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RESEARCH ARTICLE

Analysis of distributions using stochastic models with fuzzy random variables

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Abstract

The significance of this paper lies in the investigation of a novel method for comparing the expectations of stochastic models in fuzzy settings. In order to comprehend actuarial science and economical modeling, stochastic models are necessary. The primary benefit of the paper is to comprehend the novel ideas of stochastic comparison of stochastic models built on the exponential order. We applied the fuzzy mean inactivity time order definition, solved the preservation properties and theorem, and created a new definition. Applications involving stochastic models are presented.

Keywords: Fuzzy set, Random variables, Stochastic orders, Mean residual life, Hazard rate.

Introduction

Numerous applications in applied probability, statistics, dependability, operation research, economics, and allied domains have demonstrated the value of stochastic ordering. Over the years, a variety of stochastic ordering and related features have been quickly produced. Let *X* be a positive random variable that represents a system's lifespan with density function *f*, distribution coefficient *F*, and survival parameter $\bar{F}$= 1-F. Given that the system has already survived up to t, its residual life after t is shown by the conditional random variable $X_t = (X_t, X > t), t \geq 0$. The anticipated value of X_t is the mean leftover life (function of and can be obtained by,

$$\mu_X(t) = \begin{cases} \int_t^{\infty} (\frac{\bar{F}(x)}{\bar{F}(t)}) ds, & t > 0 \\ 0, & t \leq 0. \end{cases}$$

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In several disciplines, namely survival analysis, actuarial research, and reliability engineering, the MRL function is a significant feature. It has received a great deal of attention in the literature, particularly when it comes to binary systems those in which there are just two conceivable states: successful or unsuccessful. The hazard rate (HR) function of *X,* which is provided by, is an additional helpful reliability metric.

$$r_X(t) = \frac{f(x)}{\bar{F}(x)}, \quad t \geq 0.$$

The function is very helpful in characterizing how the probability of witnessing the event varies over time and in identifying the proper failure distributions using qualitative data regarding the failure mechanism. The MRL function has been shown to be more effective in replacement and repair procedures, even though the shape of the HR function is still significant. The HR function only accounts for any potential of a sudden failure at any given time. Stochastic comparisons of residual lives and inactive periods at quantiles can be used to distinguish a family of stochastic orderings known as transform stochastic orderings in the literature (Arriaza M.A., *et al*., 2017). Following that was a brand-new stochastic order known as the star order, which falls in between the convex order and the other two transform orderings. A novel idea for the comprehensive and straightforward characterization of situations where one Beta distribution is smaller than another based on the convex transform order has been proposed (Arab, I., *et al*., 2021). They derive monotonicity properties for the probability of a random variable that is beta distributed and exceeds its distribution's

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mean or mode as an application. Stochastic comparisons of vectors with a multivariate skew-normal distribution are made (Arevalillo, J.M. and Navarro,H.A., 2019). The novel ordering is based on the canonical transformation linked with the multivariate skew-normal distribution and the well-known convex transform order applied to the single skewed component of that canonical transformation. Three functional measures of the shape of univariate distributions are proposed (Arriaza, A., *et al*., 2019). These metrics are appropriate with respect to the convex transform order. To close a gap in the literature (Belzunce F., *et al*., 2022). concentrate on giving sufficient conditions for a few well-known stochastic orders in dependability while handling their discrete forms. In particular, based on the likelihood ratio's unimodality, they found comparison criteria for two discrete random variables in specific stochastic orders. The mean residual life, the bending property of the failure rate, the reversed hazard rate, and the mean inactive duration in mixtures have all been explored (Badia, F.G. and Cha, J.H., 2017). The idea of relative spacings was first developed (Belz unce, F., *et al*., 2017). They demonstrate the relevance of this idea in several situations, such as economy and reliability, and we offer various results for evaluating relative spacings among two populations. Numerous shifting and proportional stochastic orders have been used (Belzunce, F., Ruiz, J.M., and Ruiz, M.C., 2002). To compare certain coherent structures that were formed from a set of components or from two sets of components. A new type of stochastic order has been proposed and explored (Izadkhah, S. and Kayid, M., 2013). Several fundamental and afterward fundamental preservation properties of the new stochastic order under convolution, mixture, and shock model reliability procedures are investigated. A thorough overview of the theory and applications of aging and reliance on the application of mathematical techniques to survival and reliability studies is provided (Lai, C.D. and Xie, M., 2006). The study of getting older properties of residual lifetime mixture models and stochastic comparisons has been enhanced (Patra, A. and Kundu, C., 2021). They performed stochastic comparisons of two distinguish mixture models under the likelihood ratio, hazard rate, mean residual life and variance residual life orders employing two different mixing distributions and two different baseline distributions. Recent discussions regarding the stochastic comparison and aging properties of RLRT (ITRT) based on variance residual life have led to new findings regarding the stochastic aging qualities, as noted (Patra, A. and Kundu, C., 2022). Sufficient standards for the residual life and inactive time's log-concavity and log-convexity have been given (Misra N., *et al*., 2008). In addition, we do stochastic comparisons between the inactivity time and residual life in terms of the typical stochastic order, the mean residual life order, and the failure rate order. A well-known MRL order has been introduced and examined in the literature based on the MRL function (Nanda, K., *et al*., 2010). Numerous writers have studied the MRL order's uses in survival and reliability analysis throughout the years (see Shaked, M. and Shanthikumar, J.G., 2007), (Muller, K. and Stoyan, K., 2022). However, several existing concepts of stochastic comparisons of random variables are thought to be generalized by the proportional stochastic order, according to the literature. Proportional stochastic orders have been explored by numerous researchers as enlarged versions of the prominent stochastic orders prevalent in the literature right now (Ramos-Romero, H.M. and Sordo-D´ıaz, M.A., 2001). Nanda *et al*. examined reliability models adopting the MRL function and conducted a fresh analysis on various partial ordering effects relevant to the MRL order (Nanda, K., Bhattacharjee, S., *et al*., (2010)). The probability distribution of potential outcomes is its primary concern. Examples include (Shaked, M. and Shanthikumar, J.G., 2007)., Markov models and regression models. The modal functions as a realistic case simulation to gain a deeper understanding of the system, investigate unpredictability, and evaluate uncertain scenarios that delineate all possible outcomes and the trajectory of the system's evolution. Thus, in order to optimize profitability, experts and investors can develop their business practices and make better management decisions with the aid of this modeling technique.

An Introduction to stochastic orders discusses this helpful tool, which may be used to assess probabilistic models in a range of domains, including finance, economics, survival analysis, risks associated with stock trading, and reliability. For academics and students wishing to use this data as a tool for their own research, it provides a general foundation on the subject. Along with applications to probabilistic models and discussions of basic properties of many stochastic orders in the univariate and multivariate scenarios, detailed proofs of the principal results in several sectors of interest are provided. In applied probability, stochastic ordering among random variables has shown to be an effective method for comparing system reliability. Stochastic orderings are viewed as a key tool for marketing decision-making in the face of uncertainty. In order to create a mathematical or financial model that can find every possible outcome for a particular circumstance or issue, stochastic modeling uses random input variables.

