonomous), Affiliated to Bharathidasan University, Tiruchirappalli – 620002, India, E-Mail: rithaprakash71@gmail.com

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research focuses on (i) Inventory management for reverse logistic model and closed loop supply chain and (ii) Supply chain inventory model with carbon emissions and energy usage. At first, (Schrady,1967) dealt with reverse logistics and put forward a mathematical inventory model for a single echelon system along with a product recovery process. (Nahmias and Rivera's, 1979) work, an extension of Schrady's work, introduced a model with a finite recovery rate and examined the limitations of repair and warehouse capacities.

The first inventory model was put forward by (Chung et al., 2008), which established the exploration of product return in a closed-loop supply chain system. (Yuvan et al., 2015) put forth Chung's model by presenting a new mechanism to control the production and remanufacturing, that is, (P, R) policy. Later (Taletzadeh and Moshtagh., 2019) suggested a multi-level closed-loop supply chain system comprising a supplier, a retailer, a producer, and a collector with a dispatch stock policy and imperfect production and remanufacturing processes. Ben-Daya et al., 2019 directed a two-echelon closed-loop supply chain composed of a vendor and multi-buyers under a shipment stock policy. Ullah et al., 2021 advanced a model for manufacturer-multi-buyers collector systems to ascertain the optimal remanufacturing decisions and packaging capacity.

Materials and Methods

Pentagonal Fuzzy Number

A fuzzy number $\tilde{P}$ is a pentagonal fuzzy number denoted by $\tilde{P} = (p_1, p_2, p_3, p_4, p_5)$ where p_1, p_2, p_3, p_4, p_5 are real numbers, and its membership function is defined as,

$$\mu_{\tilde{P}}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-p_1}{p_2-p_1}\right) & for\ p_1 \le x \le p_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-p_2}{p_3-p_2}\right) & for\ p_2 \le x \le p_3 \\ 1-\frac{1}{2}\left(\frac{x-p_3}{p_4-p_3}\right) & for\ p_3 \le x \le p_4 \\ \frac{1}{2}\left(\frac{p_5-x}{p_5-p_4}\right) & for\ p_4 \le x \le p_5 \\ 0 & for\ x \prec p_1 and\ x \succ p_5 \end{cases}$$

Ranking Pentagonal Fuzzy Numbers Using Incentre of Centroids

Take into account a pentagonal fuzzy number with five distinct points $A, M, O, N \& E$. The points $M \& N$ meet at points $B \& D$ and join at the points $BO \& DO$. Currently, the pentagon has been partitioned into two

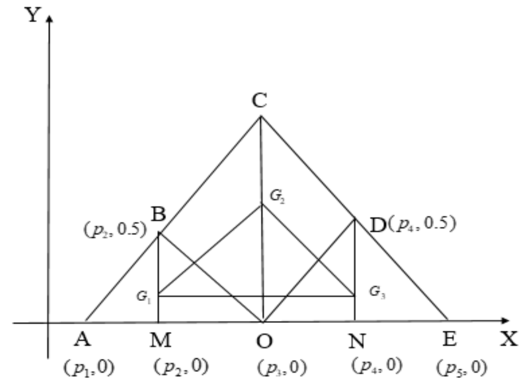

Figure 1: The scoring of pentagonal fuzzy number using incentre of centroids

triangles and one quadrilateral, $ABO, ODE \& OBCD$ respectively, yielding three centroids namely $\ddot{u}$ ü, & . The centroids are defined via ranking of normalised pentagonal fuzzy number using their incenter $\ddot{u}$ ü, & as

$$G_1 = \left\{\left[\frac{p_1+p_2+p_3}{3}\right], \frac{1}{6}\right\}, G_2 = \left\{\left[\frac{p_2+2p_3+p_4}{4}\right], \frac{1}{2}\right\} \&$$

$$G_3 = \left\{\left[\frac{p_3+p_4+p_5}{3}\right], \frac{1}{6}\right\}$$

The incentre of centroids for pentagonal fuzzy number is defined by the relation stated below,

