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RESEARCH ARTICLE

Neural net influenced magdm problem with modified choquet integral aggregation operators and correlation coefficient for triangular fuzzy intuitionistic fuzzy sets

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Abstract

With respect to multiple attribute group decision-making (MAGDM) problems in which attribute values take the form of triangular fuzzy intuitionistic fuzzy set (TrFIFS) values, a new decision-making analysis method is developed. First, a novel correlation coefficient for the TrFIFS is proposed and then utilized in the decision-making process for the ranking of the best alternatives. Several theorems substantiate the new correlation coefficient proved to establish its effectiveness. Then, two TrFIFS choquet integral aggregation operators are developed and utilized in solving the MAGDM problem. The triangular fuzzy intuitionistic fuzzy improved choquet integral averaging (TrFIFIMCOA) operator for TrFIFS and the triangular fuzzy intuitionistic fuzzy improved choquet integral geometric (TrFIFIMCOG) operator for TrFIFS are proposed and some desirable properties are studied. The prominent characteristic of the operators is that they can not only consider the importance of the elements or their ordered positions but also reflect the correlation among the elements or their ordered positions. Using the proposed two operators, the input vector is produced for an artificial neural network (ANN), which is solved to provide an effective solution for MAGDM problem. The newly proposed correlation coefficient and the aggregation operators are effectively utilized to solve real-life decision problems. Finally, an illustrative example has been given to show the developed method and comparisons are made with existing methods.

Keywords: MAGDM, Choquet integral operators, Correlation coefficient, Triangular fuzzy intuitionistic fuzzy sets, ANN.

Introduction

There is a lot of fuzziness in real-world management settings. First proposed an intuitionistic fuzzy set can simultaneously express three states: Neutrality, resistance, and support. It does this by describing fuzziness using both membership and non-membership degrees or functions. As a result, when uncertainty like hesitation degree is present, the intuitionistic fuzzy set may represent information that is more plentiful and adaptable than the fuzzy set. For this reason, it appears to be appropriate for handling the inherent characteristics of physical phenomena in complicated management scenarios. An effective generalization of the fuzzy set is the intuitionistic fuzzy set. The fuzzy set has already had remarkable success in both theoretical and real-world applications. It follows that the intuitionistic fuzzy set should be able to mimic human decision-making processes as well as any task involving human knowledge, experience, and expertise—all of which are inherently erratic or incomplete. Therefore, it is widely accepted that the intuitionistic fuzzy set offers a wide range of potential applications in the business, environment, management, economics, and military domains Atanassov, K. (1986).

The literature contains a variety of aggregation operator types for information aggregation. Numerous writers and academics have already addressed decision-making difficulties in general, and numerous solution strategies have been put out using an enormous variety of aggregation operators. In daily life, multi attribute group decision making (MAGDM) problems arise frequently. Selecting a desired solution from a small pool of viable possibilities that have

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been assessed according to a range of quantitative and qualitative criteria is the aim of a MAGDM problem. There is a lot of room for research on MAGDM problems using digitally improved AI techniques, and there are many opportunities to combine ANN techniques with decision support systems (DSS). In this research work, the intuitionistic fuzzy set data are utilized to create the input for ANN especially when the decision-making situation is vague in nature. Choquet integral operators were much discussed in many articles. The concept of fuzzy number intuitionistic fuzzy sets was developed and the correlation coefficient for the same in a trapezoidal environment was proposed. Recently, the authors have worked on solving MAGDM problems through ANN techniques by employing several learning rules and have proposed different techniques for producing inputs for the ANN for solving those MAGDM problems. The rest of the work is organized as follows: i) Correlation coefficient for TrFIFS is proposed, ii) Two improved Choquet integral operators are proposed for MAGDM, iii) Numerical illustration comprising MAGDM algorithm and ANN algorithm is performed, and finally iv) Comparison is made with the proposed methods and some existing methods Behroozifar, M., Agahi, H. (2018), Bottero, M., (2018), Greco, S., (2013), Leonishiya, A., (2023), Liu, F. and Yuan, X.H. (2007), Pereira, M. A., (2020), Robinson, J.P., (2014), Robinson, J. P., (2024), Robinson, J. P., (2024), Wang, X.F. (2008a), Wang, X.F. (2008b), Wei, G., (2014), Xu, Z.S. (2010).

Arithmetic Operations For Trfifss

Definition 1

Triangular Fuzzy Number (TrFN)

$A = (a, b, c)$ is called a triangular fuzzy number if the membership function $\mu_A : R \to [0,1]$ is expressed as:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & a \le x \le b \\ \frac{x-c}{b-c} & b \le x \le c \\ 0 & \text{otherwise} \end{cases}$$

Where $x \in R, \quad 0 \le a \le b \le c \le 1.$

Definition 2

Triangular Fuzzy Intuitionistic Fuzzy Number (TrFIFN)

Let *X* be a non-empty set.

Then $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in X\}$ is called a Triangular Fuzzy Intuitionistic Fuzzy Number (TrFIFN) if $\mu_A(x) = \left(\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x)\right)$ and $\gamma_A(x) = \left(\gamma_{A_1}(x), \gamma_{A_2}(x), \gamma_{A_3}(x)\right)$ are triangular fuzzy numbers, which can express the membership degree and the non-membership degree of *x* in *X*, and fulfill $0 \le \mu_{A_3}(x) + \gamma_{A_3}(x) \le 1, \quad \forall x \in X.$

Suppose,

$$A = \{\langle x, [\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x)], [\gamma_{A_1}(x), \gamma_{A_2}(x), \gamma_{A_3}(x)]\rangle / x \in X\},$$

$$B = \{\langle x, [\mu_{B_1}(x), \mu_{B_2}(x), \mu_{B_3}(x)], [\gamma_{B_1}(x), \gamma_{B_2}(x), \gamma_{B_3}(x)]\rangle / x \in X\}$$

are two TrFIFNs, then the operation rules of TrFIFNs are as follows:

$$A + B = \{\langle [\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x)], [\gamma_{A_1}(x), \gamma_{A_2}(x), \gamma_{A_3}(x)]\rangle + \langle [\mu_{B_1}(x), \mu_{B_2}(x), \mu_{B_3}(x)], [\gamma_{B_1}(x), \gamma_{B_2}(x), \gamma_{B_3}(x)]\rangle\}$$

$$= \left\{\left\langle \begin{bmatrix} \mu_{A_1}(x) + \mu_{B_1}(x) - \mu_{A_1}(x)\cdot\mu_{B_1}(x), \mu_{A_2}(x) + \mu_{B_2}(x) - \mu_{A_2}(x)\cdot\mu_{B_2}(x), \\ \mu_{A_3}(x) + \mu_{B_3}(x) - \mu_{A_3}(x)\cdot\mu_{B_3}(x) \end{bmatrix}, \\ [\gamma_{A_1}(x)\cdot\gamma_{B_1}(x), \gamma_{A_2}(x)\cdot\gamma_{B_2}(x), \gamma_{A_3}(x)\cdot\gamma_{B_3}(x)] \right\rangle\right\}$$

$$A \cdot B = \left\{ \langle [\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x)], [\gamma_{A_1}(x), \gamma_{A_2}(x), \gamma_{A_3}(x)]\rangle \cdot \langle [\mu_{B_1}(x), \mu_{B_2}(x), \mu_{B_3}(x)], [\gamma_{B_1}(x), \gamma_{B_2}(x), \gamma_{B_3}(x)]\rangle \right\}$$

$$= \left\{\left\langle \begin{matrix} [\mu_{A_1}(x)\cdot\mu_{B_1}(x), \mu_{A_2}(x)\cdot\mu_{B_2}(x), \mu_{A_3}(x)\cdot\mu_{B_3}(x)], \\ \begin{bmatrix} \gamma_{A_1}(x) + \gamma_{B_1}(x) - \gamma_{A_1}(x)\cdot\gamma_{B_1}(x), \gamma_{A_2}(x) + \gamma_{B_2}(x) - \gamma_{A_2}(x)\cdot\gamma_{B_2}(x), \\ \gamma_{A_3}(x) + \gamma_{B_3}(x) - \gamma_{A_3}(x)\cdot\gamma_{B_3}(x) \end{bmatrix} \end{matrix} \right\rangle\right\}$$

