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RESEARCH ARTICLE

Coefficient of absorption cross-section of RN black holes

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Abstract

This study investigates the absorption cross-section of Reissner Nordstrom (RN) black holes at high energies.It also examines the graphical relationship between absorption cross-section, mass and frequency and compares them to those of the Schwarzschild black holes. Some standard approximations like Regge pole technique, Eikonal approximation and Sinc approximation have been used in this study. Using these approximations a relation has been established between shadow radius and absorption cross-section of black holes. The graphical analysis in this study was performed with Origin software. An oscillatory pattern was found in the absorption cross-section of the Reissner-Nordstrom metric for charges Q = 0 to 1.0. The graphical dependence of the total absorption cross-section on frequency indicates an inverse relationship between the charge and total absorption cross-section. For the small value of shadow radii the high energy absorption cross-section of Reissner-Nordstrom (RN) black holes,as approximated by the sinc function, exhibits a slow increase with fluctuations(shadow radius, $R_S < 6$). However, for large value of shadow radii of RN black holes absorption cross-section rises in an exponential manner($R_S < 120$). The total absorption cross-section and mass variables exhibit an exponential correlation with respect to Schwarzschild and RN black holes. The connection between the sinc approximation and the absorption cross-section of Reissner Nordstrom black holes is calculated, which is crucial for unraveling the physics of black holes.A comparative graphical analysis of absorption cross-section of different types of black holes offers a visual understanding of their similarities and differences.

Keywords: Sinc Approximation , Absorption cross-section, Regge Pole technique, Black hole shadow, Black holes space time, Eikonal limit, Reissner Nordstrom Metric.

Introduction

The study of block holes has been very active topic of field of research area after the existence of such objects, which predicts from Einstein's general theory of relativity. The first physicist who studied the shadow of black holes was Bardeen who established that shadows cast by spherically symmetric black holes are perfectly circular where as those cast by spinning black holes are distorted and deformed. The Event Horizon Telescope (EHT) captured the first image of a black hole at the centre of a Messier 87 (M87*) galaxy in 2019. This observation opens a new window in understanding of black holes. The study of absorption cross-section of black holes gives us a better understanding of space-time properties.

Regge pole approximation is useful to describe the the properties of the waves trapped near the photon sphere for Schwarzschild black hole metric. Approximation methods are mathematical tools of the evaluation of black holes Physics (Decanini *et al.*, 2011). Sinc approximation plays a crucial role in both mechanics and quantum mechanics, with significant geometrical implications. The absorption cross-section is found to be dependent on the sinc function and shadow radius of black holes (Gearhart & Shultz 1977). The WKB method used to determine the partial absorption cross-section of charged AdS black holes. The metric of charged AdS black holes is different from RN black holes (Chen *et al.* 2024). Here we have used the traditional metric of RN black holes. In linear electrodynamics,the absorption cross-section of RN black holes was calculated,however, the effect of the shadow radius was not considered (de Paula *et al.* 2024). Most of the existing papers do not provide any relation between shadow radius and total absorption cross-section. A preprint by most existing studies have concentrated on Schwarzschild black holes,with only a limited number of studies examining Reissner Nordstrom (RN) black holes.

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The partial wave method is also used to calculate geometric absorption cross-section of RN black holes (Paula *et al.* 2020). Born approximation and partial wave analysis do not give exact results. Therefore,we propose to apply sinc approximation, eikonal and regge pole approximation to study the high energy cross-section with the shadow radius of Ressiner Nordstorm (RN) black holes.We have obtained better results using these approximation. A Comparative study between absorption cross-section of Schwarzschild and RN black holes has not be done as yet. Comparing the absorption cross-section of different type of black holes, we can gain a deeper understanding of similarities and differences between the various black holes. However, In this study we employed an approximation method to calculate the absorption cross-section in terms of shadow radius.

This paper is organized as follows: in section 2 we have described some approximation method and with the help of this we have evaluated absorption cross-section for high energy and absorption coefficient for different mass and frequency. Section 3 is dedicated to results and discussion. Section 4 is dedicated to conclusion of this work. The last sections 5 and 6 provide acknowledgement and references of this paper.