$$\tilde{I}(\tilde{x}_0, \tilde{y}_0) = \left[\frac{\frac{\tilde{\alpha}[p_1+p_2+p_3]}{3}+\frac{\tilde{\beta}[p_2+2p_3+p_4]}{4}+\frac{\tilde{\gamma}[p_3+p_4+p_5]}{3}}{\tilde{\alpha}+\tilde{\beta}+\tilde{\gamma}}\right], \left[\frac{\tilde{\alpha}\left[\frac{1}{6}\right]+\tilde{\beta}\left[\frac{1}{2}\right]+\tilde{\gamma}\left[\frac{1}{6}\right]}{\tilde{\alpha}+\tilde{\beta}+\tilde{\gamma}}\right]$$

where,

$$\tilde{\alpha} = \sqrt{\frac{(p_4+4p_5-2p_3-3p_2)^2+16}{12}}, \tilde{\beta} = \sqrt{\frac{(p_1+p_2-p_4-p_5)^2}{3}} \&$$

$$\tilde{\gamma} = \sqrt{\frac{(p_2+4p_1-2p_3-3p_4)^2+16}{12}}$$

Here, $\tilde{R} = \sqrt{\tilde{x}_0^{\ 2} + \tilde{y}_0^{\ 2}}$ is defined as the incentre of the centroid with Euclidean distance and is a set of real numbers that maps the set of all fuzzy numbers. Figure 1 represents the Scoring of Pentagonal Fuzzy number using Incentre of centroids. Table 1 below provides the data collected from the highest waste-generating cities in India, including the amount of municipal waste and construction waste produced, the number of dumpsites, and the capacity of each landfill

Specification for Vendor

D^v – Cost of exigency of vendor for unit items

σ^v – Standard deviation of exigency

O^v – Ordaining cost

T^v – Freightage expenses for carrying items

H^v – Cost for possessing items

B^v – Reordering cost

$T^v_{d_t}$ – Conveyance measure for vendor

C^v_E – Amount of carbon emitted from vendor's depot

$E_{ü}$ – Quota of tariff reimbursed for emission

$\vartheta_{e_{f_c}}$ – Emission factor allied to fuel consumption

$\vartheta_{e_{P_w}}$ – Emission factor kindred to the weight of the product

T_{f_c} – Fuel exhaustion for transporter

J^v_f – Travel distance made by the transporter from the fabricator to vendor

P_w – Product's heft

Specification for Fabricator

S^f_c – Pre-operating cost

H^f_c – Carrying cost for unit items

$H^f_{ü}$ – Cost for carrying corrigible items

$k^f_{ü}$ – Monetary for impregnability of corrigible items

k^f_2 – Appraisal for n^th^ shipment of vendor

C^f_E – Quantity of carbon spewed out from fabricator's warehouse

a_{E_f} – Parameter for carbon discharged from the manufacturing process in ton $h^2/unit^3$

b_{E_f} – Parameter for carbon belched from reworking process in ton $h/unit^2$

c_{E_f} – Parameter for carbon released from the production process in ton/unit

S^f – Cost for concealment

R^f_c – Cost of revamping items

ε – Rate of manufacturing proportion to revamping process

γ – Snag fare

β_f – Parameter for surmising the quality function during the manufacturing process

ω_f – Parameter for mending the quality in the production system

H_{p_f} – Quantum of leccy devoured during the revamping process in the idle situation

ρ – Unflagging leccy consumed during the production time

E_{c_f} – Expense for leccy utilized

P_{c_f} – Procuring cost

U_{c_f} – Cost of wield items

W_{c_f} – Detritus jettisoning cost

I_{c_f} – Shielding cost for wield items

S_{r_f} – Famine cost for corrigible items

$P_{c_{e_f}}$ – Parameter for assorting feat

$\xi_{ü}$ – The paramount outlay for ethical investment

β – Rate of resumption

θ – Maximal trifle in bringing down the emanation, $0 \prec \theta \prec 1$

ζ – Parameter for ethical investment

$P_{ü}$ – Maximum rate of production

$P_{ü}$ – Minimum rate of production

n_s – Number of shipments made

Q_l – Consignment deal

P_r – Unit rate of production

ξ – Investment for carbon-free atmosphere

τ – Rate for assorting the items

k^r_1 – Appraisal for first delivery

Assumptions

To advance the prospective model, the assumptions used are,

- The entrepreneurs make use of continual contemplation to control the inventory levels in their storage system.
- The ordering cycle of all entrepreneurs is equivalent. ie.,

$$Q_l/D = Q'_l/D'$$

- The manufacturing nexus can be utilized to manufacture products and to retrieve used products accumulated from the market.
- The quality of new products and the remanufactured products will be the same where both are sold to the same market.

