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RESEARCH ARTICLE

Multi-fuzzy set similarity measures using S and T operations

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Abstract

This paper introduces some similarity measures on multi-fuzzy set, enhancing multi-fuzzy set analysis through a weighting mechanism using S and T operations. Using a fuzzy matrix, we define a weighted relation, summarizing the characteristics of multi-fuzzy set M in relation to a weighted column matrix A. The relation $\leq_A$ is established by comparing membership grades and weighted elements, expressing similarity or dissimilarity within elements of X.

Keywords: Multi-fuzzy sets, Weighted multi-fuzzy sets, Similarity measure, T- operation, S- operation, Semi-quasi similarity measure.

2020 Mathematics Subject Classification: 03E72, 08A72, 15B15, 28E10.

Introduction

The fuzzy set theory provides a framework for handling uncertainty and vagueness in information by allowing elements to have degrees of membership rather than being strictly in or out of a set. Multi-fuzzy sets introduced by (Sebastian and Ramakrishnan, 2011), which extend traditional fuzzy sets to accommodate multiple membership grades for each element, offer a generalized framework for representing complex relationships in various domains. After that (Priyanka, Sebastian, Haseena and Sangeeth, 2024) introduce multi-fuzzy extensions of crisp functions using fuzzy matrices as bridge functions and fuzzy matrices are used for the comparative study of different characterization problems. Later(Sebastian, Ramakrishnan, Gireesan and Sangeeth, 2024) illustrate the integration of fuzziness and neural networks to generate artificial intelligence through multi-fuzzy membership functions. Recognizing the significance of individual elements within these sets, our work aims to enhance the accuracy of multi-fuzzy set comparisons through the integration of a weighting mechanism. The focus of our study includes in the development of weighted similarity measures that utilize a refined weighting mechanism based on S and T operations defined in (Begam, Vimala, Selvachandran, Nagan and Sharma, 2020). One of the key findings of our work is the exploration of norm variations and their impact on similarity measures(Lee-Kwang, Song and Lee, 1994). S-norm and t-norm defined in (Klir and Yuan, 2008) play pivotal roles in fuzzy set operations, and our comparative analysis study into the consequences of varying these norms on the overall similarity assessment. Similarity measures have a key role in decision-making processes (Bouchon-Meunier and Coletti, 2020) by providing a quantitative assessment of the resemblance or closeness between different entities or alternatives. In this paper, we present a comprehensive study on weighted multi-fuzzy sets and similarity measures (Pappis and Karacapilidis, 1993) it.

Definition 1.1

Let X be a set of elements, and let M denote a multi-fuzzy set in X of dimension n. The multi-fuzzy set M is represented as $M = \{\langle x, (\mu_1(x), \mu_2(x), ..., \mu_n(x))\rangle : x \in X\}$, where $\mu_j(x)$ signifies the membership grade of element x in the fuzzy set for the j^{th} criterion. The collection of all multi-fuzzy sets in X having dimension m is denoted by $M^m FS(X)$.

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Definition 1.2

Let M , N be multi-fuzzy sets in *X*. The S and T operations

T $(M,N) = \{\langle x, t(\mu_1(x),\zeta_1(x)), t(\mu_2(x),\zeta_2(x)),\dots,t(\mu_n(x),\zeta_n(x)),\dots\rangle : x \in X\}$;

S $(M,N) = \{\langle x, s(\mu_1(x),\zeta_1(x)), s(\mu_2(x),\zeta_2(x)),\dots,s(\mu_n(x),\zeta_n(x)),\dots\rangle : x \in X\}$, where *n* is a positive integer, t and s denote the t-norm and s-norm respectively.

Theorem 1.3

Let M, N, P be multi-fuzzy sets in X of dimension n. Then

- $S(M,0) = M$, where $0 = \{\langle x,(0,0,\dots,0)\rangle\}$;
- $N \le P$ implies $S(M,N) \le S(M,P)$;
- $S(M,N) = S(N,M)$;
- $S(M,S(N,P)) = S(S(M,N),P)$.

Theorem 1.4

Let M, N, P be multi-fuzzy sets in X of dimension n. Then

- $T(M,1) = M$, *where* $1 = \{\langle x,(1,1,\dots,1)\rangle\}$;
- $N \le P$ *implies* $T(M,N) \le T(M,P)$;
- $T(M,N) = T(N,M)$;
- $T(M,T(N,P)) = T(T(M,N),P)$.