Methodology

Regge pole technique, Eikonal approximation and Sinc approximation have been used to find the relation between shadow radius and absorption cross-section of black holes. The absorption cross-section was calculated at high frequency (Anacleto *et al.*, 2020 & Sanchez, 1978). At high frquencies, the absorption cross-section fluctuates around a limiting values. The graphical analysis presented in this study was generated using Origin software.

Some Standard Approximation Method

Sinc approximation

This approximation is a mathematical tool to derive absorption cross-section of black holes. This method is highly efficient numerical method. This approximation has extensive applications in numerous area of mathematical physics like numerical analysis, solution of integral, ordinary differential and partial differential equations. This approximation is commonly referred to as the sampling function, a crucial concept that plays a significant role in signal processing and Fourier analysis. The full name of the function is “sine cardinal” but it is commonly referred by its abbreviation “sinc”. It is represented by Stenger, (1993, 2000)

$$\text{sinc}(x) = (\sin x)/x \text{------}(2.1)$$

For this approximation we considered a function f whose values are a discrete set of points that lie on the real axis. So the set of values is represented by nh, where n=0,±1,±2,………for h>0. (Whittaker 1915)

Then we can be generated a new function C(x) where C(x) is given by.

$$C(x) = \sum_{-\infty}^{\infty} f(x) \sin \pi\left(\frac{x}{h} - n\right) \text{-----------}(2.2)$$

The above series is convergent. The expression represents the expansion of sinc function for ‘f’. The root of this function is multiple of π and Sinc(0)=1

Eikonal approximation

The eikonal approximation was first introduced in quantum scattering theory by Moliere and later developed by Glabuer. This method is applicable when de Broglie wavelength of the incident particle is much shorter than the range of potential. This method applies under two conditions: the short wave length condition and the potential V(r) must be smooth and relatively weak,with values much smaller than the energy E everywhere. This approximation method is an extension of Born approximation, providing very accurate results for scattering at small angle. The eikonal scattering wave function is obtained by linearizing the Green’s function in momentum space, as derived from the Lippmann-Schwinger equation (Singh & Sammy, 2022). The general condition in this case is ka >>1 (the short wave length condition) and the potential V (r) varies smoothly and is small everywhere in comparison with the energy E. The eikonal scattering wave function is derived by a linearization the momentum space Green’s function, which is obtained from the Lippmann-Schwinger equation . Therefore, we observe that this approach is an extension of the Born approximation method, addressing and resolving the convergence issues associated with the Born series. We may get a relation between the successive terms of the Born and Eikonal scattering series. However the terms of the eikonal scattering series are alternately real and imaginary and it is superior to Born approximation in the sense that it does not limit the value of the parameter which must be small compared to unity in order that the Born approximation is to be valid. Even for strong coupling situation, for which the Born series is useless, the eikonal approximation remains very accurate at small angles provided that ka >>1.

Regge poleapproximation

Regge-pole approximation has proved to be practical for describing elastic atom-atom scattering. This approximation used for derive relation between complex angular momentum and energy. Tullio Regge found that the angular momentum in the Schrodinger wave equation is not restricted to real values, but can also be complex. The Regge trajectory is characterized by a mathematical function that establishes a relationship between angular momentum of aparticle or system and its energy. It can be characterized by the the asymptotic behavior of the S matrix (Singh & Sammy, 2022).

This approximation is also applied to dirty black holes (A black hole surrounded by a thin shell of matter) and used to evaluate scalar cross-section for various shell configurations (Torres *et al.* 2023).

Absorption Cross-section of Reissner Nordstorm (RN) Black Holes

Now we will establish the relation between the absorption cross-section and the shadow radius of black holes.For this first of all we take the metric of Ressiner black hole is (Chandrasekhar, 1983)

$$ds^2 = -\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)dt^2 + \left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}dr^2 + r^2(d\theta^2+\sin^2\theta d\phi^2) \text{--(2.2.1)}$$

Where M=Mass of black hole , Q=Charge and $f(r) = \left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)$

To calculate the photon sphere of RN black holes (r_c) we use the condition (Hawking, 1971)

$$\frac{f'(r_c)}{f(r_c)} = \frac{2}{r_c} \text{ - - - - - - - - - - (2.2.2)}$$ where f(r) = metric of black holes