$$\lambda A = \lambda([\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x)], [\gamma_{A_1}(x), \gamma_{A_2}(x), \gamma_{A_3}(x)])$$

$$= \left\{\left\langle \begin{matrix} \left[1 - \left(1 - \mu_{A_1}(x)\right)^{\lambda}, 1 - \left(1 - \mu_{A_2}(x)\right)^{\lambda}, 1 - \left(1 - \mu_{A_3}(x)\right)^{\lambda}\right], \\ \left[\left(\gamma_{A_1}(x)\right)^{n}, \left(\gamma_{A_2}(x)\right)^{n}, \left(\gamma_{A_3}(x)\right)^{n}\right] \end{matrix} \right\rangle\right\}$$

$$A^{\lambda} = ([\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x)], [\gamma_{A_1}(x), \gamma_{A_2}(x), \gamma_{A_3}(x)])^{\lambda}$$

$$= \left\{\left\langle \begin{bmatrix} \left(\mu_{A_1}(x)\right)^{n}, \left(\mu_{A_2}(x)\right)^{n}, \\ \left(\mu_{A_3}(x)\right)^{n} \end{bmatrix}, \begin{bmatrix} 1 - \left(1 - \gamma_{A_1}(x)\right)^{\lambda}, 1 - \left(1 - \gamma_{A_2}(x)\right)^{\lambda}, \\ 1 - \left(1 - \gamma_{A_3}(x)\right)^{\lambda} \end{bmatrix} \right\rangle\right\}, \lambda \ge 0$$

For the above operation rules, the following are true:

- $A + B = B + A$
- $A \cdot B = B \cdot A$
- $\lambda(A + B) = \lambda A + \lambda B, \quad \lambda \ge 0$
- $\lambda_1 A + \lambda_2 A = (\lambda_1 + \lambda_2)A, \quad \lambda_1, \lambda_2 \ge 0$
- $A^{\lambda_1} \cdot A^{\lambda_2} = (A)^{\lambda_1 + \lambda_2}, \quad \lambda_1, \lambda_2 \ge 0$

Correlation Coefficient of Trfifs

In this section, a new method is proposed for the correlation coefficient of TrFIFSs. The correlation coefficient of TzFIFS was earlier proposed and utilized for decision making problems by the authors in [7]. Based on the method in [7], a new correlation coefficient for TrFIFNs is proposed here without the inclusion of the hesitation degree.

Correlation Coefficient of TrFIFS

Let $X = \{x_1, x_2, \ldots, x_n\}$ be the finite universal set and *A*, *B* ∈ TrFIFS(*X*) be given by:

$$A = \{\langle x, [\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x)], [\gamma_{A_1}(x), \gamma_{A_2}(x), \gamma_{A_3}(x)]\rangle / x \in X\},$$

$$B = \{\langle x, [\mu_{B_1}(x), \mu_{B_2}(x), \mu_{B_3}(x)], [\gamma_{B_1}(x), \gamma_{B_2}(x), \gamma_{B_3}(x)]\rangle / x \in X\}.$$

Then the correlation coefficient of Triangular Fuzzy Intuitionistic Fuzzy Numbers (TrFIFNs) is defined as follows:

The Triangular Fuzzy Intuitionistic Fuzzy Energy is defined as:

$$E_{TrFIFS}(A) = \frac{1}{3}\sum_{i=1}^{n}[\mu_A^2(x_i) + \gamma_A^2(x_i)]$$

$$= \frac{1}{3}\Sigma_{i=1}^{n}\left[\left(\mu_{A_1}^2(x_i) + \mu_{A_2}^2(x_i) + \mu_{A_3}^2(x_i)\right) + \left(\gamma_{A_1}^2(x_i) + \gamma_{A_2}^2(x_i) + \gamma_{A_3}^2(x_i)\right)\right]$$

$$E_{TrFIFS}(B) = \frac{1}{3}\sum_{i=1}^{n}[\mu_B^2(x_i) + \gamma_B^2(x_i)]$$

$$=\frac{1}{3}\sum_{i=1}^{n}\left[\left(\mu_{B_1}^2(x_i)+\mu_{B_2}^2(x_i)+\mu_{B_3}^2(x_i)\right)+\left(\gamma_{B_1}^2(x_i)+\gamma_{B_2}^2(x_i)+\gamma_{B_3}^2(x_i)\right)\right]$$

The correlation between $A, B \in$TrFIFS(X) is defined as:

$$C_{TrFIFS}(A,B)=\frac{1}{3}\sum_{i=1}^{n}[\mu_A(x_i)\mu_B(x_i)+\gamma_A(x_i)\gamma_B(x_i)]$$

$$=\frac{1}{3}\sum_{i=1}^{n}\left\{\left[\left(\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\mu_{A_2}(x_i)\mu_{B_2}(x_i)+\mu_{A_3}(x_i)\mu_{B_3}(x_i)\right)\right]+\left[\left(\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)\right)\right]\right\}$$

The correlation coefficient $A, B\in$TrFIFS(X) is given as:

$$K_{TrFIFS}(A,B)=\frac{C_{TrFIFS}(A,B)}{\sqrt{E_{TrFIFS}(A)\cdot E_{TrFIFS}(B)}},\quad 0\le K_{TrFIFS}(A,B)\le 1.$$

Proposition 1

For $A, B \in$ TrFIFS(X), the following are true:

- $0\le K_{TrFIFS}(A,B)\le 1,$
- $C_{TrFIFS}(A,B)=C_{TrFIFS}(B,A),$
- $K_{TrFIFS}(A,B)=K_{TrFIFS}(B,A).$

Theorem 1

For $A, B\in$TrFIFS(X), then $0\le K_{TrFIFS}(A,B)\le 1.$

Proof

Since $C_{TrFIFS}(A,B)\ge 0$, it can be proved that $K_{TrFIFS}(A,B)\le 1.$

For any arbitrary real number ξ, the following relation is true:

$$0\le\sum_{i=1}^{n}\left\{\left(\mu_{A_1}(x_i)-\xi\mu_{B_1}(x_i)\right)^2+\left(\mu_{A_2}(x_i)-\xi\mu_{B_2}(x_i)\right)^2+\left(\mu_{A_3}(x_i)-\xi\mu_{B_3}(x_i)\right)^2+\left(\gamma_{A_1}(x_i)-\xi\gamma_{B_1}(x_i)\right)^2+\left(\gamma_{A_2}(x_i)-\xi\gamma_{B_2}(x_i)\right)^2+\left(\gamma_{A_3}(x_i)-\xi\gamma_{B_3}(x_i)\right)^2\right\}$$

$$=\begin{array}{l}\sum_{i=1}^{n}\left\{\left(\mu_{A_1}^2(x_i)+\gamma_{A_1}^2(x_i)\right)+\left(\mu_{A_2}^2(x_i)+\gamma_{A_2}^2(x_i)\right)+\left(\mu_{A_3}^2(x_i)+\gamma_{A_3}^2(x_i)\right)-\right.\\ 2\xi\left(\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)\right)-2\xi\left(\mu_{A_2}(x_i)\mu_{B_2}(x_i)+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)\right)-\\ 2\xi\left(\mu_{A_3}(x_i)\mu_{B_3}(x_i)+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)\right)+\xi^2\left(\mu_{B_1}^2(x_i)+\gamma_{B_1}^2(x_i)\right)+\xi^2\left(\mu_{B_2}^2(x_i)+\right.\\ \left.\left.\gamma_{B_2}^2(x_i)\right)+\xi^2\left(\mu_{B_3}^2(x_i)+\gamma_{B_3}^2(x_i)\right)\right\}\end{array}.$$