Fuzziness

There are two typical scenarios in the real world when an observed variable gets fuzzy. In the first scenario, the response variable cannot be measured exactly due to technical measurement conditions. As a result, data cannot be recorded explicitly with precise (non-fuzzy) numbers; instead, it can only be done in linguistic terms to demonstrate the necessary tolerance to errors in measurement. The second scenario involves the response to the variable being given in linguistic forms, such as a farmer's report about his products or an expert's linguistic report, which is not numeric. To analyze the experiment, the data in both scenarios may be represented as a nation of fuzzy sets. Fuzzy sets theory has to be used to model and manage the findings from experiments of many applied fields since the values obtained from experiment outcomes are often fuzzy. Many people have utilized the fuzzy sets theory in a variety of scientific areas since Zadeh (1965) introduced it to the scientific world.

The purpose of this study is to develop hyperbolic stochastic theories of fuzzy random variables.

A fundamental concept of fuzzy sets and stochastic ordering, as well as a definition of fuzzy random variables and fuzzy random vectors, is provided in Section 2, along with a few definitions and equations. Section 3 defines stochastic comparison using the stop-loss premium of the convex order along with the properties of the convex ordering of the set of fuzzy random variables. The convex ordering of the set of fuzzy random variables was graphically depicted in a clear and understandable manner. 3.13. Stochastic model comparisons aid in the process of making investment decisions by forecasting results in unpredictable circumstances, particularly those involving the stock market. It is regarded as an insurance company that, for example, based its price list on the exponential principles of premium computation, using a distribution function in the literature called inactivity time. This section covers the exponential of the inactivity order of a random variable and preservation features under specific dependability operations. With the aid of a theorem and proof, the continuous non-negative fuzzy random variable with probability density function is elaborated. Numerous fields, including agriculture, systems biology, production, weather forecasting, and biochemistry, have benefited from the extensive applications of stochastic models in real life. Ultimately, the question of what practical uses for stochastic ordering under fuzzy random variables there are is resolved.

Preliminaries

Definition

Let χ be a set of all values. Next a fuzzy set A˜ ={(x,μ_A (x))/ xϵχ} of χ is determined by the role it plays in membership $\mu_A : \chi \to [0,1]$.

The α-cut of the set of is indicated by it's for every $(0 \le \alpha \le 1)\ \tilde{A}_\alpha = \{x \in \chi; \mu_{\tilde{A}}(x) \ge \alpha\}$.

- For each $\alpha \epsilon (0,1]$ both $[\chi^U_\alpha, \chi^U_\beta]$ defined as $\chi^L_\alpha(\omega)(x) = inf\{x \in \Omega;\ \chi^L_\alpha(\omega)(x) \ge \alpha\}$ and $\chi^U_\beta = sup\{x \in \Omega;\ \chi^U_\alpha(\omega)(x) \ge \beta\}$ are finite real valued random variables defined on such (Ω,A,P) that the mathematical expectations $E(\chi^L_\alpha)$ *and* $E(\chi^L_\alpha)$ exist.
- For each, $\omega \epsilon\ \Omega$ and, $\alpha \epsilon (0,1]$, $\chi^L_\alpha(\omega)(x) \ge \alpha$ and $\chi^U_\alpha(\omega)(x) \ge \beta$.

If χ˜and ϒ˜ be fuzzy random variables with fuzzy cumulative distribution function F˜and G˜ respectively then

- $\chi \le_{st} \Upsilon \Leftrightarrow \tilde{F}(t) \ge \tilde{G}(t)\ \forall\, t$.
- $\chi \le_{st} \Upsilon \Leftrightarrow \{P(\chi^L_\alpha \ge t) \vee P(\chi^U_\alpha \ge t)\} \le \{P(Y^L_\alpha \ge t) \vee P(Y^U_\alpha \ge t)\}$
- $\chi \le_{st} \Upsilon \Leftrightarrow E[f(\chi^L_\alpha)] \vee E[f(\chi^U_\alpha)] \le E[f(Y^L_\alpha)] \vee E[f(Y^U_\alpha)]$, for all increasing functions *f*.

Stochastic Comparison of the Exponential Convex Orders

Definition

Consider two consecutive sequence set of fuzzy random variables $\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}$ and $\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}$ such that, $e^{t\chi} E[\varphi\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}] \le \boldsymbol{e}^{t\Upsilon} E[\varphi\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}]$, for all convex functions φ, provided expectations exists. Then the sequence $\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}$ is said to be stochastically dominant of $\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}$ in the convex order denoted as $\chi \le_{FCO} Y$. $\chi = [\wedge\&\vee_{\alpha \le \beta \le 1} \{x^L_\alpha, x^U_\beta\}]$ and $Y = [\wedge\&\vee_{\alpha \le \beta \le 1} \{y^L_\alpha, y^U_\beta\}]$.

Properties of Convex Ordering of Set Fuzzy Random Variables

Let $\chi = [\chi^l_\alpha, \chi^u_\alpha]$ and $Y = [\Upsilon^l_\alpha, \Upsilon^u_\alpha]$ be two set of fuzzy random variables. Then the following conditions are satisfied,

- If $\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}$ said to be stochastically dominant of $\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}$ in convex order sense that if $X \le_{FCO} Y$. Then, $E[\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}] \le_{FCO} E[\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}]$ and $Var[\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}] \le_{FCO} Var[\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}]$.
- If $[\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}] \le_{FCO} [\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}]$ and $[\{Z_1, Z_2, Z_3, \dots, Z_n\}]$ is independent of $[\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}]$ and $[\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}]$. Then, $[\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}] + [\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}] \le_{FCO} [\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}] + [\{Z_1, Z_2, Z_3, \dots, Z_n\}]$.
- Let $\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}$ and $\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}$ two set of non negative consecutive fuzzy random variables. Then,
- $\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\} \le_{FCO} \{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\} \Leftrightarrow -\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\} \le_{FCO} -\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}$.
- Let χ and ϒ two set of non negative consecutive fuzzy random variables such that $E(\chi) = E(\Upsilon)$. Then $\chi \le_{FCO} \Upsilon$, If and only if $|E\chi - \delta| \le_{FCO} |E\Upsilon - \delta|$ for all δ∈∅.
- The convex order closed under mixtures: let χ and ϒ and Z be random variables such that
- $[\chi/Z = \emptyset] \le_{FCO} [\Upsilon/Z = \emptyset]$ for all ∅ in the support of Z. $\chi \le_{FCO} \Upsilon$. Then
- The convex order closed under convolution: let $\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}$ and $\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}$ be set of independent fuzzy random variables. If $\chi_i \le_{FCO} \Upsilon_i$, for $i = 1,2,3, \dots, n$. Then $\sum_j^m \chi_j \le_{FCO} \sum_j^m \Upsilon_j$.
- Let χ_1 *and* Υ_2 be a pair of consecutive independent fuzzy random variables and let Υ_1 *and* Υ_2 be a pair of consecutive independent fuzzy random variables. If $\sum_j^m \chi_j \le_{FCO} \sum_j^m \Upsilon_j$, $j = 1,2$ then, $[\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}, \{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}] \le_{FCO} [\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}, \{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}]$.

A clear illustration of the properties of convex ordering of set fuzzy random variables by graphical representation can be found in the following Figures 1 and 2.