Theorem 1.5

For all M,N$\in M^nFS(X)$, $T_{min}(M,N) \le T(M,N) \le \min(M,N)$ and $\max(M,N) \le S(M,N) \le S_{max}(M,N)$, where

$T_{min}(M,N) = \{\langle x, t_{min}(\mu_1(x),\zeta_1(x)),\dots,t_{min}(\mu_n(x),\zeta_n(x))\rangle : x \in X\}\}$ and $S_{max}(M,N) = \{\langle x, s_{max}(\mu_1(x),\zeta_1(x)),\dots,s_{max}(\mu_n(x),\zeta_n(x))\rangle : x \in X\}\}$, t_{min} and s_{max} denote the drastic intersection and drastic union respectively.

Fuzzification of Multi-fuzzy Set

Definition 2.1

Let *X* be a set of elements, and let *M* bea multi-fuzzy set in *X* of dimension *n*. Consider *A*, a column matrix $[a_1,a_2,\dots,a_n]^T$, where each a_i is an element belonging to the unit interval *I* and $\sum_{i=1}^{n} a_i = 1$. A weighted multi-fuzzy set with respect to a fuzzy matrix *A* is a fuzzy set

$\{\langle x, [\mu_1(x),\mu_2(x),\dots,\mu_n(x)]_A\rangle : x \in X\}$, where

$$[\mu_1(x),\mu_2(x),\dots,\mu_n(x)]_A = [\mu_1(x),\mu_2(x),\dots,\mu_n(x)]\begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{bmatrix}$$

$= s(t(\mu_1(x),a_1), t(\mu_2(x),a_2),\dots,t(\mu_n(x),a_n))$ where *s* and *t* represent the s–norm and t-norm, respectively.

Weighted Order in Multi-fuzzy Sets

Definition 3.1

Let *X* be a set of elements, and let *M* denote a multi-fuzzy set in *X* of dimension *n*. Consider *A*, a column matrix $[a_1,a_2,\dots,a_n]^T$, where each a_i is an element belonging to the unit interval *I* and $\sum_{i=1}^{n} a_i = 1$.

Let $M = \left\{\left\langle x,(\mu_i(x))_1^n : x \in X\right\rangle\right\}$, where $(\mu_i(x))_1^n$ is the multi-fuzzy membership function of $x \in X$.

For $1 \le i,j \le n$, define a relation $\le_A$ such that for any $x, y \in X$, $\left\langle x,(\mu_i(x))_1^n\right\rangle <_A \left\langle y,(\mu_i(y))_1^n\right\rangle$

if and only if $\left[\mu_i(x)_1^n\right]\left[(a_j)_1^n\right]^T < \left[\mu_i(y)_1^n\right]\left[(a_j)_1^n\right]^T$.

Furthermore, $\left\langle x,(\mu_i(x))_1^n\right\rangle =_A \left\langle y,(\mu_i(y))_1^n\right\rangle$ if and only

if $\left[\mu_i(x)_1^n\right]\left[(a_j)_1^n\right]^T = \left[\mu_i(y)_1^n\right]\left[(a_j)_1^n\right]^T$.

Definition 3.2

Let $M = \left\{\left\langle x,(\mu_i(x))_1^n : x \in X\right\rangle\right\}$ and $N = \left\{\left\langle x,(\varsigma_i(x))_1^n : x \in X\right\rangle\right\}$.

If $\left\langle x,(\mu_i(x))_1^n\right\rangle <_A \left\langle x,(\varsigma_i(x))_1^n\right\rangle$ for all $x \in X$, then we say that $[M]_A \subset [N]_A$.

If $\left\langle x,(\mu_i(x))_1^n\right\rangle =_A \left\langle x,(\varsigma_i(x))_1^n\right\rangle$, for all $x \in X$, then we say that $[M]_A = [N]_A$.