- Due to defective manufacturing nexus, an obstreperous situation will be caused, resulting in the production of defective products.
- The emission rate due to production and reworking processes is governed by the production rate, which is determined using a quadratic function.
- Presume the production rate to be reworked and assorted between $P_{ü}$ and $P_{ü}$. This presumption is made used by many scholars like Aldurgam. et al. (2017), Herbon (2020), and Dey. et al. (2021).

Mathematical Formulation

Vendor's Model

By drawing on the foundational work of Hsia(2008), who developed a joint economic lot sizing (JELS) with stochastic demand and two variable lead times, a mathematical model for vendors is developed in this section of the paper. For vendor v, $LT_1^v = Q_l/P_r + T_{dt}^v$ and $ü\ _2^v = \ _{dt}^v$ gives the lead time for the first delivery and lead time for delivery, $2, \ldots\ldots n$ respectively. If $D^v\left(Q_l/P_r + T_{dt}^v\right)$ denote the demand during the lead time and $\sigma^v\left(\sqrt{Q_l/P_r + T_{dt}^v}\right)$ denote the standard deviation of demand for first delivery then the appraisal for the stock is provided by $k_1^v\sigma^v\left(\sqrt{Q_l/P_r + T_{dt}^v}\right)$. $D^v T_{dt}^v, \sigma^v\sqrt{T_{dt}^v}$ and $k_2^v\sigma^v\left(\sqrt{T_{dt}^v}\right)$ respectively indicate the demand that arises during the lead time, the standard deviation and the appraisal for delivery $2, \ldots\ldots n$. Consequently, the association between k_1^v and k_2^v can be laid out by $k_2^v = k_1^v\left(\sqrt{\frac{Q_l/P_r + T_{dt}^v}{T_{dt}^v}}\right)$. The predicament outlines the anticipated cost of famine items for the first and subsequent deliveries, $\sigma^v\left(\sqrt{Q_l/P_r + T_{dt}^v}\psi(k_1^v)\right)$ and $\sigma^v\left(\sqrt{T_{dt}^v}\psi(k_1^v)\right)$ where

$\psi(k_1^v) = f_s(k_1^v) - k_1^v\left[1 - F_s(k_1^v)\right]$ and

$\psi(k_2^v) = f_s(k_2^v) - k_2^v\left[1 - F_s(k_2^v)\right]$ when the vendor's inventory strikes the reorder boundary, an order for $n_s Q_l^v$ items is placed $H^v\left(\frac{Q_l^v}{2} + k_1^v\sigma^v\sqrt{Q_l/P_r + T_{dt}^v}\right)$, providing the cost for carrying items at unit time. For every kilogram of CO_2 produced, the store pays a carbon emission fee. The vendor's system calculates the carbon emission from two activities namely transportation and storage. The equation

$$E_{tax}\left(\frac{D^v}{Q_l^v}\vartheta_{e_{f_c}}\varepsilon J_f^v + \vartheta_{e_{P_w}}\alpha D^v\right) +$$
$$E_{tax}C_E^v\left(\frac{Q_l^v}{2} + k_1^v\sigma^v\left(\sqrt{Q_l/P_r + T_{dt}^v}\right)\right)$$

outlines the cost of carbon incurred by the store. The debut term represents the cost associated with the activity of relocating the items, while the second term provides the cost of carbon associated with the activity of storing. By adopting a common cycle as stated in the assumption, the total cost for all shops, which includes the ordaining and relocating, is given by,

$$R_{T_C} = \frac{D}{n_s Q_l}\sum_{v=1}^{V}\left(O^v + n_s T^v\right) +$$
$$\sum_{v=1}^{V}\left(H^v + E_{tax}C_E^v\right)\left(\frac{Q_l}{2}\frac{D^v}{D} + k_1^v\sigma^v G_1\right) +$$
$$\frac{D}{n_s Q_l}\sum_{v=1}^{V}\pi^v\sigma^v\left[G_1\psi(k_1^v) + (n_s - 1)\sqrt{T_{dt}^v}\psi(k_2^v)\right] +$$
$$\sum_{v=1}^{V}E_{tax}\left(\frac{D}{Q_l^v}\vartheta_{e_{ü_c}}T_f J_f^v + \vartheta_{e_{\ w}}\alpha D^v\right), where\, G_1$$
$$= \sqrt{Q_l/P_r + T_{dt}^v}$$