Hence,

$$\left\{\sum_{i=1}^{n}\left[\left(\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\mu_{A_2}(x_i)\mu_{B_2}(x_i)+\mu_{A_3}(x_i)\mu_{B_3}(x_i)+\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)\right)\right]\right\}^2$$

$$\le\left(\sum_{i=1}^{n}\left\{\begin{array}{l}\left(\mu_{A_1}^2(x_i)+\gamma_{A_1}^2(x_i)\right)\\+\left(\mu_{A_2}^2(x_i)+\gamma_{A_2}^2(x_i)\right)\\+\left(\mu_{A_3}^2(x_i)+\gamma_{A_3}^2(x_i)\right)\end{array}\right\}\times\sum_{i=1}^{n}\left\{\left(\begin{array}{l}\left(\mu_{B_1}^2(x_i)+\gamma_{B_1}^2(x_i)\right)\\+\left(\mu_{B_2}^2(x_i)+\gamma_{B_2}^2(x_i)\right)\\+\left(\mu_{B_3}^2(x_i)+\gamma_{B_3}^2(x_i)\right)\end{array}\right)\right\}\right)$$

The above inequality can be written as:

$$\frac{\left\{\sum_{i=1}^{n}\left[\begin{array}{c}\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\mu_{A_2}(x_i)\mu_{B_2}(x_i)+\mu_{A_3}(x_i)\mu_{B_3}(x_i)+\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)\\+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)\end{array}\right]\right\}^2}{\left(\sum_{i=1}^{n}\left\{\begin{array}{l}\left(\mu_{A_1}^2(x_i)+\gamma_{A_1}^2(x_i)\right)\\+\left(\mu_{A_2}^2(x_i)+\gamma_{A_2}^2(x_i)\right)\\+\left(\mu_{A_3}^2(x_i)+\gamma_{A_3}^2(x_i)\right)\end{array}\right\}\times\sum_{i=1}^{n}\left\{\begin{array}{l}\left(\mu_{B_1}^2(x_i)+\gamma_{B_1}^2(x_i)\right)\\+\left(\mu_{B_2}^2(x_i)+\gamma_{B_2}^2(x_i)\right)\\+\left(\mu_{B_3}^2(x_i)+\gamma_{B_3}^2(x_i)\right)\end{array}\right\}\right)}\le 1$$

Therefore $\frac{[C_{TrFIFS}(A,B)]^2}{E_{TrFIFS}(A)\cdot E_{TrFIFS}(B)}\le 1,$

Hence $K_{TrFIFS}(A,B)=\frac{C_{TrFIFS}(A,B)}{\sqrt{E_{TrFIFS}(A)\cdot E_{TrFIFS}(B)}}\le 1$

Theorem 2

Prove that $K_{TrFIFS}(A,B)=1\Leftrightarrow A=B.$

Proof

Considering the inequality in the proof of theorem 1, then the equality holds if and only if the following satisfy:

- $\mu_{A_1}(x_i)=\xi\mu_{B_1}(x_i),\quad\mu_{A_2}(x_i)=\xi\mu_{B_2}(x_i),\quad\mu_{A_3}(x_i)=\xi\mu_{B_3}(x_i),$
- $\gamma_{A_1}(x_i)=\xi\gamma_{B_1}(x_i),\quad\gamma_{A_2}(x_i)=\xi\gamma_{B_2}(x_i),\quad\gamma_{A_3}(x_i)=\xi\gamma_{B_3}(x_i),$

  for some positive real ξ.

As

$$\mu_{A_1}(x_i)+\gamma_{A_1}(x_i)+\pi_{A_1}(x_i)=\mu_{B_1}(x_i)+\gamma_{B_1}(x_i)+\pi_{B_1}(x_i)=1,$$
$$\mu_{A_2}(x_i)+\gamma_{A_2}(x_i)+\pi_{A_2}(x_i)=\mu_{B_2}(x_i)+\gamma_{B_2}(x_i)+\pi_{B_2}(x_i)=1,$$
$$\mu_{A_3}(x_i)+\gamma_{A_3}(x_i)+\pi_{A_3}(x_i)=\mu_{B_3}(x_i)+\gamma_{B_3}(x_i)+\pi_{B_3}(x_i)=1,$$

where $\pi_{A_1}(x_i)$ and $\pi_{B_1}(x_i)$ are the hesitation degree in the context of an ordinary Intuitionistic Fuzzy Set, then it means ξ = 1, and therefore $A = B$.

Theorem 3

$C_{TrFIFS}(A,B)=0\Leftrightarrow A$ and B are non-fuzzy sets and satisfy the condition $\mu_A(x_i)+\mu_B(x_i)=1$ or $\gamma_A(x_i)+\gamma_B(x_i)=1$ or $\pi_A(x_i)+\pi_B(x_i)=1,\quad\forall x_i\in X.$

Proof

For all $x_i \in X$, the following are true:

$$\left(\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)+\pi_{A_1}(x_i)\pi_{B_1}(x_i)\right)\ge 0,$$
$$\left(\mu_{A_2}(x_i)\mu_{B_2}(x_i)+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)+\pi_{A_2}(x_i)\pi_{B_2}(x_i)\right)\ge 0,$$
$$\left(\mu_{A_3}(x_i)\mu_{B_3}(x_i)+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)+\pi_{A_3}(x_i)\pi_{B_3}(x_i)\right)\ge 0.$$

Hence,

$$\left\{\begin{array}{l}\left(\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)+\pi_{A_1}(x_i)\pi_{B_1}(x_i)\right)+\\\left(\mu_{A_2}(x_i)\mu_{B_2}(x_i)+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)+\pi_{A_2}(x_i)\pi_{B_2}(x_i)\right)+\\\left(\mu_{A_3}(x_i)\mu_{B_3}(x_i)+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)+\pi_{A_3}(x_i)\pi_{B_3}(x_i)\right)\end{array}\right\}\ge 0$$

$$\left\{\begin{array}{c}\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\mu_{A_2}(x_i)\mu_{B_2}(x_i)+\mu_{A_3}(x_i)\mu_{B_3}(x_i)+\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)\\+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)+\pi_{A_1}(x_i)\pi_{B_1}(x_i)+\pi_{A_2}(x_i)\pi_{B_2}(x_i)\\+\pi_{A_3}(x_i)\pi_{B_3}(x_i)\end{array}\right\}\ge 0$$

If $C_{TrFIFS}(A,B)=0$ for all $x_i \in X$, then the following should be true:

$$\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\mu_{A_2}(x_i)\mu_{B_2}(x_i)+\mu_{A_3}(x_i)\mu_{B_3}(x_i)=0,$$
$$\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)=0,$$
$$\pi_{A_1}(x_i)\pi_{B_1}(x_i)+\pi_{A_2}(x_i)\pi_{B_2}(x_i)+\pi_{A_3}(x_i)\pi_{B_3}(x_i)=0.$$

and

If $\mu_{A_1}(x_i)=1$ then $\mu_{B_1}(x_i)=0$ and $\gamma_{A_1}(x_i)=\pi_{A_1}(x_i)=0,$
If $\mu_{A_2}(x_i)=1$ then $\mu_{B_2}(x_i)=0$ and $\gamma_{A_2}(x_i)=\pi_{A_2}(x_i)=0,$
If $\mu_{A_3}(x_i)=1$ then $\mu_{B_3}(x_i)=0$ and $\gamma_{A_3}(x_i)=\pi_{A_3}(x_i)=0.$

Also,

If $\mu_{B_1}(x_i)=1$ then $\mu_{A_1}(x_i)=0$ and $\gamma_{B_1}(x_i)=\pi_{B_1}(x_i)=0,$
If $\mu_{B_2}(x_i)=1$ then $\mu_{A_2}(x_i)=0$ and $\gamma_{B_2}(x_i)=\pi_{B_2}(x_i)=0,$

