



## RESEARCH ARTICLE

# Fuzzy inventory model with warehouse limits and carbon emission

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## Abstract

The theory of optimization deals with some techniques to find optimal solutions for a given mathematical problem. This paper examines the use of the Yager ranking method to minimize the total cost in an EOQ. The assortment of goods and different costs with demand must be balanced for an inventory management system to operate without any issues. The sustainable approach takes carbon emissions to be considered with constraints on warehousing capacity. The order lot quantity is not allowed to exceed the capacity limit available warehouse. The parameters are fuzzified by using a trapezoidal fuzzy number. The Yager ranking method is used to find the total cost. Finally numerical example is carried out.

**Keywords:** Carbon emission, Economic order quantity, Trapezoidal method, Yager ranking method.

## Introduction

Inventory models deal with the size of the order as well as the timing of placing orders for certain commodities. This study focuses on methods for optimizing this decision while taking into account the costs of obtaining the commodities, holding an inventory, and the cost of shortages. The key to the inventory model is to minimize the total cost and maximize the profit. It is determining the EOQ that minimizes the total costs. The EOQ model was created by Harris in the year 1913. It minimizes a various cost such as ordering costs, carrying costs, and storage costs. It discusses the optimal order quantity that a company should place with a supplier to reduce inventory expenses while balancing inventory carrying and average fixed order costs determined by using the EPQ model. Park had introduced in the year 1983 the fuzzy set with EOQ. It discusses an inventory model in a fuzzy sense by considering the shortages found using various fuzzy numbers Sanjukta

Malakar and Nabendu Sen in the year 2015; Dolgui *et al.*, (2010).

To prevent the rise of global temperatures, conducted research on ways to decrease carbon emissions. And also looked into the viability of carbon emission trading as a market-based approach. Utilization of advanced technological tools can reduce emissions. In order to reduce carbon emissions, the greenhouse company sector is required by carbon emission rules to re-create its optimal choices. Investment in green technologies meets its goal of maximizing profit. Carbon emissions generated by supply chain activities, particularly by transport, contributed greatly to global warming. It is discussed that reducing carbon emissions can increase production quantity and also increase profits Hua *et al.* (2011) (Mishra *et al.*, 2020a) (Micheli & Mantella, 2018) Cao and Yu (2018).

It is discussed Yager's ranking approach for linear programming using trapezoidal fuzzy numbers. In order to determine the best solution for a residential project and, the Yager ranking function is used along with a fuzzy network model and project management. This model considers the EOQ of carbon emissions, taking into account the limit of warehouse capacity. This proposed model is modified from the model. This research is expected can solve inventory planning problems optimally based on carbon emissions and limit of warehouse capacity Karyati(2018), Huda Fadhl Abbass (2022), K. Kalaiarasi (2020).

## Definition

### Fuzzy Set

The fuzzy set  $\tilde{A}$  is the set of ordered pairs  $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ . The mapping  $\mu_{\tilde{A}} : X \rightarrow [0,1]$  is membership of  $x \in X$  in  $\tilde{A}$ .

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**α-cut**

The α-cut of Y is defined by  $Y_\alpha = \{ x \in X : \mu_y(x) \geq \alpha \}$  where given  $\alpha \in [0, 1]$ .

**Yager Ranking Method**

If any fuzzy number's α-cut of  $\tilde{Y}$  is  $[Y_L(\alpha) + Y_g(\alpha)]$  then Yager ranking Index is

$$I(\tilde{Y}) = \frac{1}{2} \int_0^1 [Y_L(\alpha) + Y_g(\alpha)] d\alpha$$

**Notation**

- D – Demand/year
- Q – Total quantity of delivery
- S - Shortage cost
- $D_T$  - Cost of delivering the products
- $D_I$  – Inventory cost for products/unit/year
- $C_I$  – Carbon emissions of products in transportation/unit
- $C_T$  – Carbon emissions of products in inventory/unit/year
- μ – Tax rate proportion
- U - Unit room for usage product
- $C_s$  - Maximum storage capacity
- $\tilde{D}$  – Fuzzy Demand/year
- $\tilde{Q}$  – Fuzzy total quantity of delivery
- $\tilde{S}$  - Fuzzy shortage cost
- $\tilde{D}_T$  - Fuzzy Cost of delivering the products
- $\tilde{D}_I$  – Fuzzy Inventory cost for products/unit/year
- $\tilde{C}_I$  – Fuzzy carbon emissions of products in transportation/unit
- $\tilde{C}_T$  – Fuzzy carbon emissions of products in inventory/unit/year
- $\tilde{\mu}$  – Fuzzy tax rate proportion
- $\tilde{U}$  - Fuzzy unit room for usage product
- $\tilde{C}_s$  - Fuzzy maximum storage capacity