Fabricator Model

Both production and remanufacturing strategies are handled by the manufacturing system on the manufacturer's end. A pre-operating cost S_c^f is incurred by the producer, who turns out $n_s Q_l$ units for each production run. Once Q_l the manufacturing system has generated units, delivery has been completed. The amount of inventory owned by a fabricator is equal to their accumulated production minus their total accumulated deliveries. The pre-operating and carrying cost that the equation shows the fabricator incurred

$$\frac{DS_c^f}{n_s Q_l} + H_c^f\frac{Q_l}{2}\left(n_s\left[1 - \frac{D}{P_r}\right] - 1 + \frac{2D}{P_r}\right)$$

The fabricator's production process, as mentioned in the part before, is flawed and results in the fabrication of some defective goods. Despite producing good products, the system has a chance of going out of control, producing defective products with a probability of γ . We assume that the period until the production system enters an uncontrollable state will follow an exponential distribution. By using Rosenblatt and Lee (1986) as a reference, we assume that the system will remain in an out-of-control state until the batch has been created. As a result, the quantity of defective products produced by the manufacturing system is determined as follows,

$$E(N_{def}{}^{f}) = \gamma L(P_r)\frac{(n_s Q_l)^2}{2P_r}$$

where $L(P_r) = \beta_f + \omega_f P_r^2$ and $1/L(P_r)$ is the period when the production system becomes uncontrollable. The defective pieces will subsequently be worked to raise the caliber of the final output. The cost of rework is provided by

$$C_R = R_c{}^{f}\frac{D}{n_s Q_l}E(N_{def}{}^{f})$$

To keep the production process running electrical energy is needed. The production rate determines the amount of electrical energy required by the process $\ddot{u}_{\ddot{u}_f} = \quad_{0,} \quad + \rho$ and $\ddot{u}_{\ddot{u}_f} = \quad_{0,} \quad + \rho\varepsilon$, respectively, gives the total electrical power needed by the production process and the reworking process. The total power required when the production or reworking process is underway and the power required by the system when it is in an idle position H_0. The equation $EC_P = \frac{H_{0,p} + \rho P_r}{P_r}$ $EC_R = \frac{H_{0,r} + \rho\varepsilon P_r}{\varepsilon P_r}$ provides the specific energy consumption per unit for the production process and the reworking process. Returns for used items have a mean D and standard deviation, $\sigma^f\sqrt{D}$ which indicates that they are regularly distributed. The number of recoverable items of each year may be less than the requirement for remanufacturing due to the uncertainty associated with returns. As a result, the fabricator must maintain some appraisal to guard against famine items in the supply chain. The equation draws the appraisal,

$$S_{rec}{}^{f} = \beta\tau n_s Q_l - N_{rec}{}^{f}$$

The following is a possible revision to the formulation of appraisal for famine items.

$$S_{rec}{}^{f} = k_{rec}{}^{f}\sigma^f\sqrt{\frac{\beta\tau n_s Q_l}{D}}$$

The quantity of salvageable items required for remanufacturing is,

$$N_{rec}{}^{f} = \beta\tau n_s Q_l - k_{rec}{}^{f}\sqrt{\frac{\beta\tau n_s Q_l}{D}}$$

Consequently, the fabricator's level of inventory for recoverable items is,

$$INV_{rec}{}^{f} = \frac{D}{n_s Q_l}\left\{S_{rec}{}^{f}\frac{n_s Q_l}{D} + N_{rec}{}^{f}\frac{n_s Q_l}{2D} + N_{rec}{}^{f}\frac{n_s Q_l}{2P_r}\right\}$$

$$= \frac{1}{2}\beta\tau\ddot{u}_s \quad_l + \frac{1}{2} \quad_{rec}{}^{f}\sigma^f\sqrt{\frac{\beta\tau n_s Q_l}{D}} + \frac{D}{2P_r}$$

$$\left(\beta\tau\ddot{u}_s \quad_l - \quad_{rec}{}^{f}\sigma^f\sqrt{\frac{\beta\tau n_s Q_l}{D}}\right)$$

