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RESEARCH ARTICLE

Some properties of maximal product of two picture fuzzy soft graph

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Abstract

In this manuscript, the concept of a picture fuzzy soft graph is formally established. Subsequently, we explore the notion of maximal product between two picture fuzzy soft graphs. Additionally, we investigate the complement of the maximal product of picture fuzzy soft graphs, along with the maximal product involving strong and complete picture fuzzy soft graphs. Furthermore, a discussion of various outcomes stemming from this exploration is presented.

Keywords: Picture fuzzy soft graph, Maximal product, Strong, complete, complement.

Introduction

Fuzzy set theory, introduced by Zadeh, addresses the challenges associated with handling uncertainties. Since its inception, many researchers have explored fuzzy sets and fuzzy logic to tackle various real-world problems characterized by ambiguity and uncertainty. The authors investigated the complement of the max product of intuitionistic fuzzy graphs. The authors explored the maximal product of two fuzzy graphs. The authors described concepts related to soft graphs, fuzzy soft graphs, and fuzzy soft sets. The authors introduced the picture fuzzy set, an enhanced version of the fuzzy set. The authors further developed this concept by introducing the picture fuzzy graph. In this paper, we define the picture fuzzy soft graph and subsequently explore concepts such as the maximal product of two picture fuzzy soft graphs, the complement of the maximal product of picture fuzzy soft graphs, and the maximal product of strong and complete picture fuzzy soft graphs. Additionally, we discuss several associated results, Akram, M., & Nawaz, S. (2015), Cen, Z., Pal, A., & Dey, A. (2019), Cuong, B. C., & Kreinovich, V. (2013), Muhammad Akram, & Nawaz, S. (2016), Maji, P. K., Roy, A. R., & Biswas, R. (2001), Radha, K., & Arumugam, S. (2015), Yahya Mohamed, S., & Mohamed Ali, A. (2021), Zadeh, L. A. (1965).

Preliminaries

Definition: 2.1

A fuzzy set is a collection of elements with varying degrees of membership, characterized by a membership function that assigns each element a value between 0 and 1. This value indicates the element's degree of belonging to the set, allowing for partial membership, Zadeh, L. A. (1965).

Definition: 2.2

A pair (J,K) is called a soft set (over D) if and only if J is a function that maps each element in K to a subset of D. In other words, J is a mapping from K to the power set of D (denoted as $\wp(D)$), which is the set of all possible subsets of D, Muhammad Akram & Nawaz, S. (2016).

Definition: 2.3

A pair (J,K) is called fuzzy soft set over D, where J is a mapping from set of parameters to the collection of all fuzzy subset of D, Akram, M., & Nawaz, S. (2015), Maji, P. K., Roy, A. R., & Biswas, R. (2001).

Definition: 2.4

The picture fuzzy set is characterized by three membership functions: positive, neutral, and negative membership degrees. These membership values consistently fall within

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the range of 0 to 1, Cen, Z., Pal, A., & Dey, A. (2019), Cuong, B. C., & Kreinovich, V. (2013).

Picture Fuzzy Soft Graph

Definition: 3.1

The graph $G'^{*} = (W, Y, (K,\lambda_A), (K,\delta_A), (K,\varphi_A), (K,\lambda_B), (K,\delta_B), (K,\varphi_B))$ is defined as a picture fuzzy soft graph, wherein:

Positive membership functions map from a parameter set to the collection of all picture fuzzy subsets in vertices, with each pair (K,λ_A) representing a picture fuzzy soft vertex of positive membership function.

Similarly, positive membership functions map from a parameter set to the collection of all picture fuzzy subsets in edges, with each pair (K,λ_B) representing a picture fuzzy soft edge of positive membership function.

- Neutral and negative membership functions operate similarly.

These functions adhere to the following conditions:

- The positive membership function of a picture fuzzy soft edge is no greater than the minimum of the positive membership functions of the two picture fuzzy soft vertices it connects.
- The neutral membership function of a picture fuzzy soft edge is no greater than the minimum of the neutral membership functions of the two picture fuzzy soft vertices it connects.
- The negative membership function of a picture fuzzy soft edge is no less than the maximum of the negative membership functions of the two picture fuzzy soft vertices it connects.

Furthermore, all three membership values range between 0 and 1 and it is denoted by $G'^{*}_{K,W,Y}$.