Theorem 3.3

For all M,N$\in M^nFS(X)$, the following properties hold:

- $[M \cup M]_A = [M]_A$
- $[M \cap M]_A = [M]_A$
- $[M]_A \subseteq [M \cup N]_A$
- $[M \cap N]_A \subseteq [M]_A$
- $[M \cup N]_A = [M]_A$ if and only if $[N]_A \subseteq [M]_A$
- $[M \cap N]_A = [M]_A$ if and only if $[M]_A \subseteq [N]_A$

Main Results

A New Weighted Similarity Measure Between Multi-Fuzzy Sets

Definition 4.1

Suppose that $X = \{x_1,x_2,\dots,x_n\}$ is a crisp set, and $M \in M^nFS(X)$.

Weighted similarity measure $S^*: M^nFS(X) \times M^nFS(X) \to [0,\infty)$ is

$$S^*(M,N) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{\min(\mu_M(x_i),\mu_N(x_i))}{\max(\mu_M(x_i),\mu_N(x_i))}\right]_A,$$

where the weighted matrix *A* is any $n \times 1$ fuzzy matrix $A = [a_1,a_2,\dots,a_n]^T$ and $a_i \in I$, i= 1,2,...,n.

Note that $\left[\frac{\min(\mu_M(x_i),\mu_N(x_i))}{\max(\mu_M(x_i),\mu_N(x_i))}\right]_A$ denotes

$$\frac{\left[\min(\mu_M(x_i),\mu_N(x_i))\right]_A}{\left[\max(\mu_M(x_i),\mu_N(x_i))\right]_A}.$$

In order to avoid the denominator being zero, we set $\frac{0}{0} = 1$.

Theorem 4.2

$S^*(M,N)$ is a similarity measure.

Proof

- Commutativity property

$$\min\left(\mu_M(x_i),\mu_N(x_i)\right)=\min\left(\mu_N(x_i),\mu_M(x_i)\right)$$
$$\max\left(\mu_M(x_i),\mu_N(x_i)\right)=\max\left(\mu_N(x_i),\mu_M(x_i)\right),\text{for all }\mu_M,\ \mu_N\in I.$$

Hence $S^*(M,N)=S^*(N,M)$, for all $M,N\in M^nFS(X)$.

- Let Z^* be a crisp set. Thus $Z^*=\{\langle x,(1,1,...,1)\rangle : x\in X\}$.
  Applying the property of min and max, we can write

$$\min\left(\mu_{A^*}(x_i),\mu_{(A^*)^C}(x_i)\right)=\min\left((1,1,...,1),(0,0,...,0)\right)=(0,0,...,0)$$
$$\max\left(\mu_{A^*}(x_i),\mu_{(A^*)^C}(x_i)\right)=\max\left((1,1,...,1),(0,0,...,0)\right)=(1,1,...,1)$$

  for all $x_i\in X$.
  Hence,

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\left[\min\left(\mu_M(x_i),\mu_N(x_i)\right)\right]_A}{\left[\max\left(\mu_M(x_i),\mu_N(x_i)\right)\right]_A}=0.$$

- By the property of min and max operations,we have

$$S^*(M,N)=\frac{1}{n}\sum_{i=1}^{n}\frac{\left[\min\left(\mu_M(x_i),\mu_N(x_i)\right)\right]_A}{\left[\max\left(\mu_M(x_i),\mu_N(x_i)\right)\right]_A}\le 1=\frac{1}{n}\sum_{i=1}^{n}\frac{\left[\min\left(\mu_P(x_i),\mu_P(x_i)\right)\right]_A}{\left[\max\left(\mu_P(x_i),\mu_P(x_i)\right)\right]_A}=S^*(P,P),$$

  for all $M,N\in M^nFS(X)$.
  That is, $S^*(M,N)\le S^*(P,P)$, for all $M,N\in M^nFS(X)$.
- Suppose that $M\subseteq N\subseteq P$.

Then $\mu_M(x_i)\le\mu_N(x_i)\le\mu_P(x_i)$ and $min(\mu_M(x_i),\mu_P(x_i))=\mu_M(x_i)$ and $max(\mu_M(x_i),\mu_P(x_i))=\mu_P(x_i)$.