Comparisons of Stochastic Models Fuzzy Random Variables

The integral form has applications in actuarial science, reliability, and economics in numerous stochastic comparison relations. A class F of measurable functions generates a stochastic order relation referred to as an integral stochastic comparison or $\leq_{\mathcal{F}}$. In particular, given two sets of fuzzy random variables, $\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}$ and $\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}$ is said to be stochastically dominance than $\{Y_1, Y_2, Y_3, \dots, Y_n\}$, in the F sense, expressed as

$$E[\varphi\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}] \leq_{\mathcal{F}} E[\varphi\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}] \text{ for all the functions } \varphi \in \mathcal{F},$$

For as long as the presumption in the equation above is met. Marshal and Muller looked at such stochastic evaluations in a fairly broad context. The distributions function $f_{\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}}$ and $f_{\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}}$ corresponding X to and Y are ordered, not the particular configurations of these fuzzy random variables, as should be noted. Here, we revisit the exponential order as one of these analogies.

Let $\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}$ and $\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}$ be two sets of fuzzy random variables with distributions $f_{\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}}$ *and* $f_{\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}}$ and denote their survival functions by, $\bar{F}_{\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}} = (1 - f_{\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}})$ *and* $\bar{F}_{\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}} = (1 - f_{\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}})$ respectively. Their exponential functions are defined as, for all S>0.

$$\varphi_{\{\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}\}}(S) = E[e^{S\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}}] \text{ and } \varphi_{\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}}(S) = E[e^{S\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}}]$$

Let us considerand $\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}$ and $\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}$ be two sets of fuzzy random variables, χ is said to be smaller than Υ in the exponential order denoted as ,

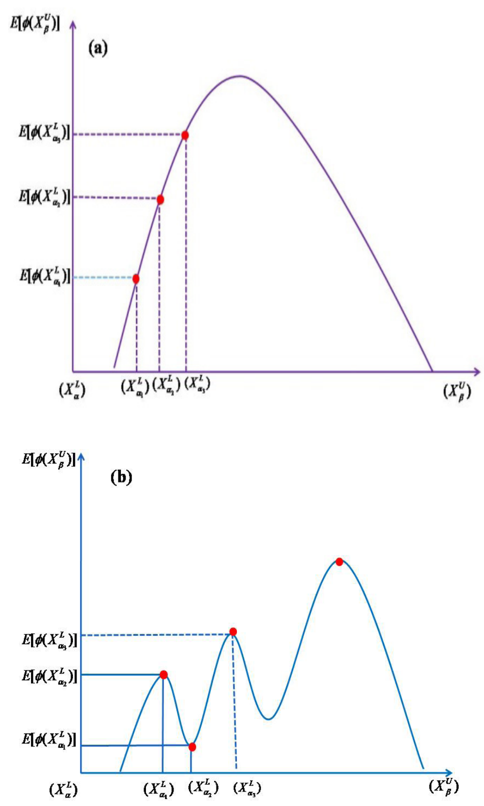

Figure 1: (a)- Convex orderings set of fuzzy random variables (b)- non convex orderings set of fuzzy random variables

$\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\} \leq_{FEO} \{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}$ if $E(e^{t_0 \Upsilon})$ is finite for some $t_0 > 0$, and

$\varphi_{\{\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}\}}(S) \leq \varphi_{\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}}(S)$, for all $S > 0$.

Notice also that,

$$\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\} \leq_{FEO} \{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\} \Leftrightarrow \int_0^\infty e^{SU} \bar{F}_{\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\}}(U) dU \leq_{FEO} \int_0^\infty e^{SU} \bar{F}_{\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\}}(U) dU$$

This is more important in the idea of reliability. From a probabilistic standpoint, the exponential order mandates that the moment-generating functions of the non-negative random factors X and Y be laid out in chronological order. Additionally, the exponential order communicates the collective preferences of all decision-makers via utility features from the $\delta(\chi) = 1 - e^{-S\chi}$. Exponential orders of fuzzy random variables have numerous meanings in an actuarial framework. For instance, allows us to consider a financial institution that uses the quadratic premium calculation principle as its basis for its cost list. In this instance, the premium amount $\sum \tau_S(\chi)$ related to the risk χ is provided by $\sum \tau_S(\chi) = 1/S \; lnE(e^{t\chi})$ From the above equation,

$$\{\chi_1, \chi_2, \chi_3, \dots, \chi_n\} \leq_{FEO} \{\Upsilon_1, \Upsilon_2, \Upsilon_3, \dots, \Upsilon_n\} \Leftrightarrow \sum \tau_S(\chi) = 1/S \; lnE(e^{t\Upsilon}) \leq_{FEO} \sum \tau_S(\Upsilon) = 1/S \; lnE(e^{t\Upsilon})$$

Here are other interpretations, features, and uses of the exponential order as reported by Stoyan and Muller. Variables of the type $\chi_t = [t - \chi / \chi \leq t]$ are of significance in many reliability engineering problems for fixed $t \epsilon (0, L_X)$ and $L_X = \sup \{t: F_{\chi(t)} < 1\}$, with a distribution function $F_t(S) = P[t - \chi \leq S / \chi \leq t]$ and a known inactivity time in the literature.

Definition

Both χ and Υ are two continuous nonzero fuzzy random factors with the following attributes: f and g are their probability density functions; F and G are their distribution functions; and F¯ and G¯ are their survival values. After that, the progression of the,

- Exponential of the likelihood ratio defined by $\chi \leq_{ELRO} \Upsilon$,

$$\underset{\alpha \leq \beta \leq 1}{\wedge \& \vee} E(e^{t\chi}) \left(\frac{f_X}{g_X} (x_\alpha^l, x_\beta^u) \right) \leq_{ELR} \underset{\alpha \leq \beta \leq 1}{\wedge \& \vee} E(e^{t\Upsilon}) \left(\frac{f_Y}{g_y} (y_\alpha^l, y_\beta^u) \right)$$

- Exponential order of mean inactivity as stated by $\chi \leq_{EMITO} \Upsilon$,

$$\underset{\alpha \leq \beta \leq 1}{\wedge \& \vee} E(e^{t\chi}) \left(\frac{\int_0^t \overline{F_X(x_\alpha^l, x_\beta^u) dx}}{F_t(x_\alpha^l, x_\beta^u)} \right) \leq_{EMITO} \underset{\alpha \leq \beta \leq 1}{\wedge \& \vee} E(e^{t\Upsilon}) \left(\frac{\int_0^t \overline{G_y(y_\alpha^l, y_\beta^u) dx}}{G_t(y_\alpha^l, y_\beta^u)} \right)$$

- The reversed Hazard rate order's exponential, stated by $\chi \leq_{EMITO} \Upsilon$,

$$\underset{\alpha \leq \beta \leq 1}{\wedge \& \vee} E(e^{t\chi}) \left(\frac{\overline{F_X(x_\alpha^l, x_\beta^u)}}{f_X(x_\alpha^l, x_\beta^u)} \right) \leq_{ERHRO} \underset{\alpha \leq \beta \leq 1}{\wedge \& \vee} E(e^{t\Upsilon}) \left(\frac{\overline{G_X(y_\alpha^l, y_\beta^u)}}{g_y(y_\alpha^l, y_\beta^u)} \right)$$

- Exponential of the Hazard rate order defined by $\chi \leq_{EMITO} \Upsilon$,

$$\underset{\alpha\le\beta\le1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{f_X(x^l_\alpha,x^u_\beta)}{\overline{F_X(x^l_\alpha,x^u_\beta)}}\right) \le_{EHRO} \underset{\alpha\le\beta\le1}{\overset{\wedge\&\vee}{}} E(e^{t\Upsilon})\left(\frac{g_Y(y^l_\alpha,y^u_\beta)}{\overline{G_Y(y^l_\alpha,y^u_\beta)}}\right)$$

