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RESEARCH ARTICLE

Properties on semi-ring of fuzzy matrices with compatible norm

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Abstract

Objectives: Using applying the concept of compatible norm, this study develops an entirely new type of fuzzy matrix semi-ring.

Methods: The main goal of the current work is to present a novel fuzzy matrix idea on compatible norm SFMc.

Findings: This study introduces fuzzy matrices and explains which ones multiply left and right over addition. Addition and multiplication are the two binary operations (+, ⊙) that occur in the semi-ring (S,+,⊙) of the set S.

Novelty: The idea of generalized semi-ring of fuzzy matrix are SFM_c studied. Using the fuzzy algebra & vector space over [0,1]. Forms a distributive law and comparable of semi-ring fuzzy matrices.

Keywords: Determinant of fuzzy matrices, semi-ring, Distribute law, Complement of Matrices, Compatible norm $\|.\|_c$, (SFM_c)

AMS Mathematics Subject Classification (2020): 03E72,15B15,15A60

Introduction

The authors introduced the idea of fuzzy sets, Zadeh, L. A. (1965).

The authors introduced the mathematical concepts of determinants and adjoins for a square fuzzy matrix. The authors presented the bi-normed sequences in a fuzzy matrix, Nagoorgani, A., & Pappa, A. (2019).

The authors several principles of matrix theory along with their applications in the field of fuzzy matrix, Meenakshi, A. R. (2008).

The authors presented the concept of compatible norms in the theory of matrix mathematics, including formulas and facts, Bernstein, S. D. (2009).

Determinants for the non-square fuzzy matrices with compatible norms were defined by Nagoorgani, A., & Pappa, A. (2019).

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Some important results on adjoins of non-square fuzzy matrices along with compatible norms were introduced by Muruganantham, P., Nagoor Gani, A., & Pappa, A. (2020).

Semi-ring of fuzzy matrices introduced by Sriram S. and Murugadas P. In the current paper our prime intention is the introduction of a novel concept of fuzzy matrix on compatible norm SFM_c.

The authors generalized a semi-ring fuzzy matrix and we recall the definition explained. In the semi-ring properties on SFM_c explained and some important theorems are proved, Pappa, A., Muruganantham, P., & Nagoor Gani, A. (2023), Pal, M. (2024), Nagoorgani, A., & Kalyani, G. (2003), Ragab, M. Z., & Emam, E. G. (1995), Sriram, S., & Murugadas, P. (2010).

In this section, we prove generalized semi-ring, fuzzy matrices and basic definitions have been studied.

Definition 1.1

Generalized semi-ring

A "semi-ring is an algebraic structure (R, +) that satisfies the properties of an abelian monoid with (identity 0)(R*) and a monoid (identity 1). Further, it has the property of distributive,

over + from both side r0 = 0r = 0 for all r € R and 0 ≠ 1.

Definition 1.2

An $m \times n$ matrix with A= $[a_{ij}]$ where the components are in [0,1] unit interval is known as a fuzzy matrix.

Definition 1.3

$|A|$ is the determinant of an $n \times n$ fuzzy matrix A that is defined as under given:

$|A| = \sum_{\sigma \in S_n} a_{1\sigma(1)} a_{2\sigma(2)} \dots a_{n\sigma(n)}$ Where, S_n denotes the symmetric group, which consists of all possible permutations of the indices (1, 2, ..., n").

Definition 1.4

(compatible fuzzy matrix) $\mathbf{FM_c}$

Multiplication of fuzzy matrices is only possible when the number of columns in the first matrix is equal to the number of rows in the second matrix.

Definition 1.5

(Compatible Norm $\|\cdot\|_c$)

Suppose $\mathcal{F}_{nn}$ is the set of all $n \times n$ at $\mathcal{F}$ = [0,1] . Norms $\|\cdot\|_c$ are defined.

Methodology

we prove that SFM_c in fuzzy algebra as well as form a fuzzy vector space under the addition, which is done component-wise, multiplication that is done component-wise, as well as scalar multiplication (+,⨀), is associative and distributive in $\mathcal{F}_{nn}$. Also associative and commutative.