Putting the value of $f(r) = \left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)$ we get expression for r_c

$$r_c = \frac{1}{2}\sqrt{9M^2-8Q^2} + \frac{3M}{2} \text{ - - - - - - (2.2.3)}$$

To calculate the radius of black hole shadow at the point r_0 we used given relation (Decanini *et al.*, 2011**)**

$$R_s = r_c\sqrt{\frac{f(r_0)}{f(r_c)}} \text{ - - - - - - - - - - - (2.2.4)}$$

For large distance we take f(r_0) = 1 above equation reduce to

$$R_s = \frac{r_c}{\sqrt{f(r_c)}} \text{ - - - - (2.2.5)}$$

Using equation (5) and metric f(r) we get shadow radius of Reissner Nordstorm black holes

$$R_s = \frac{\frac{1}{2}\left(\sqrt{9M^2-8Q^2}+3M\right)}{\sqrt{1-\frac{4M}{\sqrt{9M^2-8Q^2}+3M}+\frac{8Q^2}{\left(\sqrt{9M^2-8Q^2}+3M\right)^2}}} \text{ - - - - (2.2.6)}$$

Now we evaluate absorption cross-section for a Ressiner Nordstorm black hoke. High energy absorption cross-section of a black hole spacetime is written as (Higuchi, 2001, Konoplya, 2020 & Okyay & Ovgun, 2021)

$$\sigma_{abs}(\omega) \approx \sigma_{lim} + \sigma_{osc} \text{-----------------------(2.2.7)}$$

$$\sigma_{lim} = \sigma_{geo} = \frac{\pi {b_c}^2}{\Gamma 2} \text{------------(2.2.8)}$$

$$\sigma_{abs}{}^{osc} = -8\pi\eta_c e^{-\pi\eta_c} sinc(\omega T_c)\sigma_{abs}{}^{geo} \text{----------(2.2.9)}$$

Where as

σ_{geo} =Geometrical absorption cross-section

σ_{osc} = Oscillatory absorption cross-section

$sinc = \frac{sinx}{x}$ and orbital period $T_c = 2\pi b_c$

$b_c = R_s = \frac{r_c}{\sqrt{f(r_c)}}$ and $\eta_c = \sqrt{f(r_c) - \frac{1}{2}{r_c}^2 f(r_c)''}$

Using equation (3.8),(3.9) and above parameters we get the final expression of total absorption cross-section for Ressiner Nordstorm black hole

$$\sigma_{abs}(\omega) = -8\pi\eta_c e^{-\pi\eta_c} sinc(\omega 2\pi R_s)\pi {R_s}^2 + \pi {R_s}^2 \text{----------(2.2.10)}$$

Results and Discussion

In this paper we have studied the absorption cross-section of RN black holes with standard metric. We have applied the approximation method to calculate absorption cross-section of RN black holes. This section highlights a selection of key results.

We have established the relation between absorption cross-section and shadow radius of RN black hole starting with the black hole metric and using large distance approximation. Recent studies (de Paula *et al.* 2024 & Paula *et al.* 2020) have investigated the metric of Ayon-Beato and Gracia (ABG) black holes and compare their results with experimental observations of black holes M87* and Sgr A*.This analysis revelaed a strong connection between absorption cross-section and size of black hole's shadow.In equation (2.2.10), we have explicity derived the correlation between shadow radius and total absorption cross-section of RN black hole metric. We have obtained graphs between Oscillatory absorption cross-section () and total absorption cross-section with frequency ω for charge Q ranging from Q = 0.0 to Q = 1.0 Table 1 and 2 give the value of oscillatory absorption cross-section and total absorption cross-section respectively. Figures 1 and 2 are plots between oscillatory absorption cross-section with frquency. Figure 1 shows the graphical relation of absorption cross-section of Schwatrzschild and RN black holes for charge Q=0.5 and Q=1.0.In Figure 2 we have compared the oscillatory absorption cross-section for charges in the range 0 to 1.From these two graphs we can conclude that the oscillatory pattern of absorption cross-section fluctuate within the short range of frequency (ω=1 to ω=25).For higher frquency its fluctuation is diminished. Figure 3 displays the frequency dependence of the total absorption cross-section for Schwarzschild and Reissner black holes.From this figure we can say that the charge is inversely proportional to absorption cross-section. A similar plot and comparable results were obtained but no calculations were performed for oscillatory absorption cross-section (Övgün 2021). Figure 3 is also plotted for Q = 0.5. While previous work only calculated total absorption cross-section for lower frequency ranges (ω = 0.5–3.0) (Övgün 2021). Our study extends this to evaluate total absorption cross-section across a wide range of higher frequencies (ω=0.1 to 6.0) [Table 2]. After plotting this data with respect to frequency we have found absorption cross-section does not fluctuate with higher frquency. A recent study present similar graphical representation of the relationship between normalized absorption cross-section and frequency for a new class of regular black holes, with critical charges ranging from 0.1 to 0.2 (Karmakar 2024). However,we have derived this evaluation and tabulated the resulting value of oscillatory and total absorption cross-section in Tables 1 and 2.