- The mean residual life order exponential, as described by $\chi \le_{EMRO} \Upsilon$,

$$\underset{\alpha\le\beta\le1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{\int_0^t \overline{F_X(x^l_\alpha,x^u_\beta)}dx}{F_t(x^l_\alpha,x^u_\beta)}\right) \le_{EMRLO} \underset{\alpha\le\beta\le1}{\overset{\wedge\&\vee}{}} E(e^{t\Upsilon})\left(\frac{\int_0^t \overline{G_Y(y^l_\alpha,y^u_\beta)}dx}{G_t(y^l_\alpha,y^u_\beta)}\right)$$

- Exponential serves as the decreasing order typical residual life order computed by $\chi \le_{DEMRO} \chi_\nabla$,

$$\underset{\alpha\le\beta\le1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{\int_0^t \overline{F_X(x^l_\alpha,x^u_\beta)}dx}{F_t(x^l_\alpha,x^u_\beta)}\right) \le_{EDMRLO} \underset{\alpha\le\beta\le1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{\int_0^t \overline{G_X(x^l_\alpha,x^u_\beta)}dx}{G_t(x^l_\alpha,x^u_\beta)}\right)$$

Following figure describes the graphical representation of stochastic orders of fuzzy random variables.

These graphical representations provide these visual tools in comparing random variable distributions and stochastic ordering. They offer a natural comprehension of the strengths and weaknesses and chances associated with the variables under evaluation. In this study, we look into the exponential order of the mean inactive time in a fuzzy setting. The preservation characteristics of the exponential in activity time order under convolution and combined operations are next discussed. Subsequently, we provide multiple applications of shock models and outline a few basic instances of their use to determine situations in which the random variables are similar in this series.

We observed that the phrases "increasing" and "decreasing" were employed rather than monotone non-decreasing and monotone non-increasing, respectively, throughout the entire paper. Furthermore, all fuzzy random variables under discourse are believed to be perfectly continuous, with 0 and 1 being the usual left points of their supports, and all expectancies are implicitly assumed to be limiting whenever they appear.

Preservation Properties

Dependability theory places importance on an order's preservation properties under certain dependability operations. Some features of the exponential of the inactivity order of a random variable are covered in this section.

Let χ and Υ be two continuous non-negative fuzzy random factors that have the following traits that distribution functions F and G, survival measures $\bar{F}$ and $\bar{G}$, and probability density functions f and g, respectively.

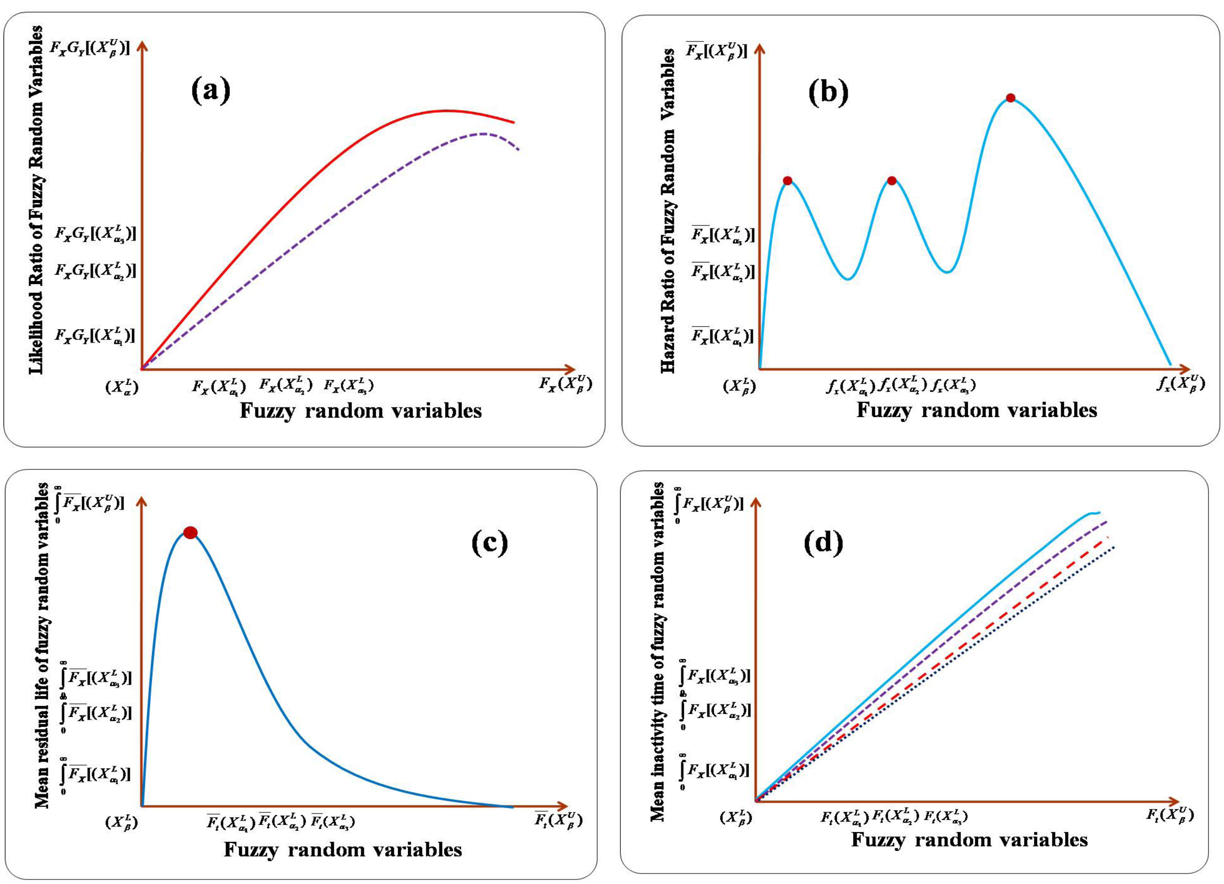

Figure 2: (a-d) Graphical representation of stochastic orders of fuzzy random variables

$$\psi^*_{\chi_t}(x^l_\alpha, x^u_\alpha) = \underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{-t\chi}) \left(\frac{\int_0^t \overline{F_x(x^l_\alpha, x^u_\beta)}dx}{\overline{F_t(x^l_\alpha, x^u_\beta)}} \right)$$

$$And$$

$$\psi^*_{\Upsilon_t}(y^l_\alpha, y^u_\alpha) = \underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{-t\Upsilon}) \left(\frac{\int_0^t \overline{F_x(x^l_\alpha, x^u_\beta)}dx}{\overline{F_t(x^l_\alpha, x^u_\beta)}} \right)$$

This condition holds true by the previous definition, then $\chi \leq_{EMITO} \Upsilon \Leftrightarrow \psi^*_{\chi_t}(x^l_\alpha, x^u_\alpha) \leq_{EMITO} \psi^*_{\Upsilon_t}(y^l_\alpha, y^u_\alpha)$.

