The Scientific Temper (2025) Vol. 16 (1): 3621-3630
E-ISSN: 2231-6396, ISSN: 0976-8653
Doi: 10.58414/SCIENTIFICTEMPER.2025.16.1.06
https://scientifictemper.com/

RESEARCH ARTICLE

Conglomerate Charge and Merchandise Swayed Inventory Model for Fragile Vendibles

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Abstract

Current business policy on marketing is very much challenging and competitive, and every seller has to attract customers by using various types of discount strategies. Apart from the common practice of trade and quantity discounts, the discount due to the pre-payment facility becomes a new trend in the market dynamics which is a motivation for every customer to buy goods. Inventory management considers customer demand as a key parameter that depends on certain factors like selling price, availability of the stock, time, quality of product, eco-friendliness of the product, impreciseness, and so on. In the mathematical modeling for perishable goods, the deterioration rate depends on certain factors like time, temperature, humidity, randomness, etc. The optimal strategy of an inventory system for perishable goods with hybrid demand dependent on selling price and stock under the partial backlogging with a certain fixed ratio is fulfilled through the Ranking Index method on Cloudy Normalised Triangular Fuzzy Number.

Keywords: Conglomerate charge, Cloudy Fuzzy Number, Convex combination-linear and non-linear price dependent demand, Fragile vendibles, Hybrid price, Pre-Payment

Introduction

Prevailing business policy relies on a combative marketing stratagem. It is a challenging task for every businessman or seller to program his/her survival. This can be achieved if only the potential customers are transformed into regular buyers. To attract customers, certain discount strategies like price discounts, quality discounts, trade discounts, seasonal discounts, and discounts due to advance payment are to be implemented. The discount due to the pre-payment facility being a new trend in the market dynamics, if offered to the sellers, will be a motivation for every customer to buy goods.

In the thesis of inventory management, customer demand is a prime criterion that depends on factors like selling price, availability of stock, time, quality of the product, eco-friendliness of the product, impreciseness, and so on. So, the inquisitor considers the demand rate as a composite charge or the inventory level.

In mathematical modelling for fragile vendibles, the putrefaction rate plays a vital role in the optimal policy. This led to analysing a model for fragile vendibles, besides the product's demand varying with time. To check this hasty putrefaction, technologists have developed a technique called 'Preservation technology.'

Advance remittance is another popular business policy in business management. This paper focusses on an inventory model under the scrutiny of hybrid market price and stock level dependent demand coupled with pre-payment and discount facility under the preservation environment. In this work, a convex combination of linear and non-linear price-dependent hybrid demand is taken into consideration.

Literature Review

Afshar-Nadjafi and Ghasemi, 2019 have proposed an EPQ model with constant demand. Khanna et al., 2016 dealt with credit financing for deteriorating imperfect quality items with allowable shortages under a constant demand rate and a constant credit period. In real life, the constant

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How to cite this article: Saarumathi R., Ritha W. (2025). Conglomerate Charge and Merchandise Swayed Inventory Model for Fragile Vendibles. The Scientific Temper, **16**(1):3621-3630.

Doi: 10.58414/SCIENTIFICTEMPER.2025.16.1.06

Source of support: Nil

Conflict of interest: None.

Received: 08/01/2025 **Accepted:** 20/02/2025 **Published** : 20/03/2025

demand rate is infelicitous for many products like clothes, mobile phones, etc. So, of late, many researchers have focussed on the time-dependent demand concept as well as the linearly increasing or decreasing demand approach. Guchhait et al., 2013 procured a production inventory model for a damageable item where different demand rates were foreseen for different types of items. Later, (Pervin et al., 2017) worked on a two-echelon inventory model with stock-dependent demand. In recent times, (Shaikh et al., 2019) perceived the retailer's optimal replacement policy of a two-warehouse inventory model for non-instantaneous items with interval-valued inventory costs and stock-dependent demand under inflationary conditions. Lately (Pervin et al., 2019) put forward a deteriorating inventory model with preservation technology under price-dependent demand, in which the production rate was viewed as a time-dependent function. (Khanna et.al., 2017) resoluted order lot size, backorder rate, and selling price for an inventory model of imperfect quality items with selling price-dependent demand under credit financing.