**Assumption**

- The demand is constant and with certainty known, so demand in 1-year is fixed.
- Transportation costs, costs save, the cost of carbon emissions is fixed;
- Warehouse capacity does not change.
- Models are used to complete a single product.

**Crisp Sense**

$$T_c = \frac{DD_T}{Q} + \frac{QSD_I}{2} + \mu \left[ \frac{DC_T}{Q} + \frac{QC_I}{2} \right] + QU - C_s$$

$$T_c = \frac{DD_T}{Q} + \frac{QSD_I}{2} + \frac{\mu DC_T}{Q} + \frac{\mu QC_I}{2} + QU - C_s \quad \text{--- (1)}$$

Partially differentiate equation (1) & equate to 0

$$\frac{\partial T_c}{\partial Q} = 0 \Rightarrow -\frac{DD_T}{Q^2} + \frac{SD_I}{2} - \frac{\mu DC_T}{Q^2} + \frac{\mu C_I}{2} + U$$

The economic order quantity is

$$\Rightarrow Q^* = \sqrt{\frac{2D[D_T + \mu C_T]}{SD_I + \mu C_I + 2U}}$$

**Fuzzy Sense**

Consider the proposed model in a fuzzy sense. The parameters  $D, S, D_T, D_I, C_T, C_I, \mu, U, C_s$  be the fuzzy number.

$$D(\alpha D) = [L^{-1}D(\alpha D), R^{-1}D(\alpha D)]$$

$$D_T(\alpha D_T) = [L^{-1}D_T(\alpha D_T), R^{-1}D_T(\alpha D_T)]$$

$$D_I(\alpha D_I) = [L^{-1}D_I(\alpha D_I), R^{-1}D_I(\alpha D_I)]$$

$$C_T(\alpha C_T) = [L^{-1}C_T(\alpha C_T), R^{-1}C_T(\alpha C_T)]$$

$$C_I(\alpha C_I) = [L^{-1}C_I(\alpha C_I), R^{-1}C_I(\alpha C_I)]$$

$$\mu(\alpha \mu) = [L^{-1}\mu(\alpha \mu), R^{-1}\mu(\alpha \mu)]$$

$$S(\alpha S) = [L^{-1}S(\alpha S), R^{-1}S(\alpha S)]$$

$$U(\alpha U) = [L^{-1}U(\alpha U), R^{-1}U(\alpha U)]$$

$$C_s(\alpha C_s) = [L^{-1}C_s(\alpha C_s), R^{-1}C_s(\alpha C_s)]$$

$T_c$  can be written as

$$K_1[\alpha D_T, \alpha D] = \frac{1}{4} \left\{ \left[ \int_0^1 L^{-1}D_T(\alpha D_T) d\alpha D_T \int_0^1 L^{-1}D(\alpha D) d\alpha D \right] + \left[ \int_0^1 R^{-1}D_T(\alpha D_T) d\alpha D_T \int_0^1 R^{-1}D(\alpha D) d\alpha D \right] \right\}$$

$$K_2[\alpha C_T, \alpha D, \alpha \mu] = \frac{1}{4} \left\{ \left[ \int_0^1 L^{-1}C_T(\alpha C_T) d\alpha C_T \int_0^1 L^{-1}D(\alpha D) d\alpha D \int_0^1 L^{-1}\mu(\alpha \mu) d\alpha \mu \right] + \left[ \int_0^1 R^{-1}C_T(\alpha C_T) d\alpha C_T \int_0^1 R^{-1}D(\alpha D) d\alpha D \int_0^1 R^{-1}\mu(\alpha \mu) d\alpha \mu \right] \right\}$$