If $\mu_{B_3}(x_i) = 1$ then $\mu_{A_3}(x_i) = 0$ and $\gamma_{B_3}(x_i) = \pi_{B_3}(x_i) = 0$.
Hence

$\mu_{A_1}(x_i) + \mu_{B_1}(x_i) = 1, \quad \mu_{A_2}(x_i) + \mu_{B_2}(x_i) = 1, \quad \mu_{A_3}(x_i) + \mu_{B_3}(x_i) = 1,$

Conversely, When *A* and *B* are non-fuzzy sets and $\mu_{A_1}(x_i) + \mu_{B_1}(x_i) = 1, \quad \mu_{A_2}(x_i) + \mu_{B_2}(x_i) = 1, \quad \mu_{A_3}(x_i) + \mu_{B_3}(x_i) = 1,$

If $\mu_{A_1}(x_i) = 1$ then $\mu_{B_1}(x_i) = 0$ and $\gamma_{A_1}(x_i) = \pi_{A_1}(x_i) = 0$,
If $\mu_{A_2}(x_i) = 1$ then $\mu_{B_2}(x_i) = 0$ and $\gamma_{A_2}(x_i) = \pi_{A_2}(x_i) = 0$,
If $\mu_{A_3}(x_i) = 1$ then $\mu_{B_3}(x_i) = 0$ and $\gamma_{A_3}(x_i) = \pi_{A_3}(x_i) = 0$.
Also,
If $\mu_{B_1}(x_i) = 1$ then $\mu_{A_1}(x_i) = 0$ and $\gamma_{B_1}(x_i) = \pi_{B_1}(x_i) = 0$,
If $\mu_{B_2}(x_i) = 1$ then $\mu_{A_2}(x_i) = 0$ and $\gamma_{B_2}(x_i) = \pi_{B_2}(x_i) = 0$,
If $\mu_{B_3}(x_i) = 1$ then $\mu_{A_3}(x_i) = 0$ and $\gamma_{B_3}(x_i) = \pi_{B_3}(x_i) = 0$.

Therefore $C_{TrFIFS}(A, B) = 0$.

The case $\gamma_A(x_i) + \gamma_B(x_i) = 1$ can be proved similarly.

Theorem 4

Prove that $C_{TrFIFS}(A, A) = 1 \Leftrightarrow A$ is a non-fuzzy set.

Proof

If *A* is a non-fuzzy set, then $C_{TrFIFS}(A, A) = 1$ is obvious.

Conversely, it can be proved by the method of contradiction.

Assume *A* is not a non-fuzzy set.
Then $0 \le \mu_A(x_i) < 1, 0 \le \gamma_A(x_i) < 1$ for some x_i.
Hence $\mu_A^2(x_i) + \gamma_A^2(x_i) < 1$.
That is,
$\mu_{A_1}^2(x_i) + \gamma_{A_1}^2(x_i) < 1, \quad \mu_{A_2}^2(x_i) + \gamma_{A_2}^2(x_i) < 1$
and $\mu_{A_3}^2(x_i) + \gamma_{A_3}^2(x_i) < 1$.
Also,
$\mu_{A_1}^2(x_i) + \mu_{A_2}^2(x_i) + \mu_{A_3}^2(x_i) + \gamma_{A_1}^2(x_i) + \gamma_{A_2}^2(x_i) + \gamma_{A_3}^2(x_i) < 1.$

Then

$$C_{TrFIFS}(A, A) = \frac{1}{3}\sum_{i=1}^{n} \begin{matrix}\{\mu_{A_1}^2(x_i) + \mu_{A_2}^2(x_i) + \mu_{A_3}^2(x_i) + \\ \gamma_{A_1}^2(x_i) + \gamma_{A_2}^2(x_i) + \gamma_{A_3}^2(x_i)\}\end{matrix} < 1.$$

This is contradictory, and so *A* is a non-fuzzy set.

Some Aggregating Operators Based on the Choquet Integral with Trfifn Information

Let $\mu(x_i)(i = 1,2,3,\ldots,n)$be the weight of the elements $x_i \in X(i = 1,2,3,\ldots,n)$, where μ is a fuzzy measure, defined as follows:

Definition 3

A fuzzy measure μ on the set X is a set function $\mu: \theta(x) \to [0,1]$, satisfying the following axioms:

- $\mu(\varphi) = 0, \mu(X) = 1$;
- $A \subseteq B$ implies $\mu(A) < \mu(B), \forall A, B \subseteq X$;
- $\mu(A \cup B) = \mu(A) + \mu(B) + \rho\mu(A)\mu(B), \forall A, B \subseteq X$ and $A \cap B = \phi$, where $\rho \in (-1, \infty)$.

Especially, if $\rho = 0$, then the condition $\tilde{A} = (\langle x_i, \tilde{t}_A(x_i), \tilde{f}_A(x_i)\rangle / x_i \in X)$reduces to the axiom of additive measure

$\mu(A \cup B) = \mu(A) + \mu(B), \forall A, B \subseteq X$ and $A \cap B = \phi$.

If all the elements in X are independent, we have $\mu(A) = \sum_{i=1}^{n} \mu(\{x_i\}), \forall A \subseteq X$.

Definition 4

Let f be a positive real-valued function on X, and μ be a fuzzy measure on X. The discrete Choquet integral of f with respect to μ is defined by,

$C_\mu(f) = \sum_{i=1}^{n} f_{\sigma(i)}[\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)})]$,

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1,2,\ldots,n)$, such that $f_{\sigma(i-1)} \ge f_{\sigma(i)}\ \forall j = 2,3,\ldots,n$,

$A_{\sigma(k)} = \{x_{\sigma(i)}/j \le k\}$, for $k \ge 1$, and $A_{\sigma(0)} = \phi$.

Definition 5

Let

$\tilde{a}(x_i) = \langle (a(x_i), b(x_i), c(x_i)), (l(x_i), m(x_i), p(x_i)) \rangle \forall (i = 1,2,\ldots,n)$

be a clump of TrFIFNs and let $TrFIFWA: Q^n \to Q$ $TrFIFWA: Q^n \to Q$, if

$$TrFIFWA_\omega(\tilde{a}(x_1), \tilde{a}(x_2), \ldots, \tilde{a}(x_n)) = \sum_{i=1}^{n} \omega_i \tilde{a}(x_i)$$

$$= \left\langle \left(1 - \prod_{i=1}^{n}(1 - a(x_i))^{\omega_i}, 1 - \prod_{i=1}^{n}(1 - b(x_i))^{\omega_i}, 1 - \prod_{i=1}^{n}(1 - c(x_i))^{\omega_i}\right), \left(\prod_{i=1}^{n} l(x_i)^{\omega_i}, \prod_{i=1}^{n} m(x_i)^{\omega_i}, \prod_{i=1}^{n} p(x_i)^{\omega_i}\right) \right\rangle,$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $\tilde{a}(x_i)(i = 1,2,\ldots,n)$, and $\omega_j > 0, \sum_{i=1}^{n} \omega_i = 1$,then $TrFIFWA$ is called the Triangular Fuzzy Intuitionistic Fuzzy Weighted Averaging operator.