Definition: 3.2

The complement of picture fuzzy soft graph $G'^{*}_{K,W,Y}$ is a picture fuzzy soft graph $\overline{G'^{*}_{K,W,Y}}$ where, $\overline{\lambda}_{A_k} = \lambda_{A_k}$, $\overline{\delta}_{A_k} = \delta_{A_k}$, $\overline{\varphi}_{A_k} = \varphi_{A_k}$ $\forall$ $w_i \in W$, $k \in K$

$$\overline{\lambda}_{B_k}(w_i, w_j) = \lambda_{A_k}(w_i) \wedge \lambda_{A_k}(w_j) - \lambda_{B_k}(w_i, w_j)$$

$$\overline{\delta}_{B_k}(w_i, w_j) = \delta_{A_k}(w_i) \wedge \delta_{A_k}(w_j) - \delta_{B_k}(w_i, w_j)$$

$$\overline{\varphi}_{B_k}(w_i, w_j) = \varphi_{A_k}(w_i) \vee \varphi_{A_k}(w_j) - \varphi_{B_k}(w_i, w_j) \ \forall$$

$(w_i, w_j) \in W$, $k \in K$.

Definition: 3.3

Picture fuzzy soft graph $G'^{*}_{K,W,Y} = (W, Y, (K,\lambda_A), (K,\delta_A), (K,\varphi_A), (K,\lambda_B), (K,\delta_B), (K,\varphi_B))$ is commonly referred to as a strong picture, fuzzy soft graph, if

$$\lambda_{B_k}(w_i, w_j) = \min(\lambda_{A_k}(w_i), \lambda_{A_k}(w_j))$$

$$\delta_{B_k}(w_i, w_j) = \min(\delta_{A_k}(w_i), \delta_{A_k}(w_j))$$

$$\varphi_{B_k}(w_i, w_j) = \max(\varphi_{A_k}(w_i), \varphi_{A_k}(w_j)) \ \forall (w_i, w_j) \in Y,$$

$k \in K$.

Maximal Product Of Picture Fuzzy Soft Graph

Definition:4.1

The maximal product of $G'^{*}_{K,W_1,Y_1} \times_M G'^{*}_{L,W_2,Y_2}$ of two picture fuzzy soft graph $G'^{*}_{K,W_1,Y_1} = (W_1, Y_1, (K,\lambda_A), (K,\delta_A), (K,\varphi_A), (K,\lambda_B), (K,\delta_B), (K,\varphi_B))$ and $G'^{*}_{L,W_2,Y_2} = (W_2, Y_2, (L,\lambda'_A), (L,\delta'_A), (L,\varphi'_A), (L,\lambda'_B), (L,\delta'_B), (L,\varphi'_B))$ is defined as (W,Y) where, $W = W_1 \times_M W_2$, $Y = \{(w_1,y_1)(w_2,y_2) : w_1 = w_2, y_1y_2 \in Y_2 (or) y_1 = y_2, w_1w_2 \in Y_1\}$ and $K, L \subseteq Q$ (parameter set) which satisfy the following,

(a) $\lambda''_{A(k,l)}(w_1, y_1) = \lambda_{A_k}(w_1) \vee \lambda'_{A_l}(y_1)$

(b) $\delta''_{A(k,l)}(w_1, y_1) = \delta_{A_k}(w_1) \vee \delta'_{A_l}(y_1)$

(c) $\varphi''_{A(k,l)}(w_1, y_1) = \varphi_{A_k}(w_1) \wedge \varphi'_{A_l}(y_1)$

for all $(w_1, y_1) \in W_1 \times_M W_2, (k,l) \in K \times_M L$

(a) $\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2)) = \begin{cases} \lambda_{A_k}(w_1) \vee \lambda'_B(y_1,y_2) f \ w_1 = w_2, y_1y_2 \in Y_2 \\ \lambda_{B_k}(w_1,w_2) \vee \lambda'_{A_l}(y_1) f \ y_1 = y_2, w_1w_2 \in Y_1 \end{cases}$

(b) $\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2)) = \begin{cases} \delta_{A_k}(w_1) \vee \delta'_B(y_1,y_2) f \ w_1 = w_2, y_1y_2 \in Y_2 \\ \delta_{B_k}(w_1,w_2) \vee \delta'_{A_l}(y_1) f \ y_1 = y_2, w_1w_2 \in Y_1 \end{cases}$