Proposition

Imagine two continuous non-negative fuzzy random parameters, χ and ϒ, with the following traits that probability density functions f and g, distribution functions F and G, and inheriting functions $\bar{F}$ and $\bar{G}$, respectively. Then $\chi \leq_{EMITO} \Upsilon \Leftrightarrow$

$$\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{t\chi}) \left(\frac{\int_0^t \overline{F_x(x^l_\alpha, x^u_\beta)}dx}{\overline{F_t(x^l_\alpha, x^u_\beta)}} \right) \leq_{EMITO} \underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{t\Upsilon}) \left(\frac{\int_0^t \overline{G_y(y^l_\alpha, y^u_\beta)}dx}{\overline{G_t(y^l_\alpha, y^u_\beta)}} \right)$$

Is decreasing $t \in (0, t_x) \cap (0, t_y)$, for all $t > 0$.

Proof

Let us observe that

$$\psi^*_{\chi_t}(x^l_\alpha, x^u_\alpha) = \underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{-t\chi}) \left(\frac{\int_0^t \overline{F_x(x^l_\alpha, x^u_\beta)}dx}{\overline{F_t(x^l_\alpha, x^u_\beta)}} \right)$$

$= \underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{-t\chi}) \left(\frac{\int_0^t \overline{F_x(x^l_\alpha, x^u_\beta)}dx}{\frac{\partial}{\partial x}\overline{F_t(x^l_\alpha, x^u_\beta)}} \right)$; therefore given $t > 0$, by previous equations $\chi \leq_{EMITO} \Upsilon \Leftrightarrow$

$$\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{-t\chi}) \left(\frac{\int_0^t \overline{F_x(x^l_\alpha, x^u_\beta)}dx}{\frac{\partial}{\partial x}\overline{F_t(x^l_\alpha, x^u_\beta)}} \right) \leq_{EMITO} \underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{-t\Upsilon}) \left(\frac{\int_0^t \overline{G_y(y^l_\alpha, y^u_\beta)}dy}{\frac{\partial}{\partial x}\overline{G_t(y^l_\alpha, y^u_\beta)}} \right)$$

$$\Leftrightarrow$$

$$\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{-t\chi}) \left(\frac{\int_0^t \frac{\partial}{\partial x}\overline{F_t(y^l_\alpha, y^u_\beta)}dy}{\overline{G_y(y^l_\alpha, y^u_\beta)}} \right) \leq_{EMITO} \underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{-t\Upsilon}) \left(\frac{\int_0^t \frac{\partial}{\partial x}\overline{F_t(x^l_\alpha, x^u_\beta)}dx}{\overline{G_y(y^l_\alpha, y^u_\beta)}} \right)$$

$$\Leftrightarrow$$

$$\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{t\chi}) \left(\frac{\int_0^t \overline{F_x(x^l_\alpha, x^u_\beta)}dx}{\overline{F_t(x^l_\alpha, x^u_\beta)}} \right) \leq_{EMITO} \underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{t\chi}) \left(\frac{\int_0^t \overline{G_x(x^l_\alpha, x^u_\beta)}dx}{\overline{G_t(x^l_\alpha, x^u_\beta)}} \right)$$

Is decreasing in $t \in (0, t_x) \cap (0, t_y)$, for all t>0.

Theorem

Let us take χ_1, χ_2 and Z be three continuous non-negative fuzzy random variables with probability density function f and g and h distribution function F and G and H survival functions $\bar{F}$ and $\bar{G}$ and $\bar{H}$, respectively. Then $\chi_1 \leq_{EMITO} \chi_2$ and Z is log-concave then $\chi_1 + Z \leq_{EMITO} \chi_2 + Z$.

Proof: By the previous preposition, it is enough to show that for all $0 \leq t_1 \leq t_2$ and $x > 0$.

$$\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{t\chi}) \left(\int_0^\infty \int_{-\infty}^{t_1} \frac{P[\chi_1 \leq u - (x^l_\alpha, x^u_\alpha)]\{f(t_1 - u)dudx}{P[\chi_2 \leq u - (x^l_\alpha, x^u_\alpha)]\{g(t_2 - u)dudx} \right) \geq_{EMITO}$$

$$\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{t\chi}) \left(\int_0^\infty \int_{-\infty}^{t_1} \frac{P[\chi_1 \leq u - (x^l_\alpha, x^u_\alpha)]\{f(t_2 - u)dudx}{P[\chi_2 \leq u - (x^l_\alpha, x^u_\alpha)]\{g(t_1 - u)dudx} \right)$$

Since Z is non negative then $g(t - u) = 0$ when $t < u$, hence the above inequality is equivalent to

$$\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{t\chi}) \left(\int_0^\infty \int_{-\infty}^{t_1} \frac{[F_{\chi_1} \leq u - (x^l_\alpha, x^u_\alpha)]\{f(t_1 - u)dudx}{[F_{\chi_2} \leq u - (x^l_\alpha, x^u_\alpha)]\{g(t_2 - u)dudx} \right) \geq_{EMITO}$$

$$\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{t\chi}) \left(\int_0^\infty \int_{-\infty}^{t_1} \frac{[G_{\chi_1} \leq u - (x^l_\alpha, x^u_\alpha)]\{f(t_2 - u)dudx}{[G_{F_{\chi_2}} \leq u - (x^l_\alpha, x^u_\alpha)]\{g(t_2 - u)dudx} \right)$$

$0 \leq t_1 \leq t_2$ or equivalently,

$$\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} \begin{vmatrix} \int_0^\infty \int_{-\infty}^{t_1} \boldsymbol{E}(\boldsymbol{e}^{t\chi})[F_{\chi_2}(u - (x^l_\alpha, x^u_\alpha))]\{g(t_2 - u)dudx\}] & \int_0^\infty \int_{-\infty}^{t_1} \boldsymbol{E}(\boldsymbol{e}^{t\chi})[F_{\chi_2}(u - (x^l_\alpha, x^u_\alpha))]\{g(t_2 - u)dudx\}] \\ \int_0^\infty \int_{-\infty}^{t_1} \boldsymbol{E}(\boldsymbol{e}^{t\chi})[F_{\chi_2}(u - (x^l_\alpha, x^u_\alpha))]\{g(t_1 - u)dudx\}] & \int_0^\infty \int_{-\infty}^{t_1} \boldsymbol{E}(\boldsymbol{e}^{t\chi})[F_{\chi_2}(u - (x^l_\alpha, x^u_\alpha))]\{g(t_1 - u)dudx\}] \end{vmatrix}$$

$$\geq \mathbf{0}$$

$$and$$

$$\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} \begin{vmatrix} \int_0^\infty \int_{-\infty}^{t_1} \boldsymbol{E}(\boldsymbol{e}^{t\chi})[F_{\chi_2}(u - (x^l_\alpha, x^u_\alpha))]\{g(t_2 - u)dudx\}] & \int_0^\infty \int_{-\infty}^{t_1} \boldsymbol{E}(\boldsymbol{e}^{t\chi})[F_{\chi_2}(u - (x^l_\alpha, x^u_\alpha))]\{g(t_2 - u)dudx\}] \\ \int_0^\infty \int_{-\infty}^{t_1} \boldsymbol{E}(\boldsymbol{e}^{t\chi})[F_{\chi_2}(u - (x^l_\alpha, x^u_\alpha))]\{g(t_1 - u)dudx\}] & \int_0^\infty \int_{-\infty}^{t_1} \boldsymbol{E}(\boldsymbol{e}^{t\chi})[F_{\chi_2}(u - (x^l_\alpha, x^u_\alpha))]\{g(t_1 - u)dudx\}] \end{vmatrix}$$

$$\geq \mathbf{0}$$

By the well known basic composition formula

$$\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} \int_{u_1<u_2}^\infty \int_{u_1<u_2}^\infty \begin{vmatrix} g(t_2 - u_1) & g(t_2 - u_2) \\ g(t_1 - u_1) & g(t_1 - u_2) \end{vmatrix} \times \begin{vmatrix} \int_0^\infty \boldsymbol{E}(\boldsymbol{e}^{t\chi})[F_{\chi_2}(u_1 - (x^l_\alpha, x^u_\alpha))] & \int_0^\infty \boldsymbol{E}(\boldsymbol{e}^{t\chi})[F_{\chi_1}(u_1 - (x^l_\alpha, x^u_\alpha))] \\ \int_0^\infty \boldsymbol{E}(\boldsymbol{e}^{t\chi})[F_{\chi_2}(u_2 - (x^l_\alpha, x^u_\alpha))] & \int_0^\infty \boldsymbol{E}(\boldsymbol{e}^{t\chi})[F_{\chi_1}(u_2 - (x^l_\alpha, x^u_\alpha))] \end{vmatrix} du_1\, du_2$$