Algebraic Properties of fuzzy matrices with compatible norm 2.1

For the fuzzy matrix compatible norm is taken into consideration, then the following properties will hold.

Let any fuzzy matrices A, B, C "then FM_c over $\mathcal{F}_{nn}$

$i)$ $||A + B||_c = ||B + A||_c$

$ii)$ $||A||_c = ||A||_C^T$

$iii)$ $||A + (B + C)||_C = ||(A + B) + C||_C$

iv) $||A + B||_C = ||A||_C + ||B||_C$

V) $||A + B||_C^T = ||A||_C^T + ||B||_C^T$

vi) $||A + A||_C = ||A||_C$

vii) $||A + O||_C = ||A||_C$

viii) $||A + J||_C = ||J||_C$

ix) $||\alpha A||_c = \alpha ||A||_c$ $for\ any\ \alpha\ in$ [0,1]

x) $||\alpha A||_c^T = \alpha ||A||_C^T$ $for\ any\ \alpha\ in$[0,1]

xi) $||\alpha(A + B)||_C = \alpha ||A||_C + \alpha ||B||_C$ $for\ any\ \alpha\ in$ [0,1]

xii) $||(\alpha + \beta)A||_c = \alpha ||A||_c + \beta ||A||_c$ $for\ all\ \alpha + \beta\ in$ [0,1]

xiii) $\alpha ||\beta A||_C = \alpha\beta ||A||_C$ $for\ any\ \alpha, \beta\ in$ [0,1]

Remark 2.2

- A+B ={max(a_{ij}, b_{ij})}"
- AB ={max{min(a_{ij}, b_{ij})}}

Definition 2.3

A Square fuzzy matrix A in $\mathcal{F}_{nn}$ is defined if A idempotent $A^2 = A$ (or)

$||A||_c^2 = ||A||_c$ where $A = [a_{ij}]$.

Example 2.4

If $||A||_c = \begin{bmatrix} 0.3 & 0.2 & 0.6 \\ 0.2 & 0.1 & 0.2 \\ 0.3 & 0.2 & 0.3 \end{bmatrix}$

Then $||A||_c = 0.2$

$$||A||_c^2 = \begin{bmatrix} 0.3 & 0.2 & 0.6 \\ 0.2 & 0.1 & 0.2 \\ 0.3 & 0.2 & 0.3 \end{bmatrix} \begin{bmatrix} 0.3 & 0.2 & 0.6 \\ 0.2 & 0.1 & 0.2 \\ 0.3 & 0.2 & 0.3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3 & 0.2 & 0.3 \\ 0.2 & 0.2 & 0.2 \\ 0.3 & 0.2 & 0.3 \end{bmatrix} = 0.2$$

Therefore $||A||_c^2 = ||A||_c = 0.2$ (idempotent).

Definition 2.5

Suppose A be in SFM $A = [a_{ij}]$ be the order of $n \times n$ over $\mathcal{F}_{nn}$ called as A^c=[1-a_{ij}] , where, $A = (a_{ij})$

For all i=1 to n ,j=1 to n.Then A^c is known as the complement Matrix of A.

If A= $\begin{bmatrix} 0.5 & 0.0 & 0.2 \\ 0.1 & 0.9 & 0.4 \\ 0.8 & 0.3 & 0.6 \end{bmatrix}$ then $A^c = \begin{bmatrix} 0.5 & 1.0 & 0.8 \\ 0.9 & 0.1 & 0.6 \\ 0.2 & 0.7 & 0.4 \end{bmatrix}$

Theorem 2.6

If A and B are SFM is "set of all $n \times n$ over $\mathcal{F}_{nn}$ .We consider $\mathcal{F}$=[0,1] and any Scalar in [0,1] we have