(c) $\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2)) = \begin{cases} \varphi_{A_k}(w_1) \wedge \lambda'_B(y_1,y_2) f \ w_1 = w_2, y_1y_2 \in Y_2 \\ \varphi_{B_k}(w_1,w_2) \wedge \lambda'_{A_l}(y_1) f \ y_1 = y_2, w_1w_2 \in Y_1 \end{cases}$

for all $(k,l) \in K \times_M L$

Example:4.1

Let $K = \{k_1, k_2\}$ and $L = \{l_1, l_2\}$ be a two-parameter sets. Let G'^{*}_{K,W_1,Y_1} and

G'^{*}_{L,W_2,Y_2} be a two-picture fuzzy soft graph is defined by $G'^{*}_{K,W_1,Y_1} = \{k_1, k_2\}$ where

Table 1(a): Picture fuzzy soft graph G'^{*}_{K,W_1,Y_1}

(a)

λ_{A_k}	w_1	w_2
k_1	0.1	0.2
k_2	0.3	0.3

(b)

δ_{A_k}	w_1	w_2
k_1	0.1	0.1
k_2	0.3	0.1

(c)

φ_{A_k}	w_1	w_2
k_1	0.4	0.5
k_2	0.3	0.2

(d)

λ_{B_k}	(w_1, w_2)
k_1	0.1
k_2	0.2

(e)

δ_{B_k}	(w_1, w_2)
k_1	0.1
k_2	0.1

(f)

φ_{B_k}	(w_1, w_2)
k_1	0.5
k_2	0.3

Table 1(b): Picture fuzzy soft graph G'^{*}_{L,W_2,Y_2}

(a)

λ'_{A_l}	y_1	y_2
l_1	0.3	0.2
l_2	0.1	0.2

(b)

δ'_{A_l}	y_1	y_2
l_1	0.2	0.3
l_2	0.2	0.1

(c)

φ'_{A_l}	y_1	y_2
l_1	0.2	0.3
l_2	0.4	0.2

(d)

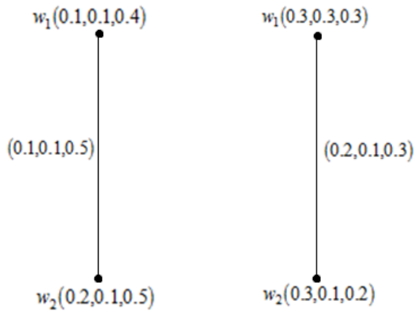
λ'_{B_l}	(y_1, y_2)
l_1	0.2
l_2	0.1

(e)

δ'_{B_l}	(y_1, y_2)
l_1	0.2
l_2	0.1

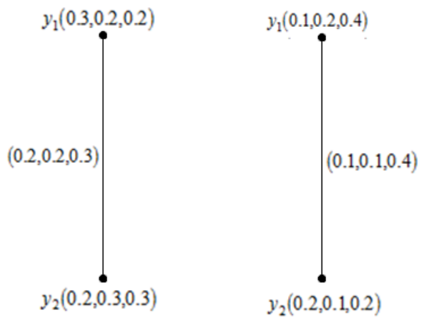
(f)

φ'_{B_l}	(y_1, y_2)
l_1	0.3
l_2	0.4



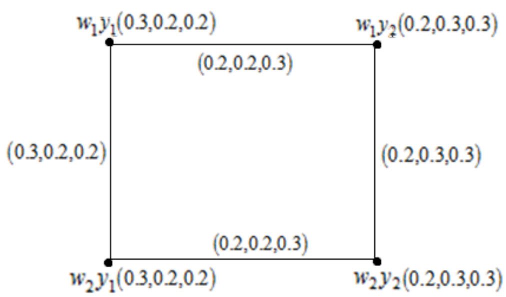
Associated with the parameter (k_1) Associated with the parameter (k_2)

Figure 1 (a): Picture fuzzy soft graph G'^{*}_{K,W_1,Y_1}

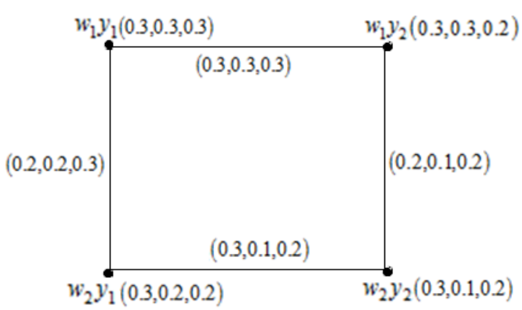


Associated with the parameter (l_1) Associated with the parameter (l_2)