Also $[min(\mu_M(x_i),\mu_P(x_i))]_A=[\mu_M(x_i)]_A$ and $[max(\mu_M(x_i),\mu_P(x_i))]_A=[\mu_P(x_i)]_A$

$$\frac{\left[\min\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}{\left[\max\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}=\frac{\left[\mu_M(x_i)\right]_A}{\left[\mu_P(x_i)\right]_A}$$

Similarly

$$\frac{\left[\min\left(\mu_M(x_i),\mu_N(x_i)\right)\right]_A}{\left[\max\left(\mu_M(x_i),\mu_N(x_i)\right)\right]_A}=\frac{\left[\mu_M(x_i)\right]_A}{\left[\mu_N(x_i)\right]_A}$$

and

$$\frac{\left[\min\left(\mu_N(x_i),\mu_P(x_i)\right)\right]_A}{\left[\max\left(\mu_N(x_i),\mu_P(x_i)\right)\right]_A}=\frac{\left[\mu_N(x_i)\right]_A}{\left[\mu_P(x_i)\right]_A}.$$

Now $M\subseteq N$ implies that $\mu_M(x_i)\subseteq\mu_N(x_i)$).
Hence $[\mu_M(x_i)]_A\subseteq[\mu_N(x_i)]_A$, for all $x_i\in X$.

Therefore $\frac{\left[\mu_M(x_i)\right]_A}{\left[\mu_P(x_i)\right]_A}\le\frac{\left[\mu_N(x_i)\right]_A}{\left[\mu_P(x_i)\right]_A}$ and

$$\frac{\left[\min\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}{\left[\max\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}\le\frac{\left[\mu_N(x_i)\right]_A}{\left[\mu_P(x_i)\right]_A}=\frac{\left[\min\left(\mu_N(x_i),\mu_P(x_i)\right)\right]_A}{\left[\max\left(\mu_N(x_i),\mu_P(x_i)\right)\right]_A}.$$

Hence $\frac{1}{n}\frac{\left[\min\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}{\left[\max\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}=\frac{1}{n}\frac{\left[\min\left(\mu_N(x_i),\mu_P(x_i)\right)\right]_A}{\left[\max\left(\mu_N(x_i),\mu_P(x_i)\right)\right]_A}.$

Hence $S^*(M,P)\le S^*(N,P)$.

Now $\mu_P(x_i)\ge\mu_N(x_i)$ implies that $\frac{\left[\mu_P(x_i)\right]_A}{\left[\mu_M(x_i)\right]_A}\ge\frac{\left[\mu_N(x_i)\right]_A}{\left[\mu_M(x_i)\right]_A}$

and so $\frac{\left[\mu_M(x_i)\right]_A}{\left[\mu_P(x_i)\right]_A}\le\frac{\left[\mu_M(x_i)\right]_A}{\left[\mu_N(x_i)\right]_A}$

$$\frac{\left[\min\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}{\left[\max\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}\le\frac{\left[\mu_M(x_i)\right]_A}{\left[\mu_N(x_i)\right]_A}=\frac{\left[\min\left(\mu_M(x_i),\mu_N(x_i)\right)\right]_A}{\left[\max\left(\mu_M(x_i),\mu_N(x_i)\right)\right]_A}$$

$$\frac{1}{n}\frac{\left[\min\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}{\left[\max\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}\le\frac{1}{n}\frac{\left[\min\left(\mu_M(x_i),\mu_N(x_i)\right)\right]_A}{\left[\max\left(\mu_M(x_i),\mu_N(x_i)\right)\right]_A}.$$

Hence $S^*(M,P)\le S^*(M,N)$.

Note: The map S^* defined in the above theorem is the weighted similarity measure between *M* and *N* and $\frac{\left[\min\left(\mu_M(x_i),\mu_N(x_i)\right)\right]_A}{\left[\max\left(\mu_M(x_i),\mu_N(x_i)\right)\right]_A}$ is the weighted similarity degree of *M* and *N* on the element x_i.

Weighted Semi-quasi Similarity Measure Between Multi-fuzzy Set

Definition 5.1

Let $X=\{x_1,x_2,...,x_n\}$ be a set, and let $M\in M^nFS(X)$.

Then $S_\wp{}^*: M^nFS(X)\times M^nFS(X)\to[0,\infty)$ as

$$S_\wp{}^*(M,N)=\frac{1}{n}\frac{\left[T\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}{\left[S\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}$$

where the weighted matrix *A* is any $n\times 1$ fuzzy matrix, $A=[a_1,a_2,...,a_n]^T$ is any column matrix and $a_i\in I$, for $i=1, 2, ..., n$ and T (*A*,*B*) and S(*A*,*B*) denote the T and S operations on *A* and *B*, respectively.