For recoverable products, the sheer amount for famine products is deducted by,

$$F_{rec}{}^{f} = \sigma^f\left(\sqrt{\frac{\beta\tau n_s Q_l}{D}}\right)\psi\left(2k_{rec}{}^{f}\right)$$

The amount resided by the fabricator on a collection of maneuvers is expressed in a quadratic form and is provided by,

$$IVT_{col}{}^{f} = \frac{1}{2}P_{c_{e_f}}\tau^2$$

Each emission exacerbated by production and storage comes up with a carbon cost for fabricators. The amount of emissions discharged as a result of production is a quadratic function that depends on the pace of production. Additionally, the fabricator invests in green technology with the objective of curbing emission $C_{e\zeta} = \theta\left[1 - e^{-\zeta\xi}\right]$ s. Estimates the percentage of emission that is curbed when the fabricator invests in environment-friendly production techniques. The cost of carbon borne by the fabricator for production and famine items is brought out by,

$$EM_c = E_{tax}\left\{1 - \theta\left[1 - e^{-\zeta\xi}\right]\right\}\left[\frac{C_E^v Q_l}{2}\left(n_s\left[1 - \frac{D}{P_r}\right] - 1 + \frac{2D}{P_r}\right) + \right.$$

$$D\left(a_{E_f}P_r^2 - b_{E_f}P_r + c_{E_f}\right) + \frac{D}{n_s Q_l}E\left(N_{def}{}^{f}\right)$$

$$\left(a_{E_f}(\varepsilon P_r^2) - b_{E_f}(\varepsilon P_r) + c_{E_f}\right) + \frac{1}{2}C_E^f$$

$$\left(\aleph\, n_s Q_l + k_{rec}{}^{f}\sqrt{\frac{\beta\tau n_s Q_l}{D}\sum_{\zeta=1}^{\zeta}\left(\quad^{v}\right)^2}\right) +$$

$$\frac{D}{P_r}\left(\aleph\,\ddot{u}_s \quad_l - \quad_{rec}{}^{f}\sqrt{\frac{\beta\tau n_s Q_l}{D}\sum_{\zeta=1}^{\zeta}\left(\quad^{v}\right)^2}\right)$$

The overall expenditure for the fabricator that encompasses waste disposal cost, raw material cost, cost associated with inspecting returned goods, and cost associated with buying back goods is disclosed by,

$$M_{T_c} = \frac{DS_c^{\ f}}{n_s Q_l} + H_c^{\ f}\frac{Q_l}{2}G_2 + E_{c_f} D\left(EC_P + EC_R\frac{E(N_{\text{ü}}^{\ f})}{n_s Q_l}\right) +$$

$$S^f D + R_c^{\ f}\frac{D}{n_s Q_l}E\left(N_{def}^{\ f}\right) + \frac{H_{rec}^{\ f}}{2} \times$$

$$\left[\beta\tau n_s Q_l + k_{rec}^{\ f}G_3 + \frac{D}{P_r}\left(\beta\tau n_s Q_l - k_{rec}^{\ f}G_3\right)\right] +$$

$$\frac{\pi_{rec}D}{n_s Q_l}G_3\psi\left(2k_{rec}^{\ f}\right) + P_{c_f}(1-\beta\tau)D + U_{c_f}\tau D + \frac{1}{2}P_{c_{e_f}}\tau^2 +$$

$$W_{c_f}(1-\beta)\tau D + I_{c_f}\tau D + E_{tax}\left\{1-\theta\left[1-e^{-m\xi}\right]\right\} \times$$

$$\left[\frac{C_E^f Q_l}{2}G_2 + D\left(a_{E_f}P_r^2 - b_{E_f}P_r + c_{E_f}\right) + \frac{D}{n_s Q_l}E\left(N_{def}^{\ f}\right) \times\right.$$

$$\left(a_{E_f}(\varepsilon P_r^2) - b_{E_f}\varepsilon P_r + c_{E_f}\right) + \frac{1}{2}C_E^f\left(\beta\tau n_s Q_l + k_{rec}^{\ f}G_3 + \frac{D}{P_r}\right)$$

$$\left. + \frac{D}{P_r}\left(\beta\tau n_s Q_l - k_{rec}^{\ f}G_3\right)\right] + \xi$$

where $G_2 = n_s\left[1-\frac{D}{P_r}\right] - 1 + \frac{2D}{P_r}$ and

$$G_3 = \sqrt{\frac{\beta\tau n_s Q_l}{D}\sum_{r=1}^{R}\left(\sigma^v\right)^2}$$