Figure 1(b): Picture fuzzy soft graph G'^{*}_{L,W_2,Y_2}



Corresponding to the parameter $(k_1 \times_M l_1)$



Corresponding to the parameter $(k_2 \times_M l_2)$

Figure 1(c): Maximal product of picture fuzzy soft graph $G'^{*}_{K,W_1,Y_1} \times_M G'^{*}_{L,W_2,Y_2}$

$W_1 = \{w_1, w_2\}$ and $Y_1 = \{(w_1, w_2)\}$ is shown in Table 1 (a) and Figure 1(a). $G'^{*}_{L,W_2,Y_2} = \{l_1, l_2\}$ where, $W_2 = \{y_1, y_2\}$ and $Y_2 = \{(y_1, y_2)\}$ is shown in the Table 1(b) and Figure 1(b).

Theorem: 4.1

If G'^{*}_{K,W_1,Y_1} and G'^{*}_{L,W_2,Y_2} are two picture fuzzy soft graph. Then maximal product $G'^{*}_{K,W_1,Y_1} \times_M G'^{*}_{L,W_2,Y_2}$ is also a picture fuzzy soft graph (Figure 1(c)).

Proof:

Let $G'^{*}_{K,W,Y}$ and $G'^{*}_{L,W,Y}$ be two picture fuzzy soft graph.

consider, if $w_1 = w_2$, $y_1y_2 \in Y_2$ and $(k,l) \in K \times_M L$

$$\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2)) = \lambda_A(w_1) \vee \lambda'_B(y_1,y_2)$$
$$\leq \lambda_A(w_1) \vee (\lambda'_A(y_1) \wedge \lambda'_A(y_2))$$
$$= (\lambda_A(w_1) \vee \lambda'_A(y_1)) \wedge (\lambda_A(w_1) \vee \lambda'_A(y_2))$$
$$= (\lambda_A(w_1) \vee \lambda'_A(y_1)) \wedge (\lambda_A(w_2) \vee \lambda'_A(y_2))$$
$$\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2)) \leq \lambda''_{A(k,l)}(w_1,y_1) \wedge \lambda''_{A(k,l)}(w_2,y_2)$$
$$\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2)) = \delta_A(w_1) \vee \delta'_B(y_1,y_2)$$
$$\leq \delta_A(w_1) \vee (\delta'_A(y_1) \wedge \delta'_A(y_2))$$
$$= (\delta_A(w_1) \vee \delta'_A(y_1)) \wedge (\delta_A(w_1) \vee \delta'_A(y_2))$$
$$= (\delta_A(w_1) \vee \delta'_A(y_1)) \wedge (\delta_A(w_2) \vee \delta'_A(y_2))$$
$$\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2)) \leq \delta''_{A(k,l)}(w_1,y_1) \wedge \delta''_{A(k,l)}(w_2,y_2)$$
$$\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2)) = \varphi_A(w_1) \wedge \varphi'_B(y_1,y_2)$$
$$\geq \varphi_A(w_1) \wedge (\varphi'_A(y_1) \vee \varphi'_A(y_2))$$
$$= (\varphi_A(w_1) \wedge \varphi'_A(y_1)) \vee (\varphi_A(w_1) \wedge \varphi'_A(y_2))$$
$$= (\varphi_A(w_1) \wedge \varphi'_A(y_1)) \vee (\varphi_A(w_2) \wedge \varphi'_A(y_2))$$
$$\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2)) \geq \varphi''_{A(k,l)}(w_1,y_1) \vee \varphi''_{A(k,l)}(w_2,y_2)$$

similarly,

If $y_1 = y_2$, $w_1w_2 \in Y_1$ and $(k,l) \in K \times_M L$.

Then by the definition, we have

$$\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2)) \leq \lambda''_{A(k,l)}(w_1,y_1) \wedge \lambda''_{A(k,l)}(w_2,y_2)$$
$$\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2)) \leq \delta''_{A(k,l)}(w_1,y_1) \wedge \delta''_{A(k,l)}(w_2,y_2)$$
$$\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2)) \geq \varphi''_{A(k,l)}(w_1,y_1) \vee \varphi''_{A(k,l)}(w_2,y_2)$$

Hence, $G'^{*}_{K,W,Y} \times_M G'^{*}_{L,W,Y}$ is a picture fuzzy soft graph.