Under the trade credit policy, Roy et al., 2019 developed a declining inventory model with fluctuating demand. A two-warehouse model with imperfect trade credit and inflation on retailers' ordering policies was examined by (Tiwari et al., 2016). An interim review fuzzy inventory model presuming fuzzy demand was bestowed by Sarkar and Mahapatra, 2017. A fuzzy inventory model with a fuzzy backorder under a foggy fuzzy demand rate was settled by (De and Mahata 2017). Later (De and Mahata, 2019) put forward a cloudy fuzzy EOQ model for imperfect quality items with allowable proportionate discounts. A fuzzy imperfect production and repair inventory model with time-dependent demand, production, and repair rates under inflationary conditions was started (Jain et al., 2018), where all cost parameters were expanded as triangular fuzzy numbers. Recently, partially back ordered shortages have received high significance.

The fuzzy lot-sizing problem was linked to the human learning effect with back orders by (Kazemi et al., 2015). A model that takes into account a constant partially back ordered rate was declared by (Pervin et al., 2018). A fuzzy inventory model for degrading items with time-dependent demand and partial backlog was described by Kumar et al. (2015) where the backlog rate was taken into consideration as a fuzzy number. A fuzzy inventory model for deteriorating items with shortages under fully backlogged conditions was developed by (Indrajit Singha et al., 2016). Later on, Shaikh et al., 2018 scrutinized a fuzzy inventory model for a deteriorating item under variable demand, permissible delay in payments, and partial backlogging under shortage in accordance with inventory policy by employing quantum-behaved particle swarm optimization techniques to solve the optimization problems.

Materials and Methods

Fuzzy Set

It X is a universe of discourse and x is a particular element X that the fuzzy set A defined on X can be written as

$$A = \left\{ \left(X, \mu_i(X);\ x \in X \right) \right\}$$

Triangular Fuzzy Number

A triangular fuzzy number $\tilde{A} = (a,b,c)$ is represented by membership function $\mu_{\tilde{A}}(x)$ s

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & if\ x \prec a\ and\ x \succ c \\ \dfrac{x-a}{b-a}, & if\ a \le x \le b \\ \dfrac{c-x}{c-b}, & if\ b \le x \le c \end{cases}$$

where the right and left α - cuts $\tilde{A}$ are defined as

$A_R(\alpha) = c - (c-b)\alpha$ and $A_L(\alpha) = a + (b-a)\alpha.$

Cloudy Normalized Triangular Fuzzy Number

A triangular fuzzy number $\tilde{A} = (a,b,c)$ is said to be a Cloudy Normalized Triangular Fuzzy Number if the set itself converges to a crisp singleton set after an infinite time; i.e., if time $t \to \infty$, then both $a \to b$ $c \to b$ and is denoted as

$$\tilde{A} = \left\langle b\left(1-\frac{\rho}{1+t}\right), b, \left(1+\frac{\gamma}{1+t}\right)\right\rangle,\ where\ 0 \prec \gamma,\ \rho \prec 1$$

Here,

$\lim_{t\to\infty} b\left(1-\dfrac{\rho}{1+t}\right) = b,\ \lim_{t\to\infty} b\left(1+\dfrac{\gamma}{1+t}\right) = b$ and