Assuming that all vendors employ the same safety factor, $k_1 = \left(k_1^1, k_1^2, k_1^3, \ldots\ldots k_1^r\right)$ the combined total cost can be determined by employing the following equation,

$$\ddot{u}_{\ddot{u}_{\ddot{u}}} = \quad +$$

The problem above can be presented as,

$$\begin{aligned} &Minimize && J_{T_c} \\ &Subject\ to && P_{r_{\min}} \le P_r \le P_{r_{\max}} \\ &&& 0 \prec \tau \prec 1 \\ &&& \xi \le \xi_{\max} \end{aligned}$$

By looking at the problem mentioned in the preceding part, the minimum joint total cost occurs at the point $\left(Q_l, S_c^{\ f}, P_r, \tau, \xi\right)$ that satisfies

$$\frac{\partial J_{T_c}}{\partial Q_l} = 0, \frac{\partial J_{T_c}}{\partial S_c^{\ f}} = 0, \frac{\partial J_{T_c}}{\partial P_r} = 0, \frac{\partial J_{T_c}}{\partial \tau} = 0 \text{ and } \frac{\partial J_{T_c}}{\partial \xi} = 0$$

simultaneously.

Eludication Strategy for Proposed Inventory Tekcik Manoeuvring Werner's Fuzzy and Operator Approach

Werner advanced the fuzzy and operator approach that is a convex combination of min-operator and arithmetical mean. Werner's fuzzy and operator approach takes the lead of being a strongly monotonically increasing function, which is a performance canon for assessing the prowess of operators.

As per the view of Tiryaki, a computationally efficient compensatory fuzzy aggregation operator, Werner's compensatory "Fuzzy and Operator" method is utilized for resolving the decentralized linear programming problem. In accordance with this study, this approach is more adequate than the other computationally effective compensatory fuzzy operators in the literature since this approach effectively leads to an extensive compendium of compensatory compromise solutions in addition to Pareto-optimal solutions for decentralized linear programming problems on the basis of compensation parameter δ predicting the value within the interval $\left[0,1\right]$. For $\delta = 1$, the fuzzy and operator leads to the min-operator, and for $\delta = 0$, the fuzzy and operator acts as the arithmetic average of the fuzzy constraints.

$$\begin{aligned} &Min\,\varphi\left(n_s, Q_l, S_c^{\ f}, P_r, \tau\right) = \delta.V_{T_c} + F_{T_c} \\ &Max\,\delta.V_{T_c} + F_{T_c} \\ &Subject\ to \quad \mu_k(x)P_{r_{\min}} \le P_r \le P_{r_{\max}} \\ &\qquad\qquad\quad 0 \prec \tau \prec 1 \\ &\qquad\qquad\quad \xi \le \xi_{\max} \end{aligned}$$

By using the conceptualization defined above, the auxiliary model can be expressed by making use of Werner's approach in addition to the non-fuzzy constraints. The compendium of the transformation process for the proclaimed problem by employing fuzzy and operator is defined as,

$$\begin{aligned} &Max && V_{T_c} + F_{T_c} \\ &Subject\ to && P_{\text{ü}} \quad .\omega_f \aleph P_r \quad E_{c_f} \quad T_{d_t}^{\ v} \\ &&& (H_p + H_r)\rho + \varepsilon \ge E_{c_f} + S_{r_f} \\ &&& P_{c_{e_f}}.\tau + (k_{rec}^f.\beta_f)\beta \le (I_{c_f} - 1)P_{c_f} \\ &&& \xi_{\text{ü}} \quad .\theta \ge \zeta.E_{tax} + \vartheta_{e_{f_c}} + \vartheta_{e_{p_w}} \end{aligned}$$