Seeing that the first determinate is non-positive because of g's log-concavity and the second determinant being non-positive due to $X_1 \leq_{EMITO} X_2$. leads us to the conclusion. Where $\chi_1 = \{\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} F_{\chi_1}(x^l_\alpha, x^u_\beta)\}$ and $\chi_2 = \{\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} F_{\chi_2}(x^l_\alpha, x^u_\beta)\}$ Which is complete the proof.

Lemma

If $\chi_1 \leq_{EMITO} \Upsilon_1$ and $\chi_2 \leq_{EMITO} \Upsilon_2$ where X_1 is an independent fuzzy random variable of χ_2 and Υ_1 is an independent fuzzy random variable of Υ_2 with probability density function *f* and g distribution function F and G and survival functions $\bar{F}$ and $\bar{G}$, respectively. Then the following statements hold:

- If χ_1 and Υ_2 have log-concave densities, then $\chi_1 + \chi_2 \leq_{EMITO} \Upsilon_1 + \Upsilon_2$
- If χ_2 and Υ_1 have log-concave densities, then $\chi_1 + \chi_2 \leq_{EMITO} \Upsilon_1 + \Upsilon_2$

Proof (i)

$$\underset{\alpha \leq \beta \leq 1}{\wedge\&\vee} E(e^{t\chi}) \left(\frac{\int_0^t \overline{F_x(x^l_\alpha, x^u_\beta)}dx}{\overline{F_t(x^l_\alpha, x^u_\beta)}} \right) \leq_{EMITO} \underset{\alpha \leq \beta \leq 1}{min} E(e^{t\chi}) \left(\frac{\int_0^t \overline{G_x(x^l_\alpha, x^u_\beta)}dx}{\overline{G_t(x^l_\alpha, x^u_\beta)}} \right)$$

The following chain inequality, which is establishing (1), follows by theorem 2.1:

$$\chi_1 + \chi_2 \leq_{EMITO} \chi_1 + \Upsilon_2 \leq_{EMITO} \Upsilon_1 + \Upsilon_2$$

Where, $\chi = \underset{\alpha\leq\beta\leq1}{\wedge\&\vee} E(e^{t\chi}) \left(\frac{\int_0^t F_x(x_\alpha^l, x_\beta^u)dx}{F_t(x_\alpha^l, x_\beta^u)}\right)$ and $\Upsilon = \underset{\alpha\leq\beta\leq1}{\wedge\&\vee} E(e^{t\Upsilon}) \left(\frac{\int_0^t G_y(y_\alpha^l, y_\beta^u)dy}{G_t(y_\alpha^l, y_\beta^u)}\right)$

Proof (ii)

The evidence for (ii) is analogous. The following outcome can be obtained by repeatedly employing lemma 3.8 and the closure property of log-concaves under convolution.

Theorem

Let us consider $\{\chi_1, \chi_2, \chi_3, \ldots, \chi_n\}$ and $\{\Upsilon_1, \Upsilon_2, \Upsilon_3, \ldots, \Upsilon_n\}$ be two sequence sets of fuzzy random variables, X_i is said to be smaller than Y_i in the exponential order denoted as, $\{\chi_1, \chi_2, \chi_3, \ldots, \chi_n\} \leq \{\Upsilon_1, \Upsilon_2, \Upsilon_3, \ldots, \Upsilon_n\}$ and have log-concave densities for all i, then $\sum_{i=1}^n \chi_i \leq_{EMITO} \sum_{i=1}^n \Upsilon_i$.
Where $i = 1,2,3, \ldots, n$

Proof

We shall employ induction to demonstrate the theorem. Certainly, the result stays true for $n = 1$. Assume that the result is true for $q = n - 1$, that is

$$\sum_{i=1}^{n-1} \chi_i \leq_{EMITO} \sum_{i=1}^{n-1} \Upsilon_i$$

Where, $\chi = \underset{\alpha\leq\beta\leq1}{\wedge\&\vee} E(e^{t\chi}) \left(\frac{\int_0^t F_x(x_\alpha^l, x_\beta^u)dx}{F_t(x_\alpha^l, x_\beta^u)}\right)$ and $\Upsilon = \underset{\alpha\leq\beta\leq1}{\wedge\&\vee} E(e^{t\Upsilon}) \left(\frac{\int_0^t G_y(y_\alpha^l, y_\beta^u)dy}{G_t(y_\alpha^l, y_\beta^u)}\right)$

Note that each of the two sides of above equation has a log-concave density. Applying previous lemma the results follows. The following concepts will be used in the sequel.

Definition

A function $F_{\chi\Upsilon}: \chi \times \Upsilon \to [0,1]$ is said to be a totally positive fuzzy set of order 2. If for all $\alpha_1 < \alpha_2$ and $\beta_1 < \beta_2$ $\{(\alpha_1, \alpha_2)\epsilon\chi, (\beta_1, \beta_2)\epsilon\Upsilon\}$

$$\begin{vmatrix} F_{\chi\Upsilon}(\alpha_1, \beta_1) & F_{\chi\Upsilon}(\alpha_1, \beta_2) \\ F_{\chi\Upsilon}(\alpha_2, \beta_1) & F_{\chi\Upsilon}(\alpha_2, \beta_2) \end{vmatrix} \geq 0$$

Let us take χ(δ) be a distribution function-containing fuzzy random variable $F_{\chi(\delta)}$ and let ϒ(δ) be another fuzzily distributed random variable with a distribution function. $F_{\Upsilon(\delta)}$, for i = 1,2, and support R^+. The following is a closure of exponential of inactivity time order under the mixture.

Theorem

Let us take X(δ) is a set of fuzzy random variables $\delta \in R^+$ and independent of $\emptyset_1$ and $\emptyset_2$. If $\emptyset_1 \leq_{FLR} \emptyset_2$ and if $\chi(\delta_1) \leq_{EMITO} X(\delta_2)$ whenever $\delta_1 \leq \delta_2$, then $\Upsilon(\emptyset_1) \leq_{EMITO} \Upsilon(\emptyset_2)$.