Definition 6

Let

$\tilde{a}(x_i) = \langle (a(x_i), b(x_i), c(x_i)), (l(x_i), m(x_i), p(x_i)) \rangle \forall (i = 1,2,\ldots,n)$

be a clump of TrFIFNs. A Triangular Fuzzy Intuitionistic Fuzzy Ordered Weighted Averaging

($TrFIFOWA$) operator of dimension n is a mapping $TrFIFOWA: Q^n \to Q$,that has an associated weight vector $w = (w_n w_1, w_2, \ldots, w_n)^T$ such that $w_j > 0, \sum_{i=1}^{n} w_i = 1$. Furthermore,

$$TrFIFOWA_w(\tilde{a}(x_1), \tilde{a}(x_2), \ldots, \tilde{a}(x_n)) = \sum_{i=1}^{n} w_i \tilde{a}(x_i)$$

$$= \left\langle \left(1 - \prod_{i=1}^{n}\left(1 - a(x_{\sigma(i)})\right)^{w_i}, 1 - \prod_{i=1}^{n}\left(1 - b(x_{\sigma(i)})\right)^{w_i}, 1 - \prod_{i=1}^{n}\left(1 - c(x_{\sigma(i)})\right)^{w_i}\right), \left(\prod_{i=1}^{n}\left(l(x_{\sigma(i)})\right)^{w_i}, \prod_{i=1}^{n}\left(m(x_{\sigma(i)})\right)^{w_i}, \prod_{i=1}^{n}\left(p(x_{\sigma(i)})\right)^{w_i}\right) \right\rangle,$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1,2,\ldots,n)$, such that $\tilde{a}(x_{\sigma(i-1)}) \ge \tilde{a}(x_{\sigma(i)})$, for all $i = 2,3,\ldots,n$.

Definition 7

Let

$\tilde{a}(x_i) = \langle (a(x_i), b(x_i), c(x_i)), (l(x_i), m(x_i), p(x_i)) \rangle \forall (i = 1,2,\ldots,n)$

be a collection of FNIFVs on X , and μ be a fuzzy measure on X , then we call

$TrFIFIMCOA_\mu(\tilde{a}(x_1), \tilde{a}(x_2), \ldots, \tilde{a}(x_n))$

$= \tilde{a}(x_{\sigma(1)})\left(\mu(A_{\sigma(1)})\right) - \left(\mu(A_{\sigma(0)})\right) \oplus \tilde{a}(x_{\sigma(2)})\left(\mu(A_{\sigma(2)})\right) - \left(\mu(A_{\sigma(1)})\right) \oplus \ldots$
$\ldots \oplus \tilde{a}(x_{\sigma(n)})\left(\mu(A_{\sigma(n)})\right) - \left(\mu(A_{\sigma(n-1)})\right)$

a Triangular Fuzzy Intuitionistic Fuzzy Improved Choquet Ordered Averaging ($TrFIFIMCOA$) operator, where $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$,

such that $\tilde{a}(x_{\sigma(j-1)}) \geq \tilde{a}(x_{\sigma(j)})$ for all $j = 2,3,\ldots,n$, $A_{\sigma(k)} = \{x_{\sigma(i)}/j \leq k\}$, for $k \geq 1$, and $A_{\sigma(0)} = \phi$.

The $TrFIFIMCOA$ operator can be transformed by induction over n:

$$TrFIFIMCOA_\mu(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n))$$
$$= \left\langle \left(1 - \prod_{i=1}^{n}\left(1 - a(x_{\sigma(i)})\right)^{\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})}, 1 - \prod_{i=1}^{n}\left(1 - b(x_{\sigma(i)})\right)^{\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})}, 1 - \prod_{i=1}^{n}\left(1 - c(x_{\sigma(i)})\right)^{\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})}\right), \left(\prod_{j=1}^{n}\left(l(x_{\sigma(i)})\right)^{\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})}, \prod_{j=1}^{n}\left(m(x_{\sigma(i)})\right)^{\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})}, \prod_{j=1}^{n}\left(p(x_{\sigma(i)})\right)^{\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})}\right)\right\rangle$$

whose aggregated value is also a TrFIFN.

Especially, if $\mu(\{x_{\sigma(i)}\}) = \mu(\{A_{\sigma(i)}\}) - \mu(\{A_{\sigma(i-1)}\}), i = 1,2,\ldots,n,$ then the $TrFIFIMCOA$ operator reduces to the $TrFIFWA$ operator. If $\mu(A) = \sum_{i=1}^{n}\mu(\{x_i\}), \forall A \subseteq X$, where $|A|$ is the number of elements in the set A, then $w_i = \mu(\{A_{\sigma(i)}\}) - \mu(\{A_{\sigma(i-1)}\}), i = 1,2,\ldots,n$ where $w = (w_1,w_2,\ldots,w_n)^T$, $w_i > 0, i = 1,2,\ldots,n,$ and $\sum_{i=1}^{n} w_i = 1$, then $TrFIFIMCOA$ operator reduces to the $TrFIFOWA$ operator.

It is easy to prove that the $TrFIFIMCOA$ operator has the following properties.

Theorem 5

(commutativity)

$TrFIFIMCOA_\mu(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n)) = TrFIFIMCOA_\mu(\tilde{a}'(x_1),\tilde{a}'(x_2),\ldots,\tilde{a}'(x_n))$,

where $(\tilde{a}'(x_1),\tilde{a}'(x_2),\ldots,\tilde{a}'(x_n))$ is any permutation of $(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n))$.

Theorem 6

(Idempotency)

If $\tilde{a}(x_j) = \tilde{a}(x)$ for all j, then

$TrFIFIMCOA_\mu(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n)) = \tilde{a}(x)$.

Theorem 7

(Monotonicity)

If $\tilde{a}(x_j) = \tilde{a}'(x)$ for all j, then

$TrFIFIMCOA_\mu(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n)) \leq TrFIFIMCOA_\mu(\tilde{a}'(x_1),\tilde{a}'(x_2),\ldots,\tilde{a}'(x_n))$.

Definition 8

Let

$\tilde{a}(x_i) = \langle(a(x_i),b(x_i),c(x_i)),(l(x_i),m(x_i),p(x_i))\rangle \forall(i = 1,2,\ldots,n)$ be a clump of TrFIFNs, and let $TrFIFWG: Q^n \to Q$ $TrFIFWG: Q^n \to Q$;if

$$TrFIFWG_\omega(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n)) = \prod_{i=1}^{n}\tilde{a}(x_i)^{\omega_i}$$
$$= \left\langle \begin{matrix}\left(\prod_{i=1}^{n}(a(x_i))^{\omega_i}, \prod_{i=1}^{n}(b(x_i))^{\omega_i}, \prod_{i=1}^{n}(c(x_i))^{\omega_i}\right), \\ \left(1 - \prod_{i=1}^{n}(1 - l(x_i))^{\omega_i}, 1 - \prod_{i=1}^{n}(1 - m(x_i))^{\omega_i}, 1 - \prod_{i=1}^{n}(p(x_i))^{\omega_i}\right)\end{matrix}\right\rangle$$

Where $\omega = (\omega_1,\omega_2,\ldots,\omega_n)^T$ is the weight vector of $\tilde{a}(x_i)(i = 1,2,\ldots,n)$,and $\omega_j > 0, \sum_{i=1}^{n}\omega_i = 1$, then $TrFIFWG$ is called the Triangular Fuzzy Intuitionistic Fuzzy Weighted Geometric ($TrFIFWG$) operator.

Definition 9

Let

$\tilde{a}(x_i) = \langle(a(x_i),b(x_i),c(x_i)),(l(x_i),m(x_i),p(x_i))\rangle \forall(i = 1,2,\ldots,n)$ be a clump of TrFIFNs. A Triangular Fuzzy Intuitionistic Fuzzy Ordered Weighted Geometric

(*TrFIFOWG*) operator of dimension n is a mapping $TrFIFOWG: Q^n \to Q$, that has an associated weight vector $w = (w_1,w_2,\ldots,w_n)^T$ such that and $w_j > 0$, $\sum_{i=1}^{n} w_i = 1$. Furthermore,

$$TrFIFOWG_w(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n)) = \prod_{i=1}^{n}\tilde{a}(x_{\sigma(i)})^{w_i}$$
$$= \left\langle\left(\prod_{i=1}^{n}\left(a(x_{\sigma(i)})\right)^{w_i}, \prod_{i=1}^{n}\left(b(x_{\sigma(i)})\right)^{w_i}, \prod_{i=1}^{n}\left(c(x_{\sigma(i)})\right)^{w_i}\right), \left(1 - \prod_{i=1}^{n}\left(1 - l(x_{\sigma(i)})\right)^{w_i}, 1 - \prod_{i=1}^{n}\left(1 - m(x_{\sigma(i)})\right)^{w_i}, 1 - \prod_{i=1}^{n}\left(1 - p(x_{\sigma(i)})\right)^{w_i}\right)\right\rangle$$

where $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$, ,such that $\tilde{a}(x_{\sigma(i-1)}) \geq \tilde{a}(x_{\sigma(i)})$ for all $i = 2,3,\ldots,n$.