Theorem: 4.2

The complement of maximal product of two picture fuzzy soft graph is a maximal product of two picture fuzzy soft graph.

Proof:

Let $G'^{*}_{K,W,Y}$ and $G'^{*}_{L,W,Y}$ be two picture fuzzy soft graph.

consider, if $w_1 = w_2$, $y_1y_2 \in Y_2$ and $(k,l) \in K \times_M L$

$$\bar{\lambda}''_{B(k,l)}((w_1,y_1)(w_2,y_2)) = \lambda''_{A(k,l)}(w_1,y_1) \wedge \lambda''_{A(k,l)}(w_2,y_2) - \lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))$$

$$=\begin{cases}(\lambda_A(w_1)\vee\lambda'_A(y_1))\wedge(\lambda_A(w_2)\vee\lambda'_A(y_2))\\ -(\lambda_A(w_1)\vee\lambda'_A(y_1))\wedge(\lambda_A(w_2)\vee\lambda'_A(y_2)),\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\lambda_A(w_1)\vee\lambda'_A(y_1))\wedge(\lambda_A(w_2)\vee\lambda'_A(y_2)),\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

$$=\begin{cases}0,\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\lambda_A(w_1)\vee\lambda'_A(y_1))\wedge(\lambda_A(w_1)\vee\lambda'_A(y_2)),\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

$$=\begin{cases}0,\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\lambda_A(w_1))\vee(\lambda'_A(y_1)\wedge\lambda'_A(y_2)),\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

$$\overline{\lambda}''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\begin{cases}0,\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\lambda_A(w_1))\vee\lambda'_B(y_1,y_2),\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

$$\overline{\delta}''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\delta''_{A(k,l)}(w_1,y_1)\wedge\delta''_{A(k,l)}(w_2,y_2)-\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))$$

$$=\begin{cases}(\delta_A(w_1)\vee\delta'_A(y_1))\wedge(\delta_A(w_2)\vee\delta'_A(y_2))\\ -(\delta_A(w_1)\vee\delta'_A(y_1))\wedge(\delta_A(w_2)\vee\delta'_A(y_2)),\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\delta_A(w_1)\vee\delta'_A(y_1))\wedge(\delta_A(w_2)\vee\delta'_A(y_2)),\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

$$=\begin{cases}0,\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\delta_A(w_1)\vee\delta'_A(y_1))\wedge(\delta_A(w_1)\vee\delta'_A(y_2)),\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

$$=\begin{cases}0,\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\delta_A(w_1))\vee(\delta'_A(y_1)\wedge\delta'_A(y_2)),\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

$$\overline{\delta}''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\begin{cases}0,\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\delta_A(w_1))\vee\delta'_B(y_1,y_2),\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

$$\overline{\varphi}''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\varphi''_{A(k,l)}(w_1,y_1)\vee\varphi''_{A(k,l)}(w_2,y_2)-\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))$$

$$=\begin{cases}(\varphi_A(w_1)\wedge\varphi'_A(y_1))\vee(\varphi_A(w_2)\wedge\varphi'_A(y_2))\\ -(\varphi_A(w_1)\wedge\varphi'_A(y_1))\vee(\varphi_A(w_2)\wedge\varphi'_A(y_2)),\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\varphi_A(w_1)\wedge\varphi'_A(y_1))\vee(\varphi_A(w_2)\wedge\varphi'_A(y_2)),\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

$$=\begin{cases}0,\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\varphi_A(w_1)\wedge\varphi'_A(y_1))\vee(\varphi_A(w_1)\wedge\varphi'_A(y_2)),\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

$$=\begin{cases}0,\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\varphi_A(w_1))\wedge(\varphi'_A(y_1)\vee\varphi'_A(y_2)),\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

$$\overline{\varphi}''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\begin{cases}0,\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\varphi_A(w_1))\wedge\varphi'_B(y_1,y_2),\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

similarly,

If $y_1=y_2$, $w_1w_2\in Y_1$ and $(k,l)\in K\times_M L$. Then we have,

$$\overline{\lambda}''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\begin{cases}0,\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\lambda_B(w_1,w_2))\vee\lambda'_A(y_1),\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

$$\overline{\delta}''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\begin{cases}0,\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\delta_B(w_1,w_2))\vee\delta'_A(y_1),\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

$$\overline{\varphi}''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\begin{cases}0,\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))>0\\ (\varphi_B(w_1,w_2))\wedge\varphi'_A(y_1),\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))=0\end{cases}$$

Hence, The complement of maximal product of two picture fuzzy soft graph is a maximal product of two picture fuzzy soft graph.