If $\| A\bar{x} \|_c \le \| A \|_c' \| \bar{x} \|_c''$

(i) $\| \bar{y}A \|_c \le \| \bar{y} \|_c' \| A \|_c''$

(ii) $\| \alpha A\bar{x} \|_c \le \alpha \| A \|_c' \| \bar{x} \|_c''$

(iii) $\| A\bar{x} + B\bar{x} \|_c = \| A\bar{x} \|_c + \| B\bar{x} \|_c$

Proof

(i) If $m = 1$ the norms $\|\cdot\|_c$, $\|\cdot\|_c'$, $\|\cdot\|_c''$, $\mathcal{F}_n, \mathcal{F}_p, \mathcal{F}_{pn}$ respectively, are compatible if for all

$A \in \mathcal{F}_{pn}$ $\bar{y} \in \mathcal{F}_p$. Let $\bar{y}$ be any fuzzy vector in $n \times n$ over $\mathcal{F}_{nn}$.

Then it is enough to prove that

$\| \bar{y}A \|_c \le \| \bar{y} \|_c \| A \|_c$

$\| \bar{y}A \|_c \le \| \bar{y} \|_c [a_{ij}]$

$\le \| \bar{y} \|_c' \| A \|_c''$.

(ii) If $n = 1$ the norms $\|\cdot\|_c$, $\|\cdot\|_c'$, $\|\cdot\|_c''$, $\mathcal{F}_m, \mathcal{F}_{mp}, \mathcal{F}_p$ respectively, are compatible if for all

$A \in \mathcal{F}_{mp}$ $\bar{x} \in \mathcal{F}_p$

If α in [0,1] then $\alpha A = [\alpha a_{ij}]$

$\| \alpha A\bar{x} \|_c \le [\alpha a_{ij}] \| \bar{x} \|_c$

$\le \alpha [a_{ij}] \| \bar{x} \|_c$

$\le \alpha \| A \|_c' \| \bar{x} \|_c''$

(iii)If $n = 1$ the norms $\|\cdot\|_c$, $\|\cdot\|_c'$, $\|\cdot\|_c''$, $\mathcal{F}_p, \mathcal{F}_p, \mathcal{F}_{pm}$ respectively, are compatible if for all

$A, B \in \mathcal{F}_{mp}$ $\bar{x} \in \mathcal{F}_p$

$\| A\bar{x} \|_c = [a_{ij}] \| \bar{x} \|_c$ and $\| B\bar{x} \|_c = [b_{ij}] \| \bar{x} \|_c$

$\| A\bar{x} + B\bar{x} \|_c = \left[[a_{ij}] + [b_{ij}] \right] \| \bar{x} \|_c$

$= [a_{ij}] \| \bar{x} \|_c + [b_{ij}] \| \bar{x} \|_c$

$= \| A\bar{x} \|_c + \| B\bar{x} \|_c$"

Results and Discussions

Semi-ring of fuzzy matrices with compatible norm 3.1

In this section, we prove that SFM_c in fuzzy algebra as well as form a fuzzy vector space under the addition, which is done component-wise, multiplication that is done component-wise as well as scalar multiplication $(+, \odot)$is associative and distributive in $\mathcal{F}_{nn}$. Also, by using the definition of comparability of SFM_c some properties are proved.

Theorem 3.1.1

The set SFM_c is fuzzy algebra under component-wise addition along with multiplication operation $(+, \odot)$ defined "as follows if A, B, C are SFM_C

- A+B ={max$(a_{ij}, b_{ij}$)}
- A $\odot$ B ={min$(a_{ij}, b_{ij}$)}

Proof

A" semi-ring (S,+,$\odot$) set S equipped with the 2 "binary operations (+ , $\odot$) and called addition and multiplication. Also,$||A + O||_C = ||A||_C$and $||A \odot J||_C = ||A||_C$ for all $\mathcal{F}_{nn}$, hence the zero matrix *O* is the additive identity and also the universal matrix *J* is the multiplicative identity. Like this identity element relative to the operations $+$ *and* $\odot$ exist.