$\tilde{A} \to \{b\}$. The membership function of Cloudy Normalized Triangular Fuzzy Number is given by,

$$\mu_{\tilde{A}}(x,t)=\begin{cases}0, & \text{if } x \prec b\left(1-\frac{\rho}{1+t}\right) \text{ and } x \succ b\left(1+\frac{\gamma}{1+t}\right)\\ \dfrac{x-b\left(1-\frac{\rho}{1+t}\right)}{\frac{b\rho}{1+t}}, & \text{if } b\left(1-\frac{\rho}{1+t}\right)\le x\le b\\ \dfrac{b\left(1+\frac{\gamma}{1+t}\right)-x}{\frac{b\gamma}{1+t}}, & \text{if } b\le x\le b\left(1+\frac{\gamma}{1+t}\right)\end{cases}$$

Ranking Index Method on Cloudy Normalized Triangular Fuzzy Number

The Ranking Index on Cloudy Normalized Triangular Fuzzy Numbers is symbolized by

$$I(\tilde{A})=\frac{1}{T}\int_{t=0}^{t=T}\int_{\alpha=0}^{\alpha=1}\frac{A_L(\alpha,t)+A_R(\alpha,t)}{2}\,d\alpha\,dt$$

where the parameters α t are independent in nature, the left and right α- cuts of $\mu_{\tilde{A}}(x,t)$ $A_L(\alpha,t)=b\left(1-\frac{\rho}{1+t}\right)+\frac{b\rho}{1+t}\alpha$ are and $A_R(\alpha,t)=b\left(1+\frac{\gamma}{1+t}\right)-\frac{b\gamma}{1+t}\alpha$. Hence we obtain the ranking value index of $I(\tilde{A})$ as $I(\tilde{A})=b\left(1+\frac{\gamma-\rho}{4}\frac{\log(1+T)}{T}\right)$ over-here

$\lim_{T\to\infty}\frac{\log(1+T)}{T}=0$ so $I(\tilde{A})\to b$ as $T\to\infty$. The factor $\frac{\log(1+T)}{T}$ is called the Cloud Index. The time horizon T cannot be infinite, and hence, the value $\frac{\log(1+T)}{T}$ can never be zero. Therefore, the value of the cloud index lies in $(0,1)$.

Assumptions

- Ordering tariffs is a continuously increasing function of time.
- The production rate is reduced than the demand rate by permitting the shortages and back ordering partially.
- Demand, which is time-dependent and fuzzy, is defined by $D(t)=\beta+\alpha t$ where $\beta\succ 0$ and β,α are fuzzy and cloudy fuzzy numbers, respectively.
- Lead time (survey time) is reviewed as zero.
- There is no restoration or renewal of perishable items during cycle time.
- The perishable rate and partially back ordered rate follow fuzzy and cloudy fuzzy numbers.
- The inflation rate tag along with a general fuzzy and cloudy fuzzy number.

Notations

I_t - Level of inventory at a given time t where $0\le t\le T$

r - Replenishment cost per order ($)

P_c - Cost of purchasing per unit ($)

H_c - Holding cost of items per unit time ($)

S_c - Shortage cost per unit time ($)

D_c - Deteriorating cost ($)

O_c - Opportunity cost of items per unit time ($)

k - Rate of backlog

θ - Rate of deterioration where $0\prec\theta\prec 1$

d_p - Percentage of discount based on advance payment

S_m - Most product amount at the outset of a cycle

ξ - Investment in preservation cost ($)

R - Highest shortage amount at the ultimate cycle

Q - Lot-size

t - Time when the inventory level becomes zero

T - Length of the cycle

Mathematical Formulation

Crisp Model

The facilitator or Ideologue places a requisition of S_m+R units of fragile merchandise by paying the integral purchase price at a time leading up to L unit time from the receiving moment (Table 1). As a reward for pre-payment, he will be showered some discount on the entire fire. When the facilitator is awarded the product, he attains the entire backlogged items spontaneously. Subsequently, the endured stock declaims due to customer satisfaction, and the deficit is partially backlogged at a rate k.