In the following, we shall develop the triangular fuzzy intuitionistic fuzzy improved choquet ordered geometric mean (TrFIFIMCOGM) operator based on the Choquet integrals developed long ago.

Definition 10

Let

$\tilde{a}(x_i) = \langle(a(x_i),b(x_i),c(x_i)),(l(x_i),m(x_i),p(x_i))\rangle \forall(i = 1,2,\ldots,n)$ be a clump of TrFIFNs on XX, and μ be a fuzzy measure on XX, then we call $TrFIFIMCOGM_\mu(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n))$

$$TrFIFIMCOGM_\mu(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n))$$
$$= \tilde{a}(x_{\sigma(1)})^{(\mu(A_{\sigma(1)}))-(\mu(A_{\sigma(0)}))} \oplus \tilde{a}(x_{\sigma(2)})^{(\mu(A_{\sigma(2)}))-(\mu(A_{\sigma(1)}))} \oplus \ldots \oplus \tilde{a}(x_{\sigma(n)})^{(\mu(A_{\sigma(n)}))-(\mu(A_{\sigma(n-1)}))}$$

a TrFIFIMCOGM operator, where $(\sigma(1),\sigma(2),\ldots,\sigma(n))$ is a permutation of $(1,2,\ldots,n)$, such that $\tilde{a}(x_{\sigma(j-1)}) \geq \tilde{a}(x_{\sigma(j)})$ for all $j = 2,3,\ldots,n$, $A_{\sigma(k)} = \{x_{\sigma(i)}/j \leq k\}$, for $k \geq 1$,and $A_{\sigma(0)} = \phi$.

With the operation of fuzzy number intuitionistic fuzzy values, the TrFIFIMCOGM operator can be transformed into following form by induction over n:

$$TrFIFIMCOGM_\mu(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n))$$
$$= \left\langle\left(\prod_{j=1}^{n}\left(a(x_{\sigma(i)})\right)^{\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})}, \prod_{j=1}^{n}\left(b(x_{\sigma(i)})\right)^{\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})}, \prod_{j=1}^{n}\left(c(x_{\sigma(i)})\right)^{\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})}\right),\right.$$
$$\left(1 - \prod_{i=1}^{n}\left(1 - l(x_{\sigma(i)})\right)^{\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})}\right.$$

$1-\prod_{i=1}^{n}\left(1-m(x_{\sigma(i)})\right)^{\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})}, 1-\prod_{i=1}^{n}\left(1-p(x_{\sigma(i)})\right)^{\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})}\Big)\Big\rangle$

whose aggregated value is also a TrFIFN.

Especially, if

$\mu(\{x_{\sigma(i)}\}) = \mu(\{A_{\sigma(i)}\}) - \mu(\{A_{\sigma(i-1)}\}), i = 1,2,\ldots,n$, then TrFIFIMCOGM operator reduces to the TrFIFWG operator. If $\mu(A) = \sum_{i=1}^{n}\mu(\{x_i\}), \forall A \subseteq X$, where $|A|$ is the number of elements in the set A, then $w_i = \mu(\{A_{\sigma(i)}\}) - \mu(\{A_{\sigma(i-1)}\}), i = 1,2,\ldots,n$ where $w = (w_1, w_2, \ldots, w_n)^T$, $w_i > 0, i = 1,2,\ldots,n$, and $\sum_{i=1}^{n} w_i = 1$, then TrFIFIMCOGM operator reduces to the TrFIFOWG operator.

It is easy to prove that the TrFIFIMCOGM operator has the following properties.

Theorem 1

(commutativity)

$TrFIFIMCOGM_{\mu}(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n)) = TrFIFIMCOGM_{\mu}(\tilde{a}'(x_1),\tilde{a}'(x_2),\ldots,\tilde{a}'(x_n))$,

where $(\tilde{a}'(x_1),\tilde{a}'(x_2),\ldots,\tilde{a}'(x_n))$ is any permutation of $(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n))$.

Theorem 2

(Idempotency)

If $\tilde{a}(x_j) = \tilde{a}(x)$ for all j, then

$TrFIFIMCOGM_{\mu}(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n)) = \tilde{a}(x)$.

Theorem 3

(Monotonicity)

If $\tilde{a}(x_j) = \tilde{a}'(x)$for all j, then

$TrFIFIMCOGM_{\mu}(\tilde{a}(x_1),\tilde{a}(x_2),\ldots,\tilde{a}(x_n)) \le TrFIFIMCOGM_{\mu}(\tilde{a}'(x_1),\tilde{a}'(x_2),\ldots,\tilde{a}'(x_n))$.

Numerical Illustration: Modified Choquet Integral Operators With Trfifn Information

Algorithm for MAGDM with Modified Choquet Integrals

Let $A = \{A_1, A_2, \ldots, A_m\}$ be a discrete set of alternatives, $G = \{G_1, G_2, \ldots, G_n\}$ be the set of attributes, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ the weighting vector of the attributes $G_j (j = 1,2,..,n)$, where $\omega_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$. Suppose that $\tilde{R} = (\tilde{r}_{ij})_{m\times n} = \langle (a_{ij}, b_{ij}, c_{ij}), (l_{ij}, m_{ij}, p_{ij})\rangle_{m\times n}$ is the TrFIFN decision matrix, where (a_{ij}, b_{ij}, c_{ij}) indicates the degree to which the alternative A_i satisfies the attribute G_j, (l_{ij}, m_{ij}, p_{ij}) indicates the degree to which the alternative does not satisfy the attribute $G_j (a_{ij}, b_{ij}, c_{ij}) \subset [0,1], (l_{ij}, m_{ij}, p_{ij}) \subset [0,1], c_{ij} + p_{ij} \le 1, i = 1,2,\ldots,m, j = 1,2,\ldots,n$.

In the following, we apply the triangular fuzzy intuitionistic fuzzy improved choquet integral averaging (TrFIFIMCOA) and TrFIFIMCOGM to multiple attribute decision making with TrFIFN information.

Step 1

If we emphasize the group's influence, we utilize the decision information given in matrix $\tilde{R}$, and the TrFIFIMCOA operator $\tilde{r}_i = \langle (a_j, b_j, c_j), (l_j, m_j, p_j)\rangle = TrFIFIMCOA\ (\tilde{r}_{i1}, \tilde{r}_{i2}, \ldots, \tilde{r}_{in})$, $i = 1,2,\ldots,m, j = 1,2,\ldots,n$, to derive the overall preference values $\tilde{r}_i (i = 1,2,\ldots,m)$ of the alternative A_i. Otherwise we utilize the decision information given in matrix $\tilde{R}$, and the TrFIFIMCOGM operator,

$\tilde{r}_i = \langle (a_j, b_j, c_j), (l_j, m_j, p_j)\rangle = TrFIFCOGM\ (\tilde{r}_{i1}, \tilde{r}_{i2}, \ldots, \tilde{r}_{in})$, $i = 1,2,\ldots,m, j = 1,2,\ldots,n$,

to derive the overall preference values $\tilde{r}_i (i = 1, 2,..,m)$ of the alternative A_i.

Step 2

Calculate the Correlation coefficient proposed in the work with the overall preference values and the Perfect-TrFIFN (P-TrFIFN, $\widetilde{r^+}_i = \langle (1,1,1), (0,0,0)\rangle$) value.