Also $||A + J||_c = ||J||_C$ and $||A \odot O||_C = ||O||_C$ Hence universal bond exists for all $A \in \mathcal{F}_{nn}$

If $||A||_C = [a_{ij}]_{n\times n}$, $||B||_C = [b_{ij}]_{n\times n}$ and $||C||_C = [c_{ij}]_{n\times n}$ over $\mathcal{F}_{nn}$

(S,+) is a commutative monoid with identity element ***O***

- $||A + (B + C)||_C = ||(A + B) + C||_C$ (Associativity)
- $||O + A||_C = ||A||_C = ||A + O||_C$ (Additive identity)
- $||A + B||_C = ||B + A||_C$" (commutative)

(S,$\odot$) is a monoid with identity element *J*.

- $||A \odot B||_C \odot ||C||_C = ||A||_C \odot ||B \odot C||_C$
- $||A \odot J||_C = ||A||_C = ||J \odot A||_C$
- $||A||_C \odot ||A + B||_C = ||A||_C$ and $||A||_C + ||A \odot B||_C = ||A||_C$ (Absorption)

Multiplication left and right distributes over addition.

- $||A \odot (B + C)||_C = ||A||_C \odot ||B + C||_C = ||A \odot B||_C + ||A \odot C||_C$
- $||(A+B) \odot C||_C = ||A + B||_C \odot ||C||_C = ||A \odot C||_C + ||B \odot C||_C$
- $||A + (B \odot C)||_C = ||A + B||_C \odot ||A + C||_C$

If A, B are SFM_c over $\mathcal{F}_{nn}$ and any scalar α, β in [0,1] we have

- $)\alpha||A + B||_C = \alpha J \odot ||A + B||_C$
$= ||\alpha(J \odot A)||_C + ||\alpha(J \odot B)||_C$
$= ||\alpha A||_C + ||\alpha B||_C$
- $(\alpha + \beta)\ ||A||_C = (\alpha + \beta)\ ||J \odot A||_C$
$= ||(\alpha J + \beta J)||_C \odot ||A||_C$
$= ||\alpha(J \odot A)||_C + ||\beta(J \odot A)||_C$
$= ||\alpha A||_C + ||\beta A||_C$

Remark 3.1.2

If SFM_c is a commutative $semi - ring$ with identity O and J

Definition 3.1.3

Comparable $\boldsymbol{SFM_c}$

Let $A = [a_{ij}]$ *and* $B = [b_{ij}]$ FM_c *over* $\mathcal{F}_{nn}$ A is "defined greater than B it $||B||_c \leq ||A||_c$ B is greater than A if $||A||_c \leq ||B||_c$ A *and* B FM_c are said to comparable it either

$||A||_c \leq ||B||_c$ *or* $||B||_c \leq ||A||_c$

Theorem 3.1.4

Let A, B FM_c over $\mathcal{F}_{nn}$. then $||A||_c \leq ||B||_c \Leftrightarrow ||A + B||_c = ||B||_c$

Proof

$||A||_c \leq ||B||_c$ then $||A||_c + ||B||_c \max\{a_{ij}, b_{ij}\} = [b_{ij}] = ||B||_c$

Conversely, if $||A + B||_c = ||B||_c$ then $a_{ij} \leq b_{ij}$ that is $||A||_c \leq ||B||_c$ thus

$||A||_c \leq ||B||_c \Leftrightarrow ||A + B||_c = ||B||_C$

Theorem 3.1.5

Let A,B be FM_c over $\mathcal{F}_{nn}$ if $||A||_c \leq ||B||_c$ then for any $C \in FM_c$ $||AC||_c \leq ||BC||_c$ and for any $D \in FM_c$ $||DA||_c \leq ||DB||_c$

Proof

If $||A||_c \leq ||B||_c FM_c$ for C is FM_c then $||AC||_c \leq ||BC||_c$ $A = (a_{ij}), B = [b_{ij}], C = [c_{ij}]$ thus $||AC||_C \leq ||BC||_c$.

$||DA||_c \leq ||DB||_c$ can be proved in this same manner.

Conclusion

This paper examines several properties of fuzzy matrices with compatible norm (SFM_c) are discussed with the examples. The concept and some properties of semi-ring fuzzy matrices are also discussed. We anticipate that the findings presented in this paper will have a profound impact on other interconnected domains, generating novel and ground-breaking outcomes.

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