The inventory holding cost is given by

$$H_c\left[\frac{\alpha(p)}{(\theta+\beta)^2}\left\{e^{(\theta+\beta)t}-(\theta+\beta)t-1\right\}\right]$$

The replenishment cost per cycle is r

Purchasing cost specifies

$$(1-d_p)P_c\left[\frac{\alpha(p)}{(\theta+\beta)}\left\{e^{(\theta+\beta)t}-1\right\}+k\alpha(p)(T-t)\right]$$

Capital cost is denoted by

$$LI_t(1-d_p)P_c\left[\frac{\alpha(p)}{(\theta+\beta)}\left\{e^{(\theta+\beta)t}-1\right\}+k\alpha(p)(T-t)\right]$$

Shortage cost signifies $\dfrac{S_c\alpha(p)k(T-t)^2}{2}$

Opportunity cost per cycle becomes

$$O_c\alpha(p)(1-k)(T-t)$$

Sales revenue cost can be marked as

$$p\alpha(p)t+\frac{p\beta\alpha(p)}{(\theta+\beta)^2}\left\{e^{(\theta+\beta)t}-(\theta+\beta)t-1\right\}+pR$$

Preservation technology cost typifies ξT

The total profit per unit of time is given by,

$$TP(t,T)=\frac{1}{T}\left[\begin{array}{l} p\alpha(p)\left\{t+(1-k)+kT\right\}+(p\beta-H_c) \\ \left[\left[\dfrac{\alpha(p)}{(\theta+\beta)^2}\left\{e^{(\theta+\beta)t}-(\theta+\beta)t-1\right\}\right]\right] \\ -r-(1+LI_t)(1-d_p) \\ P_c\left[\dfrac{\alpha(p)}{(\theta+\beta)^2}\left\{e^{(\theta+\beta)t}-1\right\}+k\alpha(p)(T-t)\right] \\ -\dfrac{S_c\alpha(p)k(T-t)^2}{2}-O_c\alpha(p)(1-k)(T-t)-\xi T \end{array}\right]$$

In favor of attaining maximum total profit *TP(t,T)* the first order partial derivatives of *TP(t,T)* with respect to *t* and *T* are procured by rearranging the terms in the above equation and equating it to zero.

Fuzzy Model and Cloudy – Fuzzy Model

Here, the replenishment cost, purchasing cost, holding cost, shortage cost, deterioration cost, and opportunity cost are first considered as general fuzzy numbers (Tables 2 and 3). This concept is extended in a Cloudy-fuzzy environment to prove the reduction in the total profit through numerical examples. Consequently r, P_c, H_c, S_c and O_c are summed up as

$$\tilde{r}=\begin{cases}\left\langle r_1,r_2,r_3\right\rangle, & \text{for NGTFN}\\ \left\langle r\left(1-\dfrac{\rho}{1+T}\right),r,r\left(1+\dfrac{\gamma}{1+T}\right)\right\rangle, & \text{for CNTFN where } 0\prec\rho,\gamma\prec T\ \&\ T\succ 0\end{cases}$$

$$\tilde{P}_c=\begin{cases}\left\langle P_{c_1},P_{c_2},P_{c_3}\right\rangle, & \text{for NGTFN}\\ \left\langle P_c\left(1-\dfrac{\rho}{1+T}\right),P_c,P_c\left(1+\dfrac{\gamma}{1+T}\right)\right\rangle, & \text{for CNTFN where } 0\prec\rho,\gamma\prec T\ \&\ T\succ 0\end{cases}$$

$$\tilde{H}_c=\begin{cases}\left\langle H_{c_1},H_{c_2},H_{c_3}\right\rangle, & \text{for NGTFN}\\ \left\langle H_c\left(1-\dfrac{\rho}{1+T}\right),H_c,H_c\left(1+\dfrac{\gamma}{1+T}\right)\right\rangle, & \\ \text{for CNTFN where } 0\prec\rho,\gamma\prec T\ \&\ T\succ 0 & \end{cases}$$