Step 3

Rank all the alternatives $A_i (i = 1,2,\ldots,m)$ and select the best one(s) in accordance with the obtained highest correlation coefficient.

Step 4

End.

Illustrative example: MAGDM with Modified Choquet Integrals

In this section, we shall present a numerical example to show potential evaluation of emerging technology commercialization with hesitant fuzzy information in order to illustrate the method proposed in this paper. The experts select four attributes to evaluate the possible emerging technology enterprises: G1 is the technical advancement; G2 is the potential market and market risk; G3 is the industrialization infrastructure, human resources and financial conditions; G4 is the employment creation and the development of science and technology. The five possible emerging technology enterprises $A_i (i = 1,2,..,5)$ are to be evaluated using the fuzzy number intuitionistic fuzzy values by the decision maker under the above four attributes, and the decision matrices $(\tilde{r}_{ij}^{(k)})_{5\times 4} (k = 1,2,3)$ as follows:

$$\tilde{R} = \begin{bmatrix} \langle (0.2,0.3,0.4),(0.3,0.4,0.4)\rangle & \langle (0.5,0.6,0.7),(0.1,0.1,0.1)\rangle & \langle (0.5,0.5,0.6),(0.1,0.1,0.2)\rangle & \langle (0.4,0.5,0.6),(0.1,0.1,0.1)\rangle \\ \langle (0.3,0.4,0.5),(0.1,0.2,0.3)\rangle & \langle (0.4,0.5,0.5),(0.1,0.1,0.2)\rangle & \langle (0.3,0.4,0.5),(0.1,0.2,0.3)\rangle & \langle (0.1,0.2,0.3),(0.3,0.4,0.5)\rangle \\ \langle (0.1,0.2,0.3),(0.3,0.4,0.5)\rangle & \langle (0.3,0.4,0.5),(0.2,0.2,0.3)\rangle & \langle (0.6,0.7,0.8),(0.1,0.1,0.1)\rangle & \langle (0.1,0.1,0.2),(0.4,0.5,0.6)\rangle \\ \langle (0.4,0.5,0.6),(0.1,0.1,0.1)\rangle & \langle (0.7,0.7,0.7),(0.1,0.1,0.1)\rangle & \langle (0.4,0.5,0.5),(0.1,0.2,0.3)\rangle & \langle (0.3,0.4,0.5),(0.2,0.3,0.3)\rangle \\ \langle (0.6,0.6,0.7),(0.1,0.1,0.1)\rangle & \langle (0.4,0.4,0.4),(0.1,0.2,0.3)\rangle & \langle (0.6,0.6,0.6),(0.1,0.1,0.1)\rangle & \langle (0.2,0.3,0.3),(0.3,0.4,0.5)\rangle \end{bmatrix}.$$

Then, we utilize the approach developed to get the most desirable emerging technology enterprise(s).

Step 1

Suppose the fuzzy measure of attribute of $G_j (j = 1,2,\ldots,n)$ and attribute sets of G as follows:

$\mu(G_1) = 0.30, \mu(G_2) = 0.35, \mu(G_3) = 0.30, \mu(G_4) = 0.22, \mu(G_1,G_2) = 0.70,$
$\mu(G_1,G_3) = 0.60, \mu(G_1,G_4) = 0.55, \mu(G_2,G_3) = 0.50, \mu(G_2,G_4) = 0.45,$
$\mu(G_3,G_4) = 0.40, \mu(G_1,G_2,G_3) = 0.82, \mu(G_1,G_2,G_4) = 0.87,$

$\mu(G_1,G_3,G_4) = 0.75, \mu(G_2,G_3,G_4) = 0.60, \mu(G_1,G_2,G_3,G_4) = 1.00$

Then, by using TrFIFCOA operator, it follows that

$$TrFIFCOA_\mu(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3,\tilde{a}_4) = \langle(1-(1-0.5)^{0.30}\times(1-0.5)^{0.70-0.30}\times(1-0.4)^{0.82-0.50}\times(1-0.2)^{1-0.60},$$
$$1-(1-0.6)^{0.30}\times(1-0.5)^{0.70-0.30}\times(1-0.5)^{0.82-0.50}\times(1-0.3)^{1-0.60},$$
$$1-(1-0.7)^{0.30}\times(1-0.6)^{0.70-0.30}\times(1-0.6)^{0.82-0.50}\times(1-0.4)^{1-0.60}),$$
$$(0.1^{0.30}\times0.1^{0.70-0.30}\times0.1^{0.82-0.50}\times0.3^{1-0.60},$$
$$0.1^{0.30}\times0.1^{0.70-0.30}\times0.1^{0.82-0.50}\times0.4^{1-0.60},$$
$$0.1^{0.30}\times0.2^{0.70-0.30}\times0.1^{0.82-0.50}\times0.4^{1-0.60})\rangle$$
$$\tilde{r}_1 = \langle(1-0.478, 1-0.399, 1-0.294),(0.059,0.066,0.087)\rangle$$
$$\tilde{r}_1 = \langle(0.522,0.601,0.706),(0.059,0.066,0.087)\rangle.$$

Similarly all the other values can be computed:

$$\tilde{r}_2 = \langle(0.364,0.486,0.572),(0.059,0.109,0.197)\rangle,$$
$$\tilde{r}_3 = \langle(0.426,0.473,0.618),(0.124,0.149,0.202)\rangle,$$
$$\tilde{r}_4 = \langle(0.683,0.672,0.715),(0.050,0.074,0.084)\rangle,$$
$$\tilde{r}_5 = \langle(0.578,0.600,0.633),(0.059,0.083,0.103)\rangle.$$

Step 2

If we emphasize the group's influence, we utilize the decision information given in matrix $\tilde{R}$, and the TrFIFIMCOA operator to obtain the overall preference values $\tilde{r}_i$ of the emerging technology enterprises $A_i (i = 1, 2, \ldots, 5)$.

$$\tilde{r}_1 = \langle(0.522,0.601,0.706),(0.059,0.066,0.087)\rangle$$
$$\tilde{r}_2 = \langle(0.364,0.486,0.572),(0.059,0.109,0.197)\rangle$$
$$\tilde{r}_3 = \langle(0.426,0.473,0.618),(0.124,0.149,0.202)\rangle$$
$$\tilde{r}_4 = \langle(0.683,0.672,0.715),(0.050,0.074,0.084)\rangle$$
$$\tilde{r}_5 = \langle(0.578,0.600,0.633),(0.059,0.083,0.103)\rangle.$$

Step 3

Compute the correlation coefficient for the overall preference values of TrFIFIMCOA and the Perfect-TrFIFN (P-TrFIFN, $\widetilde{r^+}_i = \langle(1,1,1),(0,0,0)\rangle$) value.

Step 4

Rank all the emerging technology enterprises $A_i (i = 1,2,\ldots,5)$ in accordance with the proposed correlation coefficients of the overall TrIFN preference values, we observe that $A_4 > A_5 > A_1 > A_2 > A_3$, and thus, the most desirable emerging technology enterprises is A_4.