Using sinc approximation we have also plotted graph between and R_s for ω = 0.1 to ω = 1.0 (Figure 4 and 5). For small value of shadow radius of RN black hole has high energy absorption cross-section (σ_{abs}) in sinc approximation increases slowly by fluctuations($R_S$upto 6). However, for large value of shadow radius of RN black hole it rises in exponential manner (R_S< 120). A similar plot depicted for R_S<50 (Övgün, 2021). Similarly, total absorption cross-section of RN black holes shows an exponential variation with mass , as depicted in Figure 6 (M<25). Figure 7 presents a comparative anlysis of how the total absorption cross-section of Schwarzschild (Q=0.0) and RN black holes (Q = 1.0) varies with mass. The comparison of absorption cross-section and mass for RN and Schwarzschild black holes shows a modest effect of charge.

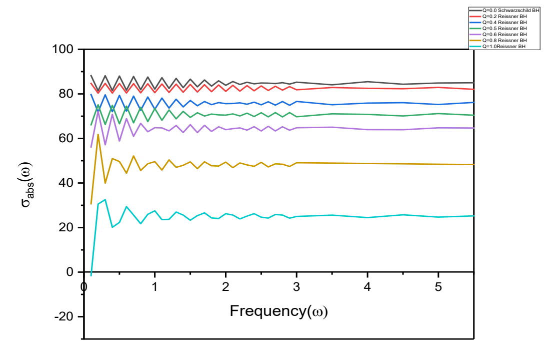

Figure 3: Graph between total absorption cross-section and frequency (ω)