$$\tilde{S}_c=\begin{cases}\left\langle S_{c_1},S_{c_2},S_{c_3}\right\rangle, & \text{for NGTFN}\\ \left\langle S_c\left(1-\dfrac{\rho}{1+T}\right),S_c,S_c\left(1+\dfrac{\gamma}{1+T}\right)\right\rangle, & \\ \text{for CNTFN where } 0\prec\rho,\gamma\prec T\ \&\ T\succ 0 & \end{cases}$$

$$\tilde{O}_c=\begin{cases}\left\langle O_{c_1},O_{c_2},O_{c_3}\right\rangle, & \text{for NGTFN}\\ \left\langle O_c\left(1-\dfrac{\rho}{1+T}\right),O_c,O_c\left(1+\dfrac{\gamma}{1+T}\right)\right\rangle, & \\ \text{for CNTFN where } 0\prec\rho,\gamma\prec T\ \&\ T\succ 0 & \end{cases}$$

The corresponding total fuzzy and cloudy-fuzzy profit per unit of time is given by,

$$\overline{Z}=\begin{cases}\left\langle Z_1,Z_2,Z_3\right\rangle, & \text{for NGTFN}\\ \left\langle Z_1{}^c,Z_2{}^c,Z_3{}^c\right\rangle, & \text{for CNTFN}\end{cases}$$

where the values of Z_1, Z_2, Z_3 and $Z_1{}^c, Z_2{}^c, Z_3{}^c$ are defined as

$$\begin{cases} Z_1 = \frac{1}{T}\left(R_{11}+R_{12}+R_{13}+B\right) \\ Z_2 = \frac{1}{T}\left(R_{21}+R_{22}+R_{23}+B\right) \\ Z_3 = \frac{1}{T}\left(R_{31}+R_{23}+R_{33}+B\right) \\ Z_1^{\ c} = \frac{1}{T}\left(R_{11}^{\ c}+R_{12}^{\ c}+R_{13}^{\ c}+B\right) \\ Z_2^{\ c} = \frac{1}{T}\left(R_{21}^{\ c}+R_{22}^{\ c}+R_{23}^{\ c}+B\right) \\ Z_3^{\ c} = \frac{1}{T}\left(R_{31}^{\ c}+R_{32}^{\ c}+R_{33}^{\ c}+B\right) \end{cases}$$

The corresponding total profit per unit of time is given by,

$$Z(T_1,T_2,T_3) = \frac{1}{T}(SR - r - H_c - S_c - O_c - CC - PTC)$$

Here R_1, R_2 and R_3 are defined as,

$$R_1 = p\aleph(p)\left\{t + \frac{\beta}{(\theta+\beta)^2}\left\{e^{(\theta+\beta)t} - (\ + \ )t - 1\right\} + \frac{R}{\alpha(p)}\right\} - \left\{H_c\left[\frac{\alpha(p)}{(\theta+\beta)^2}\left\{e^{(\theta+\beta)t} - (\theta+\beta)t - 1\right\}\right]\right\} - \left\{LI_t(1-d_p)P_c\alpha(p)\left[\frac{1}{(\theta+\beta)}\left\{e^{(\theta+\beta)t} - 1\right\} + k(T-t)\right]\right\}$$

$$R_2 = \alpha(p)(T-t)\left\{\frac{S_c k(T-t)}{2} + O_c(1-k)\right\}$$

$$R_3 = r + \xi T$$

Thereupon,

$$R_{11} = p\aleph(p)\left\{t + \frac{\beta}{(\theta+\beta)^2}\left\{e^{(\theta+\beta)t} - (\ + \ )t - 1\right\} + \frac{R}{\alpha(p)}\right\} - \left\{Đ_{c_1}\left[\frac{\alpha(p)}{(\theta+\beta)^2}\left\{ \ ^{(\theta+\beta)t} - (\theta+\beta) \ - 1\right\}\right]\right\} - \left\{LI_t(1-d_p)P_{c_1}\alpha(p)\left[\frac{1}{(\theta+\beta)}\left\{e^{(\theta+\beta)t} - 1\right\} + k(T-t)\right]\right\}$$