$$K_3[\alpha D_I, \alpha S] = \frac{1}{4} \left\{ \left[ \int_0^1 L^{-1}D_I(\alpha D_I) d\alpha D_I \int_0^1 L^{-1}S(\alpha S) d\alpha S \right] + \left[ \int_0^1 R^{-1}D_I(\alpha D_I) d\alpha D_I \int_0^1 R^{-1}S(\alpha S) d\alpha S \right] \right\}$$

$$K_4[\alpha C_I, \alpha \mu] = \frac{1}{4} \left\{ \left[ \int_0^1 L^{-1}C_I(\alpha C_I) d\alpha C_I \int_0^1 L^{-1}\mu(\alpha \mu) d\alpha \mu \right] + \left[ \int_0^1 R^{-1}C_I(\alpha C_I) d\alpha C_I \int_0^1 R^{-1}\mu(\alpha \mu) d\alpha \mu \right] \right\}$$

$$K_5[\alpha U] = \frac{1}{4} \left\{ \int_0^1 L^{-1}U(\alpha U) d\alpha U + \int_0^1 R^{-1}U(\alpha U) d\alpha U \right\}$$

$$K_6[\alpha C_s] = \frac{1}{4} \left\{ \int_0^1 L^{-1}C_s(\alpha C_s) d\alpha C_s + \int_0^1 R^{-1}C_s(\alpha C_s) d\alpha C_s \right\}$$

$$T_c(Q) = \frac{K_1[\alpha D_T, \alpha D]}{Q} + \frac{K_2[\alpha C_T, \alpha D, \alpha \mu]}{Q} + \frac{QK_3[\alpha D_I, \alpha S]}{2} + \frac{QK_4[\alpha C_I, \alpha \mu]}{2} + QK_5[\alpha U] - K_6[\alpha C_s]$$

$$\frac{\partial T_c}{\partial Q} = 0 \Rightarrow -\frac{1}{Q^2} [K_1[\alpha D_T, \alpha D] + K_2[\alpha C_T, \alpha D, \alpha \mu]] + \frac{K_3[\alpha D_I, \alpha S]}{2} + \frac{K_4[\alpha C_I, \alpha \mu]}{2} + K_5[\alpha U] = 0$$

$$Q = \sqrt{\frac{2 [K_1[\alpha D_T, \alpha D] + K_2[\alpha C_T, \alpha D, \alpha \mu]]}{K_3[\alpha D_I, \alpha S] + K_4[\alpha C_I, \alpha \mu] + 2K_5[\alpha U]}}$$

**Numerical Example**

$D = 7000, D_T = 1500/\text{delivery}, D_I = 6/\text{unit/year}, \mu = 35/$

ton,  $C_T = 0.6$  ton/delivery,  $C_I = 0.05$  ton/unit/year,  $S = 500$ ,  
 $U = 5000$ ,  $C_s = 1000$

Crisp Sense:  $Q = 40.5$

**Fuzzy Sense**

$D = (6800, 6900, 7100, 7200)$ ,  $D_T = (1300, 1400, 1500, 1600)$ ,

$D_I = (4, 5, 7, 8)$

$\mu = (33, 34, 36, 37)$ ,  $C_T = (0.4, 0.5, 0.7, 0.8)$ ,  $C_I = (0.03, 0.04, 0.06, 0.07)$ ,  $S = (490, 495, 505, 510)$

$U = (4990, 4995, 5005, 5010)$ ,

$D(\alpha D) = (6800 + 100\alpha, 7200 - 100\alpha)$

$D_T(\alpha D_T) = (1300 + 100\alpha, 1700 - 100\alpha)$

$D_I(\alpha D_I) = (4 + 1\alpha, 8 - 1\alpha)$

$C_T(\alpha C_T) = (33 + 1\alpha, 37 - 1\alpha)$

$C_I(\alpha C_I) = (0.4 + 0.1\alpha, 0.8 - 0.1\alpha)$

$\mu(\alpha\mu) = (0.03 + 0.01\alpha, 0.07 - 0.01\alpha)$

$S(\alpha S) = (490 + 5\alpha, 510 - 5\alpha)$

$U(\alpha U) = (4990 + 5\alpha, 5010 - 5\alpha)$

$$K_1[\alpha D_T, \alpha D] = \frac{1}{4} \left\{ \left[ \int_0^1 1300 + 100\alpha \int_0^1 6800 + 100\alpha \right] + \left[ \int_0^1 1700 - 100\alpha \int_0^1 7200 - 100\alpha \right] \right\} = 5261250$$