In order to avoid the denominator being zero, we set $\frac{0}{0}=1$.

Theorem 5.2

$S_\wp{}^*$ is a semi-quasi similarity measure.

Proof

- $S_\wp{}^*(M,N)=S_\wp{}^*(N,M)$
  Apply commutativity property of *T*-operation and *S*-operation, we can easily say that
  $T(\mu_M(x_i),\mu_N(x_i))=T(\mu_N(x_i),\mu_M(x_i))$, and $S(\mu_M(x_i),\mu_N(x_i))=S(\mu_N(x_i),\mu_M(x_i))$, for all $\mu_M(x_i),\mu_N(x_i)\in I$.
- Let $A_\wp{}^*$ be a crisp set.

  $A_\wp{}^*=\{\langle x,(1,1,...,1): x\in X\rangle\}$

  Apply the property of T-operation,

$$T\left(\mu_{A_\wp^*}(x_i),\mu_{(A_\wp^*)^C}(x_i)\right)=T\left((1,1,...,1),(0,0,...,0)\right)=(0,0,...,0)$$

  Apply the property of S-operation,

$$S\left(\mu_{A_\wp^*}(x_i),\mu_{(A_\wp^*)^C}(x_i)\right)=S\left((1,1,...,1),(0,0,...,0)\right)=(1,1,...,1)$$

$$\left[T\left(\mu_{A_\wp^*}(x_i),\mu_{(A_\wp^*)^C}(x_i)\right)\right]_A=0 \text{ and } \left[S\left(\mu_{A_\wp^*}(x_i),\mu_{(A_\wp^*)^C}(x_i)\right)\right]_A=(1,1,...,1)\begin{bmatrix}a_1\\a_2\\...\\a_n\end{bmatrix},$$

  which is finite, $a_i\in[0,1]$.
  Hence

$$\frac{1}{n}\frac{\left[T\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}{\left[S\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}=\frac{1}{n}\times 0=0.$$

- By property of T and S operations,

$$S_\wp^*(M,N)=\frac{1}{n}\frac{\left[T\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}{\left[S\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}\le\frac{1}{n}\frac{\left[\min\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}{\left[\max\left(\mu_M(x_i),\mu_P(x_i)\right)\right]_A}$$

  ≤ 1, for all $M,N\in M^nFS(X)$.
  If M = N, $S_\wp^*(M,N)=1$.

Weighted Semi-quasi Similarity Measure Between Elements of a Multi-fuzzy Set

According to the above concept, the similarity measure between two elements $x_i, x_j \in X$, M_k, *for* $k = 1, 2, \ldots, n$ be multi-fuzzy sets can be defined as follows:

$$S_e^*(x_i, x_j) = \frac{1}{n}\sum_{i=1}^{n}\frac{\left[T\left(\mu_{M_k}(x_i), \mu_{M_k}(x_j)\right)\right]_A}{\left[S\left(\mu_{M_k}(x_i), \mu_{M_k}(x_j)\right)\right]_A}.$$

Observations

- $S_e^*(x_i, x_j) = S_e^*(x_j, x_i), x_i, x_j \in X$;
- $S_e^*(x_m, x_m{}^c) = 0, x_i \in X$ .Here $x_m{}^c = x_i \in X, m \neq i$;
- $S_e^*(x_m, x_m) = \max\limits_{x_i, x_j \in X}(x_i, x_j)$.

Remark

If the *t*-norm is replaced by Drastic intersection $t_{min}(a,b)$ and the *s*-norm is replaced by Drastic sum $t_{max}(a,b)$, then $S^*_{DI,DS}(M,N) \leq S_{\wp}{}^*(M,N) \leq S^*(M,N)$), for all $M,N \in M^n FS(X)$

Conclusion

In this paper, we introduced an approach to multi-fuzzy set analysis using of weighted multi-fuzzy set similarity measures. The utilization of a unique weighting mechanism, together with S and T operations, enhances the precision of comparisons between multi-fuzzy sets by accounting for the individual significance of each element. The comparative analysis is presented herein for an understanding of multi-fuzzy set analysis and the broader field of fuzzy logic.

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