Table 1: Crisp and fuzzy assay of the specifications

Notation	*Crisp Epitome*	*Fuzzy Epitome*
H_c^f	0.5	(0.1,0.3,0.5,0.7,0.9)
T_{f_c}	0.3	0.3
α	0.05	0.05
$\vartheta_{e_{f_c}}$	2.6	2.6
$\vartheta_{e_{p_w}}$	2.5	2.5
$E_{ü}$	0.0618	0.0618
S_c^f	800	0.00012
a	0.00012	0.0008
b	0.0008	(400,600,800,1000,1200)
c	8.4	8.4
a_{E_f}	0.0000007	0.0000007
b_{E_f}	0.00012	0.00012
c_{E_f}	1.4	1.4
c_E^v	10	10
C_E^f	8	8
H_{p_f}	1000	1000
H_{r_f}	800	800
z	1	1
ρ	0.8	0.8
$P_{ü}$	500	500
$P_{ü}$	2500	2500
E_{c_f}	0.015	(0.005,0.01,0.015,0.02,0.025)
R_c^f	3	(2,2.5,3,3.5,4)
s	1	(0.6,0.8,1.0,1.2,1.4)
γ	0.02	0.02
β_f	0.015	0.015
ω_f	0.00001	0.00001
ε	1.2	1.2
β	0.7	0.7
P_{c_f}	10	(6,8,10,12,14)
U_{c_f}	3	(1,2,3,4,5)
$P_{c_{e_f}}$	2200	2200
W_{c_f}	0.5	(0.1,0.3,0.5,0.7,0.9)
I_{c_f}	0.2	(0.1,0.15,0.2,0.25,0.3)
$k_{ü}^f$	1.2	1.2
$H_{ü}^f$	0.3	(0.1,0.2,0.3,0.4,0.5)
$\pi_{ü}^f$	30	(10,20,30,40,50)
ζ	0.005	0.005
θ	0.8	0.8
$\xi_{ü}$	85	85
n_s	4	4
Q_l	185.69 units	168.4874 units
P_r	2.02	1.8625
ξ	1640.4 units	1543.3204 units
τ	0.755	0.6922
k_1^r	\$350.1	\$338.8432
R_{T_c}	\$1775.59	\$1447.1099
M_{T_c}	\$22731.4	\$19363.9515
J_{T_c}	\$24507	\$20841.0614

Result

This paper advances a mathematical model for governing inventories in a closed-loop supply chain system constructed of a fabricator and multi-vendor. This model aims to reduce the joint total cost by concurrently ascertaining some decision variables. An algorithm is recommended to procure the values of decision variables.

This research helps managers to check the inventory levels in the supply chain by perpetuating the joint total cost, energy consumption, and the total emissions fabricated by the system. To achieve the above, the manager has to adjust the manufacturing rate, consignment deal, and number of shipments made to control the rate of electrical energy consumed by the production and reworking processes and reduce the inadequacies and total emission produced from the supply chain. To increase the total emission generated by the fabricator, the manager should focus on ruling the roost of the production rate in order to sustain the trade-off between total energy consumption and total emissions produced. Amount invested on reducing the emissions in due to the carbon tax change.

Discussion

The study of (Wakhid Ahmad Jauhari, 2022) provides a mathematical model for inventory management in a manufacturer-multi-retailer CLSC system. The model seeks to minimize the joint total cost by concurrently identifying a few decision factors. It is prudent for managers to increase production rates for fixed power consumption in order to reduce the overall energy used by the industrial system. Nevertheless, this approach will result in a rise in the manufacturer's overall emissions. To maintain the trade-off between total energy consumption and total emissions produced, the management must focus more intently on regulating the production pace.

The shift in the carbon tax has a significant effect on the amount of funds spent on reducing emissions. If the tax becomes more expensive, the supply chain should focus on minimizing the emissions, thereby increasing the investment in ethical investment. So, by formulating a pentagonal fuzzy number and adopting the incentre of centroids through Werner fuzzy and operator approach, the implementation strategies in a closed-loop supply chain system furnish a higher impact on emission and carbon tax reduction, thereby presuming 100% through the imperfect reworking process, and permitting the inclusion of a quality investment.

Investigating the inventory decisions in a complicated system can augment the supply chain system by embracing three-party logistics, which accumulates pre-owned items from the market and sells them to the fabricator.

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