$$K_2[\alpha C_T, \alpha D, \alpha\mu] = \frac{1}{4} \left\{ \left[ \int_0^1 0.4 + 0.1\alpha \int_0^1 6800 + 100\alpha \int_0^1 33 + 1\alpha \right] + \left[ \int_0^1 0.8 - 0.1\alpha \int_0^1 7200 - 100\alpha \int_0^1 37 - 1\alpha \right] \right\} = 74748.75$$

$$K_3[\alpha D_I, \alpha S] = \frac{1}{4} \left\{ \left[ \int_0^1 4 + 1\alpha \int_0^1 490 + 5\alpha \right] + \left[ \int_0^1 8 - 1\alpha \int_0^1 510 - 5\alpha \right] \right\} = 1505.625$$

$$K_4[\alpha C_I, \alpha\mu] = \frac{1}{4} \left\{ \left[ \int_0^1 0.03 + 0.01\alpha \int_0^1 33 + 1\alpha \right] + \left[ \int_0^1 0.07 - 0.01\alpha \int_0^1 37 - 1\alpha \right] \right\} = 0.88625$$

$$K_5[\alpha U] = \frac{1}{4} \left\{ \int_0^1 (4990 + 5\alpha) + \int_0^1 (5010 - 5\alpha) \right\} = 2500$$

$Q = 40.5$

**Conclusion**

The economic order quantity, taking into account warehouse limitation of capacity and carbon emissions, is effective in solving the issue of determining the optimal value that should be ordered by the corporation so that it minimizes the total inventory cost. The trapezoidal fuzzy number is used for the fuzzification of various costs and utilizing the Yagers ranking approach, to determine the ideal order quantity. Finally the numerical comparison between crisp and fuzzy sense was done.

**References**

Darom, N. A., Hishamuddin, H., Ramli, R., Mat Nopiah, Z. (2018). "An inventory model of supply chain disruption recovery with safety stock and carbon emission consideration". *Journal of Cleaner Production*, 197, pp 1011- 1021.

Fan, J., Wang, G. (2018). "Joint optimization of dynamic lot and warehouse sizing problems". *European Journal of Operational Research*, 267(3), pp 849-854.

Ghosh, A., Jha, J. K., Sarmah, S. P. (2017). "Optimal lot-sizing under strict carbon cap policy considering stochastic demand". *Applied Mathematical Modelling*, 44, pp 688-704.

Liao, H., Deng, Q. (2018). "A carbon-constrained EOQ model with uncertain demand for remanufactured products". *Journal of Cleaner Production*, 199, pp 334-347.

Lin, T.-Y., Sarker, B. R. (2017). "A pull system inventory model with carbon tax policies and imperfect quality items". *Applied Mathematical Modelling*, 50, pp 450-462.

Ma, X., Ji, P., Ho, W., Yang, C.-H. (2018). "Optimal procurement decision with a carbon tax for the manufacturing industry". *Computers & Operations Research*, 89, pp 360-368.

Roy, T.K, M. Maiti, (1997) " A fuzzy EOQ model with demand-dependent unit cost under limited storage capacity", *European Journal of Operational Research*. 99 (2), pp 425-432.

Tang, S., Wang, W., Cho, S., Yan, H. (2018). "Reducing emissions in transportation and inventory management: (R, Q) Policy with considerations of carbon reduction". *European Journal of Operational Research*, 269(1),pp 327-340

Wang, S, B. Ye, (2018) " A comparison between just-in-time and economic order quantity models with carbon emissions", *Journal of Cleaner Production*. 187, pp 662-671.

Zadeh. L.A(1965), "Fuzzy sets", *Inf. Control*. 8 (3), pp 338-353.

Zimmerman. H.-J(1983)., "Using fuzzy sets in operational research", *European Journal of Operational Research*. 13 (3), pp 201-216