Similarly, R_{12}, R_{13} are obtained by substituting $H_c = H_{c_2}, P_c = P_{c_2}$ and $H_c = H_{c_3}, P_c = P_{c_3}$ in R_1.

$$R_{21} = \alpha(p)(T-t)\left\{\frac{S_{c_1} k(T-t)}{2} + O_{c_1}(1-k)\right\}$$

Likewise, R_{22}, R_{23} they are obtained by putting $S_c = S_{c_2}, O_c = O_{c_2}$ and $S_c = S_{c_3}, O_c = O_{c_3}$ in R_2.

$Đ_Đ = \ + \xi$

In the same manner, it R_{32}, R_{33} is procured by placing $r = r_2$ and $r = r_3$ in R_3.

Furthermore,

$$R_{11}^{\ c} = p\aleph(p)\left\{t + \frac{\beta}{(\theta+\beta)^2}\left\{e^{(\theta+\beta)t} - (\ + \ )t - 1\right\} + \frac{R}{\alpha(p)}\right\} - \left\{H_c\left(1 - \frac{\rho}{1+T}\right)\left[\frac{\alpha(p)}{(\theta+\beta)^2}\left\{e^{(\theta+\beta)t} - (\theta+\beta)t - 1\right\}\right]\right\} - \left\{LI_t(1-d_p)P_c\left(1 - \frac{\rho}{1+T}\right)\alpha(p)\left[\frac{1}{(\theta+\beta)}\left\{e^{(\theta+\beta)t} - 1\right\} + k(T-t)\right]\right\}$$

Analogously, $R_{13}^{\ c}$ it is attained by replacing $H_c = H_c\left(1 + \frac{\gamma}{1+T}\right)$ and $P_c = P_c\left(1 + \frac{\gamma}{1+T}\right)$ in R_1 and

$R_Đ^{\ c} = R$.

$$R_{21}{}^{c} = \alpha(p)(T-t)\left\{\frac{S_{c}\left(1-\frac{\rho}{1+T}\right)k(T-t)}{2} + O_{c}\left(1-\frac{\rho}{1+T}\right)(1-k)\right\}$$

In the same way, the value of $R_{23}{}^{c}$ can be found by substituting $S_c = S_c\left(1+\frac{\gamma}{1+T}\right)$ and

$$O_c = O_c\left(1+\frac{\gamma}{1+T}\right) R_2 \text{ and } R_{Đ}{}^{c} = R \ .$$

$$R_{31}{}^{c} = r\left(1-\frac{\rho}{1+T}\right)+\xi T$$

The value of $R_{33}{}^{c}$ is obtained by substituting $r = r\left(1+\frac{\gamma}{1+T}\right)$ in R_3 and $R_{Đ}{}^{c} = R$.

The membership function of the fuzzy total profit per unit time is given by,

$$\mu(\bar{Z}) = \begin{cases} 0, & if\ Z \prec Z_1\ and\ Z \succ Z_2 \\ \dfrac{Z-Z_1}{Z_2-Z_1}, & if\ Z_1 \le Z \le Z_2 \\ \dfrac{Z_3-Z}{Z_3-Z_2}, & if\ Z_2 \le Z \le Z_3 \end{cases}$$

and the corresponding fuzzy index value is calculated by

$$\begin{aligned} TP(\bar{Z}) &= \frac{Z_1+2Z_2+Z_3}{4} \\ &= \frac{1}{4T}\left[R_{11}+R_{21}+R_{31}+B+2\left(R_{21}+R_{22}+R_{32}+B\right)+R_{13}+R_{23}+R_{33}+B\right] \\ &= \frac{1}{4T}\left[\left(R_{11}+2R_{12}+R_{12}\right)+\left(R_{21}+2R_{22}+R_{23}\right)+\left(R_{31}+2R_{32}+R_{33}\right)+4B\right] \\ &= \frac{1}{4T}\left[\frac{Đ}{4}+\frac{}{4}+\frac{}{4}+B\right] \end{aligned}$$