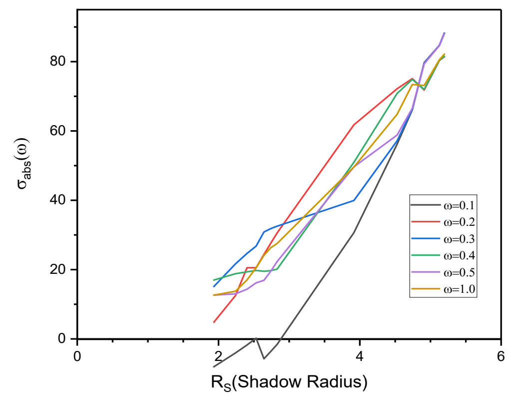

Figure 4: Graph between total absorption cross-section shadow radius

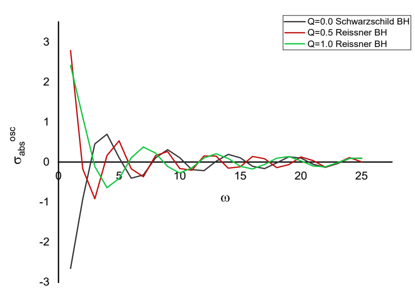

Figure 1: Graph between oscillatory absorption cross-section and frequency

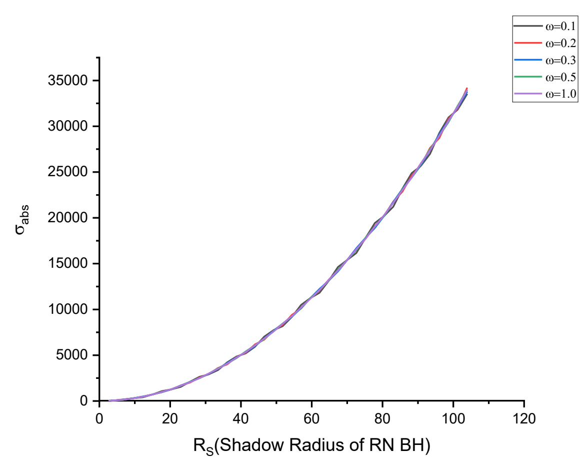

Figure 5: Graph between total absorption cross-section and shadow radius

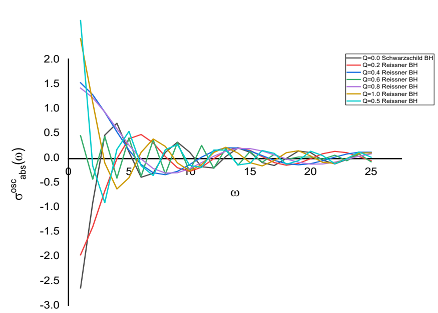

Figure 2: Graph between oscillatory absorption cross-section and frequency

Table 1: Oscillatory absorption cross-section

Frequency (ω)	*Schwarzschild BH Q = 0.0*	*Reissner BH Q = 0.2*	*Reissner BH Q = 0.4*	*Reissner BH Q = 0.5*	*Reissner BH Q = 0.6*	*Reissner BH Q = 0.8*	*Reissner BH Q = 1.0*
1	-2.64877	-1.97631	1.52057	2.78477	0.44561	1.41355	2.41441
2	-0.92039	-1.40795	1.27654	-0.16743	-0.43986	1.21699	1.12301
3	0.4565	-0.67862	0.92204	-0.91483	0.43038	0.92583	-0.10834
4	0.69813	-0.02122	0.52283	0.16622	-0.41731	0.58714	-0.6371
5	0.11424	0.38299	0.14906	0.53291	0.40085	0.25329	-0.40913
6	-0.39926	0.46889	-0.14	-0.16422	-0.38125	-0.02797	0.10757
7	-0.31943	0.29908	-0.30795	-0.36372	0.35883	-0.22221	0.37801
8	0.1052	0.02121	-0.34742	0.16143	-0.3339	-0.31381	0.22701
9	0.31344	-0.20576	-0.27901	0.26564	0.30686	-0.30748	-0.10629
10	0.11188	-0.28082	-0.14367	-0.15789	-0.2781	-0.22546	-0.2706
11	-0.18576	-0.1954	0.00898	-0.20008	0.24804	-0.10134	-0.14188
12	-0.21157	-0.02119	0.13354	0.15363	-0.21713	0.02792	0.10452
13	0.02146	0.13747	0.19938	0.15225	0.18579	0.13013	0.2098
14	0.1952	0.20004	0.19639	-0.14869	-0.15449	0.18413	0.09164
15	0.10801	0.14688	0.13496	-0.11526	0.12363	0.18314	-0.10226
16	-0.10042	0.02117	0.0406	0.14309	-0.09364	0.13452	-0.16937
17	-0.16099	-0.10122	-0.05492	0.08551	0.06491	0.05641	-0.05806
18	-0.0164	-0.15502	-0.12318	-0.1369	-0.03779	-0.02784	0.09954
19	0.13325	-0.11869	-0.1468	-0.06091	0.01259	-0.09588	0.13967
20	0.10273	-0.02114	-0.12329	0.13017	0.01039	-0.13179	0.03385
21	-0.05256	0.0787	-0.06434	0.0402	-0.03093	-0.12937	-0.09638
22	-0.12826	0.12626	0.00897	-0.12295	0.04884	-0.09283	-0.11635
23	-0.03727	0.10021	0.07315	-0.02256	-0.06398	-0.03477	-0.01553
24	0.09275	0.0211	0.10948	0.11531	0.07628	0.02772	0.09281
25	0.09617	-0.06333	0.10917	0.00745	-0.08571	0.07781	0.09717