Proof

Let F_χ be the distribution function of $\chi(\delta_i)$ with i = 1,2. We know that

$$F_{\chi_i} = \underset{\alpha\leq\beta\leq1}{\wedge\&\vee} E(e^{t\chi}) \left(\frac{\int_0^t F_{\chi(\delta_i)}(x_\alpha^l, x_\beta^u)dx}{F_t(x_\alpha^l, x_\beta^u)}\right)$$

Again, because of previous preposition, we should prove that ,

$$\psi_{\chi_i}^*(x_\alpha^l, x_\alpha^u) = \underset{\alpha\leq\beta\leq1}{\wedge\&\vee} E(e^{t\chi}) \left(\frac{\int_0^t F_{\chi(\delta_i)}(t-(x_\alpha^l, x_\beta^u))dx}{F_t(t-(x_\alpha^l, x_\beta^u))}\right)$$ is totally positive order2 in (i,t). But actually

$$= \underset{\alpha\leq\beta\leq1}{\wedge\&\vee} E(e^{t\chi}) \left(\frac{\int_0^t\int_0^\infty F_{\chi(\delta_i)}(t-(x_\alpha^l, x_\beta^u))dxdt}{F_t(t-(x_\alpha^l, x_\beta^u))}\right)$$

$$= \underset{\alpha\leq\beta\leq1}{\wedge\&\vee} E(e^{t\chi}) \left(\frac{\int_0^t\int_0^\infty F_{\chi(\delta_i)}(t-(x_\alpha^l, x_\beta^u))dG_{\Upsilon(\emptyset_i)}dx}{F_t(t-(x_\alpha^l, x_\beta^u))}\right)$$

$$= \underset{\alpha\leq\beta\leq1}{\wedge\&\vee} E(e^{t\chi}) \left(\frac{\int_0^t\int_0^\infty \psi_{\chi_i}^*(x_\alpha^l, x_\beta^u)dG_{\Upsilon(\emptyset_i)}dx}{\psi_{\chi_i}^*(x_\alpha^l, x_\beta^u)}\right)$$

$$= \underset{\alpha\leq\beta\leq1}{\wedge\&\vee} E(e^{t\chi}) \left(\frac{\int_0^t\int_0^\infty \psi_{\chi_i}^*(x_\alpha^l, x_\beta^u)\psi_{\Upsilon_i}^*(y_\alpha^l, y_\alpha^u)dxdy}{\psi_{\chi_i}^*(x_\alpha^l, x_\beta^u)\psi_{\Upsilon_i}^*(y_\alpha^l, y_\alpha^u)}\right)$$

By assumption $\chi(\delta_1) \leq_{EMITO} \chi(\delta_2)$ whenever $\delta_1 \leq \delta_2$, we have that $\psi_{\chi_i}^*(x_\alpha^l, x_\alpha^u)$ is totally positive order2 in (δ,t), while form assumption $(\emptyset_1) \leq_{FLR} (\emptyset_2)$ follows that $\psi_{\Upsilon_i}^*(y_\alpha^l, y_\alpha^u)$ is totally positive order 2 in (δ,i). Thus again assertion follow from the basic composition formula.

Let $\{\chi_1, \chi_2, \chi_3, \ldots, \chi_n\}$ be sets of fuzzy random variables with distributions $f_{\{\chi_1, \chi_2, \chi_3, \ldots, \chi_n\}}$ and denote their survival functions by, $\bar{f}_{\{\chi_1, \chi_2, \chi_3, \ldots, \chi_n\}} = (1 - f_{\{\chi_1, \chi_2, \chi_3, \ldots, \chi_n\}})$ respectively. Let $\underline{\alpha} = \{\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n\}$ and $\underline{\beta} = \{\beta_1, \beta_2, \beta_3, \ldots, \beta_n\}$ be two sets of probability vectors. A probability vector $\underline{\alpha} = \{\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n\}$ with $a_i > 0$ for $i = 1,2,3, \ldots, n$ is said to be smaller than the probability vector $\underline{\beta} = \{\beta_1, \beta_2, \beta_3, \ldots, \beta_n\}$ in the sense of discrete likelihood ratio order, denoted as $\alpha_i \leq_{DFLR} \beta_i$ if

$$\frac{\beta_i}{\alpha_i} \leq_{DFLR} \frac{\beta_j}{\alpha_j} \text{ for all } 1 \leq i \leq j \leq n$$

Let us take X and Y be two continuous non-negative fuzzy random variables with probability density functions f and g, distribution functions F and G, and survival functions $\bar{F}$ and $\bar{G}$, respectively.

$$F(\chi) = \sum_{i=1}^n \underset{\alpha\leq\beta\leq1}{\wedge\&\vee} E(e^{t\chi}) \left(\frac{\int_0^t \alpha_i F_{\chi_i}(x_\alpha^l, x_\beta^u)dx}{\alpha_i F_t(x_\alpha^l, x_\beta^u)}\right) \text{ and } G(\Upsilon) = \sum_{i=1}^n \underset{\alpha\leq\beta\leq1}{\wedge\&\vee} E(e^{t\Upsilon}) \left(\frac{\int_0^t \beta_i G_{\Upsilon_i}(y_\alpha^l, y_\beta^u)dy}{\beta_i G_t(y_\alpha^l, y_\beta^u)}\right)$$

Conditions under which χ and ϒ are analogous with regard to the exponential inactivity time order of fuzzy random variables are established by the following discovery.

Theorem

Let $\{\chi_1, \chi_2, \chi_3, \ldots, \chi_n\}$ be sets of fuzzy random variables with distributions $f_{\{\chi_1, \chi_2, \chi_3, \ldots, \chi_n\}}$ and denote their survival functions by, $\bar{F}_{\{X_1, X_2, X_3, \ldots, X_n\}} = (1 - F_{\{\chi_1, \chi_2, \chi_3, \ldots, \chi_n\}})$, such that $X_1 \leq_{EMITO} X_2 \leq_{EMITO} \cdots \leq_{EMITO} X_n$ and let $\underline{\alpha} = \{\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n\}$ and $\underline{\beta} = \{\beta_1, \beta_2, \beta_3, \ldots, \beta_n\}$ such that $\alpha_i \leq_{DFLR} \beta_i$. Let fuzzy random variable X and Y have a distribution function f_x and g_y defined by the previous equation. Then $\chi \leq_{EMITO} \Upsilon$.

Proof: Because of previous preposition, we need to establish that

$$\sum_{i=1}^{n}\left(\frac{\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{\int_0^t \beta_j F_{X_i}\left((x_\alpha^l, x_\beta^u)-t\right)dx}{\beta_j F_t\left((x_\alpha^l, x_\beta^u)-t\right)}\right)}{\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{\int_0^t \alpha_i G_{Y_i}\left((x_\alpha^l, x_\beta^u)-t\right)dx}{\alpha_j G_t\left((x_\alpha^l, x_\beta^u)-t\right)}\right)}\right) \leq \sum_{i=1}^{n}\left(\frac{\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\Upsilon})\left(\frac{\int_0^t \beta_j F_{X_i}\left((x_\alpha^l, x_\beta^u)-t\right)dx}{\beta_j F_t\left((x_\alpha^l, x_\beta^u)-t\right)}\right)}{\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\Upsilon})\left(\frac{\int_0^t \alpha_i G_{Y_i}\left((x_\alpha^l, x_\beta^u)-t\right)dx}{\alpha_j G_t\left((x_\alpha^l, x_\beta^u)-t\right)}\right)}\right)$$

The aforementioned equation can be demonstrated to be equivalent to by multiplying by the denominators and eliminating equal terms.