Using the definition of membership function of cloudy-fuzzy number, the membership function of the total profit under a cloudy-fuzzy environment is stated as,

$$\mu(\tilde{Z}) = \begin{cases} 0, & if\ Z \prec Z_1{}^{c}\ and\ Z \succ Z_3{}^{c} \\ \dfrac{Z-Z_1{}^{c}}{Z_2{}^{c}-Z_1{}^{c}}, & if\ Z_1{}^{c} \le Z \le Z_2{}^{c} \\ \dfrac{Z_3{}^{c}-Z}{Z_3{}^{c}-Z_2{}^{c}}, & if\ Z_2{}^{c} \le Z \le Z_3{}^{c} \end{cases}$$

The Ranking Index value of cloudy-fuzzy total profit is given by,

$$TP(\tilde{Z}) = \frac{1}{\psi}\int_{T=0}^{\psi}\frac{1}{4}\left(Z_1{}^{c}+2Z_2{}^{c}+Z_3{}^{c}\right)dT$$

$$= \frac{1}{4\psi}\left[\begin{aligned} &\int_{T=0}^{\psi}\frac{R_{11}{}^{c}+2R_{12}{}^{c}+R_{13}{}^{c}}{T}dT+2 \\ &\int_{T=0}^{\psi}\frac{Đ_{21}{}^{c}+2{}_{22}{}^{c}+{}_{23}{}^{c}}{T}dT+ \\ &\int_{T=0}^{\psi}\frac{Đ_{31}{}^{c}+2{}_{32}{}^{c}+{}_{33}{}^{c}}{T}dT+\int_{T=0}^{\psi}\frac{4B}{T}dT \end{aligned}\right]$$

Result

In this paper, an EPQ model is established under a cloudy-fuzzy domain where the putrefaction of items and an inflation rate of money are reviewed. Besides, shortages are allowed with partial back ordering. This model also progresses by presuming selling price and stock-linked hybridized market demand, preservation technology, advanced payment facility, and partial backlogging with a constant backlogging rate. The corresponding optimization problem related to the proposed model is described using fuzzy numbers and cloudy-fuzzy numbers instead of probability theory to satisfy the existing uncertainties.

Discussion

Sadikur Rahman Md., Al-Amin Khan Md., Mohammad Abdul Halim, Taher A. Nofal, Ali Akbar Shaikh, and Emad E. Mahmoud (2021) have examined a model of inventory for perishable items, which is based on the assumptions of partial backlog with a constant backlog rate, hybridized market demand connected to selling price and stock, and advanced payment facilities.

To address the imprecise situation that arises over the production span, the current study extends the model by utilizing preservation technology and by treating the

Table 1: Out-turn of crisp model

Notation	*Non-linear price and stock-dependent demand*	*Hybrid price and linear stock-dependent demand*	*Linear price and linear stock-dependent demand*
r	200	200	200
ϕ	0	0.5	1
a	6	6	6
b	0.5	0.5	0.5
p	60	60	60
θ	0.1	0.1	0.1
β	0.007	0.007	0.007
k	0.9	0.9	0.9
H_c	1.5	1.5	1.5
d_p	0.3	0.3	0.3
P_c	10	10	10
S_c	12	12	12
O_c	8	8	8
L	0.2 months	0.2 months	0.2 months
I_t	0.12	0.12	0.12
α	0.2	0.2	5
ξ	5	5	5
t	2.8642 months	2.5154 months	2.2705 months
T	3.0319 months	2.6249 months	2.3396 months
S	83.5176 quintals	96.2208 quintals	107.425 quintals
R	4.1308 quintals	3.5636 quintals	2.7960 quintals
$ü\ (\ )$	\$485.9250	\$666.8320	\$850.144