Table 2: Total absorption cross-section

Frequency (ω)	*Schwarzschild BH Q = 0.0*	*Reissner BH Q = 0.2*	*Reissner BH Q = 0.4*	*Reissner BH Q = 0.5*	*Reissner BH Q = 0.6*	*Reissner BH Q = 0.8*	*Reissner BH Q = 1.0*
0.1	88.20624	84.70756	79.69827	66.1026	56.20267	30.66885	-1.58035
0.2	81.37906	80.33731	71.81226	75.10234	72.18815	61.77388	30.56734
0.3	88.13903	84.68994	79.57718	66.26185	57.14255	39.93766	32.53841
0.4	81.47912	80.36366	71.99181	74.86665	70.84049	50.97787	20.12622
0.5	88.00699	84.65496	79.34165	66.57026	58.82777	49.64331	22.29914
0.6	81.64198	80.40716	72.28009	74.49039	68.90435	44.41474	29.40845
0.7	87.81476	84.60312	79.00456	67.00844	60.91715	52.14056	25.63516
0.8	81.8619	80.46716	72.66141	73.99713	66.76447	45.61543	21.71984
0.9	87.56906	84.53516	78.58416	67.54914	63.00593	48.63323	25.95265
1	82.13123	80.54282	73.11525	73.4173	64.82122	49.59145	27.53446
1.1	87.27842	84.45208	78.1029	68.15928	64.72178	45.80965	23.54324
1.2	82.44068	80.63303	73.61759	72.78601	63.39761	50.37139	23.69781
1.3	86.95281	84.35505	77.586	68.80238	65.80862	47.0373	26.98995
1.4	82.77971	80.73652	74.14245	72.14043	62.66995	47.94261	25.63064
1.5	86.60321	84.24547	77.05978	69.44127	66.17717	49.50746	23.30502

1.6	83.13698	80.85181	74.66354	71.5171	62.6391	46.44799	25.36759
1.7	86.2412	84.12488	76.55009	70.04068	65.91058	49.53332	26.62648
1.8	83.5008	80.97728	75.15586	70.94936	63.14764	47.72755	24.31949
1.9	85.87847	83.99497	76.0807	70.56974	65.2263	47.62775	24.08165
2	83.85961	81.11118	75.59725	70.4651	63.93575	49.39489	26.24306
2.1	85.52634	83.85755	75.67185	71.00402	64.40682	46.87564	25.62315
2.2	84.20241	81.25164	75.96965	70.08494	64.71853	49.00206	23.90318
2.3	85.19535	83.71452	75.33917	71.32701	63.72003	48.14263	25.13563
2.4	84.51924	81.39676	76.26006	69.82107	65.26239	47.47491	26.22942
2.5	84.89484	83.56781	75.09286	71.53093	63.3512	49.25843	24.67117
2.6	84.80149	81.54458	76.46118	69.6768	65.44018	47.21326	24.27213
2.7	84.63258	83.41938	74.93728	71.61692	63.36426	48.62003	25.86885
2.8	85.04225	81.69313	76.57158	69.64672	65.25153	48.41153	25.61279
2.9	84.41458	83.27117	74.87098	71.59439	63.70037	47.41595	24.22877
3	85.2365	81.84051	76.59551	69.7177	64.80599	49.10373	25.01171
3.5	84.05563	82.84655	75.11637	71.06591	65.04807	48.93706	25.59965
4	85.47813	82.49791	75.86689	70.79875	63.9583	48.76504	24.48295
4.5	84.32466	82.29937	76.03825	70.09289	63.91218	48.59431	25.71644
5	84.89424	82.90212	75.22569	71.16544	64.77646	48.43119	24.71092
5.5	84.98205	82.05459	76.15031	70.40984	64.69624	48.28144	25.26877
6	84.38074	82.98802	75.59917	70.46831	63.99436	48.14993	25.22762

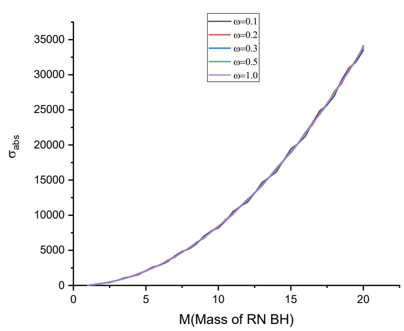

Figure 6: Graph between total absorption cross-section and mass of black holes

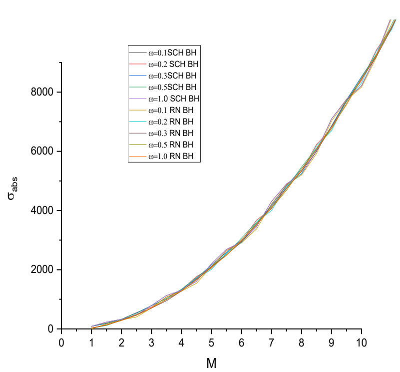

Figure 7: A comparative plot of the total absorption cross-section versus mass is shown for RN and Schwarzschild black holes

In these Figures 5,6,7 we can see that the total absorption cross-section of Schwarzschild and RN black holes show exponential behaviour with frequency. So, we can say that total absorption cross-section of these black holes show that similar pattern with mass and shadow radius of these black holes.