If we emphasize the individual influence, we utilize the decision information given in matrix $\tilde{R}$ and the TrFIFIMCOGM operator to obtain the overall preference values $\tilde{r}_i$ of the emerging technology enterprises. $A_i (i = 1,2,\ldots,5)$:

$$\tilde{r}_1 = \langle(0.241,0.322,0.431),(0.221,0.268,0.302)\rangle$$
$$\tilde{r}_2 = \langle(0.127,0.221,0.305),(0.221,0.327,0.452)\rangle$$
$$\tilde{r}_3 = \langle(0.101,0.148,0.253),(0.356,0.430,0.534)\rangle$$

Table 1: Correlation coefficient for overall preference values of TrFIFIMCOA

S. No.	*TrFIFN Energy E(A)*	*TrFIFN Correlation C(A, B)*	*TrFIFN Correlation Coefficient K(A, B)*
1	0.3825	0.6096	0.9857
2	0.2500	0.4740	0.9480
3	0.2885	0.5056	0.9413
4	0.4814	0.6900	0.9945
5	0.3719	0.6036	0.9898

Table 2: Correlation coefficient for overall preference values of TrFIFIMCOGM

S. No.	*TrFIFN Energy E(A)*	*TrFIFN Correlation C(A, B)*	*TrFIFN Correlation Coefficient K(A, B)*
1	0.1864	0.3313	0.7674
2	0.1727	0.2176	0.5237
3	0.2309	0.1673	0.3482
4	0.1991	0.3700	0.8292
5	0.1891	0.3113	0.7159

$$\tilde{r}_4 = \langle(0.287,0.378,0.445),(0.179,0.251,0.281)\rangle$$
$$\tilde{r}_5 = \langle(0.274,0.322,0.338),(0.221,0.295,0.372)\rangle.$$

Compute the correlation coefficient for the overall preference values of TrFIFIMCOGM and the Perfect-TrFIFN (P-TrFIFN, $\widetilde{r^+}_i = \langle(1,1,1),(0,0,0)\rangle$) value.

Step 4

Rank all the emerging technology enterprises $A_i (i = 1,2,\ldots,5)$ in accordance with the proposed correlation coefficients of the overall TrIFN preference values, we observe that $A_4 > A_1 > A_5 > A_2 > A_3$, and thus the most desirable emerging technology enterprises is A_4.

ANN Architecture for solving MAGDM problems with TrFIFNS with TrFIFIMCOA/TrFIFIMCOGM

Step 1

Defuzzify the overall TrIFN preference values obtained from TrFIFIMCOA/TrFIFIMCOGM operators for inputs of ANN

Step 2

Define the ANN parameters
Fix number of input features
Fix number of neurons in the hidden layer
Fix number of output features (same as input size)
Fix Learning rate for the gradient descent
Fix number of training epochs

Step 3

Initialize weights and biases
Confirm hidden_layer_size, output_size
Confirm Bias for hidden_layer_size

Step 4

Activation function (ReLU) and its derivative

define ReLU(x)
 return maximum(0, x)
define ReLU _derivative(x):
 return value, where(x > 0, 1, 0)

Step 5

Forward and backward pass functions

define forward_pass(X):
 Hidden layer computations
Output layer computations
Linear activation for the output layer
return output
def backward_pass (X)

Step 6

Compute the loss (Mean Squared Error)

Step 7

Compute gradients
Update weights and biases: return loss
Training loop
Final prediction after training

Inputs

(The overall preference values using TrFIFIMCOA operator) (Table 1):

$$\tilde{r}_1 = \langle(0.522,0.601,0.706),(0.059,0.066,0.087)\rangle$$
$$\tilde{r}_2 = \langle(0.364,0.486,0.572),(0.059,0.109,0.197)\rangle$$
$$\tilde{r}_3 = \langle(0.426,0.473,0.618),(0.124,0.149,0.202)\rangle$$
$$\tilde{r}_4 = \langle(0.683,0.672,0.715),(0.050,0.074,0.084)\rangle$$
$$\tilde{r}_5 = \langle(0.578,0.600,0.633),(0.059,0.083,0.103)\rangle.$$

Defuzzyfying the above five inputs of the alternatives using the hesitation degree for TrFIFNs, we get:

$\tilde{r}_1 = 0.3196, \tilde{r}_2 = 0.4043, \tilde{r}_3 = 0.3360, \tilde{r}_4 = 0.2406, \tilde{r}_5 = 0.3146.$

Providing these as the inputs for the above ANN algorithm, we have the final predictions for the 5 input samples: [[0.09850305 0.09937405 0.09859918 0.09918655 0.09993035]]. Hence the ranking according to the output of the ANN is $A_5 > A_2 > A_4 > A_3 > A_1$.

Inputs

(The overall preference values using TrFIFIMCOGM operator) (Table 2):

$$\tilde{r}_1 = \langle(0.241,0.322,0.431),(0.221,0.268,0.302)\rangle$$
$$\tilde{r}_2 = \langle(0.127,0.221,0.305),(0.221,0.327,0.452)\rangle$$
$$\tilde{r}_3 = \langle(0.101,0.148,0.253),(0.356,0.430,0.534)\rangle$$
$$\tilde{r}_4 = \langle(0.287,0.378,0.445),(0.179,0.251,0.281)\rangle$$
$$\tilde{r}_5 = \langle(0.274,0.322,0.338),(0.221,0.295,0.372)\rangle$$

Defuzzyfying the above five inputs of the alternatives using the hesitation degree for TrFIFNs, we get:

$\tilde{r}_1 = 0.405, \tilde{r}_2 = 0.449, \tilde{r}_3 = 0.392, \tilde{r}_4 = 0.393, \tilde{r}_5 = 0.392.$

Table 3: Comparison of the rankings from proposed methods with existing methods

S. No.	*Operator type*	*Ranking of alternatives*
1	Correlation Coefficient Ranking with TrFIFIMCOA	$A_4 > A_5 > A_1 > A_2 > A_3$
2	Correlation Coefficient Ranking with TrFIFIMCOGM	$A_4 > A_1 > A_5 > A_2 > A_3$
3	ANN Influenced MAGDM with TrFIFIMCOA	$A_5 > A_2 > A_4 > A_3 > A_1$
4	ANN Influenced MAGDM with TrFIFIMCOGM	$A_5 > A_1 > A_2 > A_3 > A_4$
5	FNIFCOA [12]	$A_4 > A_5 > A_1 > A_2 > A_3$
6	FNIFCOGM [12]	$A_4 > A_5 > A_1 > A_2 > A_3$

Providing these as the inputs for the above ANN algorithm, we have the final predictions for the 5 input samples: [[0.09902642 0.09901096 0.09898628 0.09866711 0.10022129]] (Table 3).

Hence the ranking according to the output of the ANN is $A_5 > A_1 > A_2 > A_3 > A_4$.

Discussion

In this work, two new operators proposed for MAGDM problem solving are utilized for producing the input vector for Artificial Neural Network (ANN). In recent days, the use of ANN in identifying the best alternatives for decision problems has gained much attention [5], [9] and [10]. In this regard, as a novel initiative in utilizing ANN for solving MAGDM problems, the proposed algorithm and correlation coefficient have contributed to a large extent. The numerical illustration provided above proves the efficiency and productivity in comparison with the existing methods, where the linear relationship between the decision variables where not of much concern in the earlier research. Rather, from the proposed methods, the linear relationship between the decision variables is preserved, which is an added advantage to the decision maker in providing a good consensus for the stakeholders involved in the problem. The following comparison table reveals the different options arrived in utilizing the proposed methods and the methods already existing in the literature.

Conclusion

The traditional choquet integral aggregation operators are generally suitable for aggregating the information taking the form of numerical values, and yet they will fail in dealing with TrFIFN information. In this paper, we have developed two aggregation operators based on intuitionistic fuzzy Choquet integral operations: The TrFIFIMCOA operator and the TrFIFIMCOGM operator. In this work, the two operators are used to produce the input vector for the Artificial Neural Network which in turn is expected to solve the MAGDM problem. The prominent characteristic of the operators is that they can not only account for the importance of the elements or their ordered positions but also reflect the correlation among the elements or their ordered positions. We have studied some desirable properties of both the new operators, such as commutativity, idempotency and monotonicity, and applied the operators to MAGDM problems with TrFIFN information. Alongside this, a new correlation coefficient for TrFIFNs is also proposed to preserve the linear relationship between the decision variables. The new aggregation operators are then for ANN and a comparison between the methods is made in the discussion section, which shows consistency in the method of ranking of the alternatives. In the future, several aggregation operators can be proposed for computations through ANN for ease of saving time and cost of solving any decision problem.

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