$$\sum_{i=1}^{n}\sum_{i=1}^{n}\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{\int_0^t \alpha_i\beta_j F_{X_i}\left((x_\alpha^l, x_\beta^u)-t\right)dx}{\alpha_i\beta_j F_t\left((x_\alpha^l, x_\beta^u)-t\right)}\right)\right)\times\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\Upsilon})\left(\frac{\int_0^t \alpha_i\beta_j G_{Y_i}\left((y_\alpha^l, y_\beta^u)-t\right)dx}{\alpha_i\beta_j G_t\left((y_\alpha^l, y_\beta^u)-t\right)}\right)\right)$$

$$\leq$$

$$\sum_{i=1}^{n}\sum_{i=1}^{n}\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{\int_0^t \alpha_i\beta_j F_{X_i}\left((x_\alpha^l, x_\beta^u)-t\right)dx}{\alpha_j\beta_i F_t\left((x_\alpha^l, x_\beta^u)-t\right)}\right)\right)\times\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\Upsilon})\left(\frac{\int_0^t \alpha_i\beta_j G_{Y_i}\left((y_\alpha^l, y_\beta^u)-t\right)dx}{\alpha_j\beta_i G_t\left((y_\alpha^l, y_\beta^u)-t\right)}\right)\right)$$

Where $i \neq j$. Now for each fixed pair (i,j) with $i < j$ we have

$$\left[\beta_i\alpha_j\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\Upsilon})\left(\frac{\int_0^t \alpha_i\beta_j G_{Y_i}\left((y_\alpha^l, y_\beta^u)-t\right)dx}{G_t\left((y_\alpha^l, y_\beta^u)-t\right)}\right)\right)\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{\int_0^t \alpha_i\beta_j F_{X_i}\left((x_\alpha^l, x_\beta^u)-t\right)dx}{F_t\left((x_\alpha^l, x_\beta^u)-t\right)}\right)\right)\right]$$

$$+$$

$$\left[\beta_j\alpha_i\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\Upsilon})\left(\frac{\int_0^t \alpha_i\beta_j G_{Y_i}\left((y_\alpha^l, y_\beta^u)-t\right)dx}{G_t\left((y_\alpha^l, y_\beta^u)-t\right)}\right)\right)\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{\int_0^t \alpha_i\beta_j F_{X_i}\left((x_\alpha^l, x_\beta^u)-t\right)dx}{F_t\left((x_\alpha^l, x_\beta^u)-t\right)}\right)\right)\right]$$

$$-$$

$$\left[\beta_i\alpha_j\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\Upsilon})\left(\frac{\int_0^t \alpha_i\beta_j G_{Y_i}\left((y_\alpha^l, y_\beta^u)-t\right)dx}{G_t\left((y_\alpha^l, y_\beta^u)-t\right)}\right)\right)\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{\int_0^t \alpha_i\beta_j F_{X_i}\left((x_\alpha^l, x_\beta^u)-t\right)dx}{F_t\left((x_\alpha^l, x_\beta^u)-t\right)}\right)\right)\right]$$

$$+$$

$$\left[\beta_j\alpha_i\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\Upsilon})\left(\frac{\int_0^t \alpha_i\beta_j G_{Y_i}\left((y_\alpha^l, y_\beta^u)-t\right)dx}{G_t\left((y_\alpha^l, y_\beta^u)-t\right)}\right)\right)\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{\int_0^t \alpha_i\beta_j F_{X_i}\left((x_\alpha^l, x_\beta^u)-t\right)dx}{F_t\left((x_\alpha^l, x_\beta^u)-t\right)}\right)\right)\right]$$

$$= (\beta_i\alpha_j\beta_j\alpha_i)[\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\Upsilon})\left(\frac{\int_0^t \alpha_i\beta_j G_{Y_i}\left((y_\alpha^l, y_\beta^u)-t\right)dx}{G_t\left((y_\alpha^l, y_\beta^u)-t\right)}\right)\right)\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{\int_0^t \alpha_i\beta_j F_{X_i}\left((x_\alpha^l, x_\beta^u)-t\right)dx}{F_t\left((x_\alpha^l, x_\beta^u)-t\right)}\right)\right)$$

$$-$$

$$\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\chi})\left(\frac{\int_0^t \alpha_i\beta_j F_{X_i}\left((x_\alpha^l, x_\beta^u)-t\right)dx}{F_t\left((x_\alpha^l, x_\beta^u)-t\right)}\right)\right)\left(\underset{\alpha\leq\beta\leq 1}{\overset{\wedge\&\vee}{}} E(e^{t\Upsilon})\left(\frac{\int_0^t \alpha_i\beta_j G_{Y_i}\left((y_\alpha^l, y_\beta^u)-t\right)dx}{G_t\left((y_\alpha^l, y_\beta^u)-t\right)}\right)\right)$$

This is non-negative because both terms are non-negative by assumption. This is the complete proof. The above holds true maximum value of fuzzy random variable. The same preservation properties and theorems hold true for other models like likelihood ratio order, hazard rate order, mean residual life orders.

Practical Utilization of Stochastic Models

A framework for comparing fuzzy random variable distributional properties is provided by stochastic ordering. Fuzzy random variables that indicate imprecision and uncertainty can be used in conjunction with stochastic ordering to model and analyze a number of real-world problems. The following categories of applications for stochastic ordering of fuzzy random variables are possible:

Risk analysis

It is feasible to evaluate and rank the riskiness of various financial assets or investment portfolios using stochastic mandating of fuzzy random variables. By considering the stochastic dominance relationships between fuzzy random variables, investors can make more informed decisions about risk management and asset allocation.

Quality control

Stochastic orders can be used to gauge the caliber of products or production processes when measurements are ambiguous or imprecise. Assume that a certain approach or product is stochastically superior to another. Then, utilizing fuzzy random variables and stochastic ordering, which may capture the fuzziness and variability in the quality attributes, it can be determined.

Reliability analysis

By using stochastic ordering of fuzzy random variables, reliability engineering makes it possible to compare and measure the reliability of various systems or individual components. By taking into mind the stochastic dominance relationships, engineers can evaluate the performance and robustness of different designs and make decisions about system maintenance and improvement.

Insurance and actuarial science

Stochastic orders are useful in actuarial science and insurance, especially when fuzzy risk models are being used. They can be used to evaluate the insurance companies' solvency and financial stability, as well as to weigh the risks involved in various insurance options.

Decision-making under uncertainty

Fuzzy random variables with stochastic ordering come in use whenever there is ambiguity and imprecision in the decision factors and objectives. By using stochastic dominance criteria, decision-makers can determine which options or strategies are better based on their distributional properties.

Environmental modeling

It is feasible to employ stochastic ordering of fuzzy random variables in environmental modeling and analysis. They can be utilized for illustration, for assessing the impact of confusing and imprecise factors on environmental processes or to compare and rank the pollution levels from different emission sources.

The aforementioned applications showcase the adaptability of stochastic ordering in the context of fuzzy random variables, hence facilitating the examination of imprecise and uncertain systems in diverse fields.

Conclusion

In actuarial science, one of the most important roles is the exponential order of a stochastic model. We propose different preservation features under mixture and convolution reliability processes of the fuzzy random variable with exponential stochastic order in the current study. Applications such as hazard rate order, mean residual life order, and reverse hazard rate order, all using stochastic models, are outlined. Examples are given to show how the results may be exploited to find the exponential order of mean inactivity time-ordered fuzzy random variables. Our findings also have implications for dependability, risk theory, and statistics. Future studies can take into account the extra features and uses of this novel ordering.

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