Utilizing an approximation method, we derive a more accurate value for the absorption cross-section,which exhibits a distinct dependence on the radius and mass of the black holes.

Conclusion

This study has calculated absorption coefficient at high energy limit. Our solution, using the help of sinc approximation gives a simple relation between total absorption cross-section and mass.It also drawn some graph of absorption cross-section with different mass and frequency. The graph of total absorption cross-section *versus* frequency shows that the absorption cross-section decreases as the charge increases. Conclusively, the high energy absorption cross-section using sinc approximation is related to the radius of RN black hole which could give an insight to further research in black hole Physics. The relationship between the total absorption cross-section and mass of Schwarzschild and RN black holes are related exponentially. A comparative analysis between Schwarzschild and RN black holes reveals that charge has an insignificant effect on their absorption cross-section. These findings provide valuable insights for further investigation into the absorption cross-section of other black holes metrics.

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References

Anacleto, M. A., Brito, F. A., Campos, J. A. V., & Passos, E. (2020). Quantum-corrected scattering and absorption of a Schwarzschild black hole with GUP. Physics Letters B, 135830.

Chandrasekhar, S. (1983). The mathematical theory of black holes. Clarendon Press.

Chen, H., Dong, S.-H., Hassanabadi, S. H., Heidari, N., & Hassanabadi, H. (2024). Quasi-normal modes, emission rate, and shadow of charged AdS black holes with perfect fluid dark matter. Chinese Physics C, 48.

de Paula, M. A. A., Leite, L. C. S., Dolan, S. R., & Crispino, L. C. B. (2024). Absorption and unbounded superradiance in a static regular black hole spacetime. Physical Review D, 109, 064053.

Decanini, Y., Esposito, G., & Folacci, A. (2011). Universality of high energy absorption cross-sections for black holes. Physical Review D, 83(4), 044032.

Gearhart, W. B., & Shultz, H. S. (n.d.). The function sinx/x.

Higuchi, A. (2001). Low frequency scalar absorption cross-sections for stationary black holes. Classical and Quantum Gravity, 18(20), L139-L142.

Hawking, S. (1971). Gravitational radiation from colliding black holes. Physical Review Letters, 26,

Karmakar, R. (2024). A comparative study of the absorption cross-section of static regular black holes for electromagnetic field. arXiv:2408.05310v1 [gr-qc].

Konoplya, R. A. (2020). Quantum corrected black holes: Quasinormal modes, scattering, shadows. Physics Letters B, 804, 135363.

Okyay, M., & Övgün, A. (2021). Nonlinear electrodynamics effects on the black hole shadow, detection angle, quasinormal modes and grey body factors. arXiv:2108.07766v2 [gr-qc].

Övgün, A. (2021). Connection between Sinc Approximation for High-Energy Absorption Cross-section and Shadow of Black Holes. Preprints, 2021080390.

Paula, M. A. A., Leite, L. C. S., & Crispino, L. C. B. (2020). Electrically charged black holes in linear and nonlinear electrodynamics: Geodesic analysis and scalar absorption. Physical Review D, 102(10), 104033.

Sanchez, N. G. (1978). Absorption and Emission Spectra of a Schwarzschild Black Hole. Physical Review D, 18(4), 1030.

Singh, S., & Sammy, K. (2022). Coefficient of Absorption Cross-section and Sinc Approximation in Black Holes. International Journal of Advances in Science, Engineering and Technology, 10(3), 201-205. IJASEAT-IRAJ-DOIONLINE-19168

Stenger, F. (1993). Numerical Methods Based on Sinc and Analytic Function. Springer.

Stenger, F. (2000). Summary of Sinc numerical methods. Journal of Computational and Applied Mathematics, 121, 379-420.

Torres, T., El Hadj, M. O., Hu, S.-Q., & Gregory, R. (2023). Regge pole description of scattering by dirty black holes. Physical Review D, 107, 064028.

Whittaker, E. T. (1915). On the functions which are represented by the expansions of the interpolation theory. Proceedings of the Royal Society of Edinburgh, 35, 181-194.