Theorem:4.3

If G'^{*}_{K,W_1,Y_1} and G'^{*}_{L,W_2,Y_2} are strong picture fuzzy soft graph. Then maximal product $G'^{*}_{K,W_1,Y_1}\times_M G'^{*}_{L,W_2,Y_2}$ is also a strong picture fuzzy soft graph.

Proof:

Let $G'^{*}_{K,W,Y}$ and $G'^{*}_{L,W,Y}$ be strong picture fuzzy soft graph.

consider, if $w_1=w_2$, $y_1y_2\in Y_2$ and $(k,l)\in K\times_M L$

$$\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\lambda_A(w_1)\vee\lambda'_B(y_1,y_2)$$
$$=\lambda_A(w_1)\vee(\lambda'_A(y_1)\wedge\lambda'_A(y_2))$$
$$=(\lambda_A(w_1)\vee\lambda'_A(y_1))\wedge(\lambda_A(w_1)\vee\lambda'_A(y_2))$$
$$=(\lambda_A(w_1)\vee\lambda'_A(y_1))\wedge(\lambda_A(w_2)\vee\lambda'_A(y_2))$$
$$\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\lambda''_{A(k,l)}(w_1,y_1)\wedge\lambda''_{A(k,l)}(w_2,y_2)$$
$$\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\delta_A(w_1)\vee\delta'_B(y_1,y_2)$$
$$=\delta_A(w_1)\vee(\delta'_A(y_1)\wedge\delta'_A(y_2))$$
$$=(\delta_A(w_1)\vee\delta'_A(y_1))\wedge(\delta_A(w_1)\vee\delta'_A(y_2))$$
$$=(\delta_A(w_1)\vee\delta'_A(y_1))\wedge(\delta_A(w_2)\vee\delta'_A(y_2))$$
$$\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\delta''_{A(k,l)}(w_1,y_1)\wedge\delta''_{A(k,l)}(w_2,y_2)$$
$$\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\varphi_A(w_1)\wedge\varphi'_B(y_1,y_2)$$
$$=\varphi_A(w_1)\wedge(\varphi'_A(y_1)\vee\varphi'_A(y_2))$$
$$=(\varphi_A(w_1)\wedge\varphi'_A(y_1))\vee(\varphi_A(w_1)\wedge\varphi'_A(y_2))$$
$$=(\varphi_A(w_1)\wedge\varphi'_A(y_1))\vee(\varphi_A(w_2)\wedge\varphi'_A(y_2))$$
$$\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\varphi''_{A(k,l)}(w_1,y_1)\vee\varphi''_{A(k,l)}(w_2,y_2)$$

similarly,

If $y_1=y_2$, $w_1w_2\in Y_1$ and $(k,l)\in K\times_M L$.

Then by the definition, we have

$$\lambda''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\lambda''_{A(k,l)}(w_1,y_1)\wedge\lambda''_{A(k,l)}(w_2,y_2)$$
$$\delta''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\delta''_{A(k,l)}(w_1,y_1)\wedge\delta''_{A(k,l)}(w_2,y_2)$$
$$\varphi''_{B(k,l)}((w_1,y_1)(w_2,y_2))=\varphi''_{A(k,l)}(w_1,y_1)\vee\varphi''_{A(k,l)}(w_2,y_2)$$

Hence, $G'^{*}_{K,W,Y}\times_M G'^{*}_{L,W,Y}$ is a strong picture fuzzy soft graph.

Theorem:4.4

If G'^{*}_{K,W_1,Y_1} and G'^{*}_{L,W_2,Y_2} are complete picture fuzzy soft graph. Then

maximal product $G'^{*}_{K,W_1,Y_1}\times_M G'^{*}_{L,W_2,Y_2}$ is not a complete picture fuzzy soft graph.

Conclusion

This paper aims to introduce the concept of the maximal product of picture fuzzy soft graphs as its primary objective. Furthermore, we delve into the discussion regarding the complement of maximal product of picture fuzzy soft graphs, as well as explore the maximal product of strong and complete picture fuzzy soft graphs.

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