Table 2: Out-turn of fuzzy model

Notation	*Non-linear price and stock-dependent demand*	*Hybrid price and linear stock-dependent demand*	*Linear price and linear stock-dependent demand*
r	(100,200,300)	(100,200,300)	(100,200,300)
ϕ	0	0.5	1
a	6	6	6
b	0.5	0.5	0.5
p	60	60	60
θ	0.1	0.1	0.1
β	0.007	0.007	0.007
k	0.9	0.9	0.9
H_c	(1,1.5,2)	(1,1.5,2)	(1,1.5,2)
d_p	0.3	0.3	0.3
P_c	(5,10,15)	10	10
S_c	(7,12,17)	12	12
O_c	(3,8,13)	(3,8,13)	(3,8,13)
L	0.2 months	0.2 months	0.2 months
I_t	0.12	0.12	0.12
α	0.2	0.2	5
ξ	5	5	5
t	2.8369 months	2.5002 months	2.2046 months
T	3.0308 months	2.6107 months	2.2771 months
S	80.3094 quintals	96.01 quintals	102.9832 quintals
R	3.9788 quintals	3.2819 quintals	2.349 quintals
$\ddot{u}\ (\ \ )$	\$456.6270	\$628.3411	\$804.6241

numbers as cloudy normalized triangular fuzzy numbers. From the current work's numerical analysis, it is found that an ideal assertion to solve an inventory system under uncertainty is the cloudy-fuzzy environment as it is easy to understand and convenient to make a perfect decision. The uncertainty of the total cost function can be reduced by diminishing the cloud index, which in turn increases the stability of the function. Thereby, the uncertainty of the function decreases. From the numerical data, it can be concluded that the Ranking Index method on Cloudy Normalised Triangular Fuzzy Numbers with hybridized demand is more profitable than the model with non-hybridized demand.

Future improvements to the suggested inventory model might include changeable demand based on time, quantity discount, product freshness, product green level, credit period, etc. Furthermore, adding multi-level trade credit policies will substantially enhance the model.

Table 3: Out-turn of cloudy-fuzzy model

Notation	*Non-linear price and stock-dependent demand*	*Hybrid price and linear stock-dependent demand*	*Linear price and linear stock-dependent demand*
r	(100,200,300)	(100,200,300)	(100,200,300)
ϕ	0	0.5	1
a	6	6	6
b	0.5	0.5	0.5
p	60	60	60
θ	0.1	0.1	0.1
β	0.007	0.007	0.007
k	0.9	0.9	0.9
H_c	(1,1.5,2)	(1,1.5,2)	(1,1.5,2)
d_p	0.3	0.3	0.3
P_c	(5,10,15)	10	10
S_c	(7,12,17)	(7,12,17)	(7,12,19)
O_c	(3,8,13)	(3,8,13)	(3,8,13)
L	0.2 months	0.2 months	0.2 months
I_t	0.12	0.12	0.12
α	0.2	0.2	5
ξ	5	5	5
ψ	2	2	2
γ	0.17	0.17	0.17
ρ	0.15	0.15	0.15
ε	0.2	0.2	0.2
t	2. 7089 months	2.4985 months	2.1936 months
T	3.0203 months	2.5901 months	2.0946 months
S	78.2771 quintals	88.5199 quintals	97.0642 quintals
R	3.3653 quintals	2.9678 quintals	1.8551 quintals
ü ( )	$396.9472	$587.8995	$741.9225

Acknowledgment

I would like to render my eulogize and gratitude to God Almighty for His showers of blessings throughout this research work. I express my deep and sincere gratitude to my research supervisor, Dr. W. Ritha, Associate Professor, PG and Research Department of Mathematics, Holy Cross College (Autonomous), Trichy, for gifting me a window of opportunity to do research by providing invaluable guidance. I am exorbitantly beholden to my parents and all my well-wishers for their constant support, love and prayers besides sacrifice in educating me with of Bliss of Bright Future.

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